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The Uncertainty Effect in Innovation Diffusion: An Agent-Based Market Model with Opposing Information Sources

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Abstract

When consumers are uncertain whether they have sufficient and unambiguous information regarding a new product, they might delay the adoption decision to a later point in time. This "uncertainty effect" can have a major impact on the product's market diffusion. Prior models that account for consumers' belief updating in such a setting usually capture that effect by resorting to some form of Bayesian learning and, in so doing, typically assume that the distribution of information with respect to the attributes of the new product is normal and that the information points received by individual consumers are independent draws from this distribution. Consequently, consumers' uncertainty regarding their beliefs decreases with more information (e.g., after talking with a peer). This strong assumption is convenient as it makes models of opinion dynamics analytically tractable, and it works in most instances. However, when inconsistent information is provided from two opposing sources and, thus, receiving additional information potentially increases uncertainty of individual consumers, a different approach is required. Corresponding markets, for example, can be found for radically new products for which consumers have strong opinions or as a result of a disinformation campaign by some competitor. We propose an approach that can deal with such a setting and we demonstrate the value added of this novel approach (in contrast to the traditional Bayesian approach) through computational simulation experiments with an agent-based market model of innovation diffusion.

Keywords: multi-agent systems, innovation adoption, consumer uncertainty, market diffusion, simulation experiment

1. Introduction

Innovation diffusion plays a role in a broad spectrum of applications related to, for example, strategic planning, market analysis, policy evaluation, investment decisions, or product life-cycle management. It is therefore not surprising that over the past six decades, modeling of innovation diffusion has evolved as a vivid field of research, in which numerous models have been proposed for supporting managers to make better-informed decisions, optimize strategies, and enhance the likelihood of successful market penetration of innovations (for examples, see the seminal model by Bass, 1969, or a more recent one by Stummer et al. 2021).

As the diffusion of an innovation in a market is an aggregate of individual adoption decisions and, for this decision, perceived uncertainty regarding the performance of or the risk associated with a new product can be decisive, consumer uncertainty is accounted for in many models of innovation diffusion. To this end, modelers more often than not resort to Bayesian updating and, for analytical convenience, implicitly assume that the distribution of all information with respect to the attributes of the new product is normal. Such an approach works nicely in most applications, if the information points traded between the stakeholders stem from the same information source or from several sources that provide similar information (e.g., in the model by Dawid et al., 2024). However, the basic assumption that beliefs (prior and posterior) are unimodal does not hold if contradictory information—possibly in the form of narratives as framed by Shiller (2017)—from two different sources are spread among consumers upon market introduction and the early phase of innovation diffusion.

Radical innovations such as vehicles with self-driving capabilities or AI-fueled smart services are prime examples for such markets, because consumers cannot lean on previous experience or expertise with such products (i.e., they start with an empty set of information) and, thus, they have no existing (strong) opinion based on which they would disregard diverging information. Accordingly, consumers are receptive to even small information snippets, which favors the occurrence of distinct groups of consumers who, for example, criticize the new product for being risky (or even outright dangerous with respect to safety or misuse of private data) versus consumers who praise the value proposition of the same innovation. While the members of these groups actively promote their views, many more consumers—who, in the beginning, typically form the silent majority—are uncertain regarding which group's opinion to follow. Further, more mundane, examples for such a setting are new diets (low-carb, Paleo, Keto, intermittent fasting, etc.), fracking for natural gas, crypto currencies, E10 gasoline, or the eco-friendliness of electric vehicles. It is noteworthy that a similar setting could deliberately be provoked by a competitor by running a negative word-of-mouth campaign that provides "alternative truth" in an attempt to slow down market diffusion of the competing product in order to further their own product or to win time for introducing a comparable product.

When applying the usual variant of Bayesian belief updating in such a setting (i.e., in the presence of opposing information sources), consumers are modeled in a way that they would conclude that the performance of the new product lies somewhere in the middle between the means of the two information sources (although in fact, this belief might not be supported by any of the information received) and over time—that is, after having received plenty of diverging information—consumers would become rather certain that this belief is correct and they would start purchasing the new product. Obviously, such a model outcome is counterfactual, as in reality, the real consumers' overall perceived uncertainty increases in the presence of diverging information as such information induces ambivalence on the side of the consumers (Priester and Petty, 1996). It is noteworthy that sticking with Bayesian updating and simply resorting to some (more complex) different distribution is practically infeasible, because the high number of involved stakeholders renders it impossible to derive a closed form for such a distribution and Markov Chain Monte Carlo methods do not work in the (frequent) cases in which stakeholders update their beliefs with a sample size of one.

In our research, we address the "uncertainty effect" on innovation diffusion arising from consumers who do not have sufficient and coherent (unambiguous) information regarding a new product, and, thus, feel uncertain and delay the adoption decision to a later point. Against this background, we investigate the impact arising from resorting to the common version of Bayesian updating when accounting for the above-mentioned consumer uncertainty in markets with two opposing information sources. To this end, we present a straightforward agent-based model of a market in which a generic innovation is introduced, advertisement agents spread diverging information, and consumer agents engage in word-of-mouth communication and periodically decide on adopting the new product. The latter decision hinges upon (i) the agent's current individual belief (attitude) regarding the product's performance and whether it satisfies an individual minimum utility requirement and (ii) the certainty regarding the current belief with respect to whether the consumer agent considers the extent of information received so far to be sufficient (i.e., there is no lack of knowledge) and the information to be sufficiently coherent (i.e., there is no lack of security). It is noteworthy that the degree of certainty also can determine a consumer agent's willingness to communicate the own belief to peers and, thus, influence their adoption decision. Moreover, we assume that the novel product has an objective performance value, which is disclosed through first-hand experience after adoption and, consequently, consumer agents will be correctly informed in the long run. Still, the key question remains whether and how the presence of diverging information can impact and, thus, delay innovation diffusion. In light of the limitation of the usual form of Bayesian updating (i.e., when a normal distribution of information snippets is assumed) in the investigated setting with two opposing information sources, we developed an alternative approach of capturing consumer uncertainty, which accounts (i) for the dissimilarity between the distributions of the information offered by the two information sources and a normal distribution and (ii) for the relative difference in height of the peaks of these distributions. We then integrated this approach in the agent-based market model and implemented the model in a simulation tool.

In so doing, our research contribute is twofold: First, we introduce an alternative approach for capturing consumers' perceived uncertainty regarding an innovation's attribute in markets in which consumers are exposed to diverging information from two sources. Second, we demonstrate the "uncertainty effect" in innovation diffusion by contrasting diffusion curves gained by applying our new approach and diffusion curves gained for the usual variant of Bayesian updating. Furthermore, we perform sensitivity analyses for parameters used in the model, propose theoretical as well as managerial implications, and suggest an agenda for further research.

The remainder of this work proceeds as follows. Section 2 outlines prior research on innovation diffusion, Bayesian updating, and agent-based modeling, and it relates our work to these fields of research. Section 3 introduces the constitutive entities in our agent-based model (i.e., consumer agents and advertisement agents) as well as their activities (most prominently, information exchange and the decision to adopt). Section 4 describes the parameterization of our simulation experiments. In Section 5, we outline implementation issues, before we thoroughly discuss the results from the simulation runs demonstrating the investigated effect (i.e., the difference in innovation diffusion between applying our new approach versus resorting to traditional Bayesian updating) and provide insights from the sensitivity analyses. Section 6 highlights theoretical and managerial implications, thereby suggesting directions for further research. Finally, Section 7 concludes the paper.

2. Background

2.1. Innovation diffusion

Innovation diffusion is understood as the process by which an innovation is communicated through certain channels over time among the participants in a social system (Rogers, 2003). The aim of diffusion models is to determine the level of spread of a certain innovation (Mahajan et al., 1990). Pioneering work was published already in the 1960s (e.g., Fourt and Woodlock, 1960; Chow, 1967), with the model by Bass (1969) being particularly influential. Since then, numerous further models were proposed (see reviews by, e.g., Meade and Islam, 2006; Peres et al., 2010; Guidolin and Manfredi, 2023). While many of these models primarily strive for including marketing mix-variables into the analytical Bass model (for examples, see Mahajan et al., 1990), others employ alternative approaches such as agent-based modeling (see Sect. 2.3).

As in other models of innovation diffusion, in our own model, we implement a social network, in which consumer engage in word-of-mouth communication, and we account for heterogeneity among the consumer agents. While the relevance of consumer interaction through word of mouth for innovation diffusion is obvious, this relevance is less clear for consumer heterogeneity with respect to differences in their innovativeness (e.g., willingness to purchase a new product without having extensive information), needs (regarding, e.g., a minimum utility), or—as addressed in our work—the ease of dealing with uncertainty, all of which can affect the consumers' individual propensity to adopt. Ultimately, it is the mix of different groups of consumers ranging from innovators to laggards (as framed by Rogers, 2003) that causes the typical S-shaped curve for innovation diffusion: while innovators are dominant in the takeoff phase (due to their high innovativeness and their tolerance of high uncertainty), interaction with and between the remaining consumers becomes the driving force afterwards.

Interestingly, in the afterthought of their review, Peres et al. (2010) mention two issues that nicely go along with our research. First, they suggest "the integration of cutting-edge modeling from other research domains [such as] agent-based modeling," which was a prediction that realized soon after (an early overview of such approaches was provided already by Kiesling et al., 2012). Second, Peres et al. expect that complex types of product categories will spotlight different aspects of the diffusion process and raise new research questions. We agree with this notion and suggest smart products as an example (as described by Stummer, 2024, who also mentions sustainable innovations as a further category).

Such smart products can be of particular interest for studying innovation diffusion for at least three reasons. First, from a business point of view, the customer value proposition of smart products is more fundamental than for many other new products as smart products can learn over time and, thus, increase their value for the user by adapting to their specific individual needs and habits, and they can offer new business models based on, for example, pay per use. Second, from a societal point of view, smart products have already occupied many areas of everyday living (e.g., in the form of smartphones, smart homes, or smart wearables), and will do so for more markets in the foreseeable future (e.g., in the form of autonomous cars, avatars, or service robots). Accordingly, smart products will continue to broaden their impact on how we live, work, travel, or communicate. Third, from a modeling point of view, the diffusion process of smart products is special due to substantial uncertainty that arises during the digital transformation of business and society. In particular, consumers can be uncertain whether their information—which they have received from family, friends, and peers as well as through marketing channels such as traditional advertising or social media—regarding new products or services is sufficient, accurate, and credible. Uncertainty is high as long as the extent of information is insufficient ("lack of knowledge") or ambiguous ("lack of security"). If so, consumers may postpone their purchasing decisions and pass information to others only with reservations (i.e., the impact of such recommendations is limited). In other words, an "uncertainty effect" may arise.

2.2. Bayesian updating

Bayesian updating forms a well-established stream of research. In the following, we outline assumptions that are commonly made when resorting to Bayesian updating for modeling learning ("Bayesian learning") in an economic context (for a recent, considerably more in-depth, overview, see Baley and Veldkamp, 2023), and we explicate issues that can arise in our specific application case as a consequence of these assumptions—which ultimately were the reason for proposing a different approach.

The basic idea behind Bayesian learning is that an existing belief (the "prior") is modified once new information is available (for which the agent calculates the "likelihood" of this new information conforming with the prior), which can be transformed to a new updated belief (the "posterior"). As necessary assumptions, (i) the agent knows the type of distribution (e.g., a normal distribution) which the new information stems from and (ii) she has beliefs about the distribution's parameters. As all information is assumed to be drawn from the same known distribution, the agent becomes more certain about her beliefs with every update. The complexity of this updating process highly depends on the chosen distributions. For continuous variables, it is common practice to assume that the prior and the distribution of the new information are normally distributed (with known variance) because this setting results in a normally distributed posterior, for which it is straightforward to calculate the parameters of the posterior's (normal) distribution. Even when the variance is unknown (and, hence, has prior and posterior distributions), the agent becomes increasingly certain about the mean of the distribution (posterior). In other words, the "precision" of the posterior, defined as the inverse of its variance, increases with each new piece of information, regardless of how much this information diverges from previous data, even if it leads to a higher posterior variance.

While the assumption of normally distributed information is reasonably accurate for many scenarios, it does not hold for our specific case, in which information originates from diverging information sources and, thus, the information has a bimodal distribution. When applying Bayesian updating under this assumption in our case, agents would believe that the true value is somewhere in the middle between the values advertised by the two information sources, and the agents would become more certain with each piece of information although the information received is highly contradicting and none of this information indicates the value in the middle.

In order to still using Bayesian updating, we would need to change the distribution of the new information and hence the priors, possibly by using a Gaussian mixture model. However, in our case, the number of needed normal distributions is unknown, as word-of-mouth communication can include information that differs from our (two) diverging information sources. Hence, the priors of each agent would need to include one prior for each of the weights, means, and possibly variances for a large number of (sub-) distributions. Consequently, the posteriors can no longer be computed in closed form, and Markov Chain Monte Carlo (MCMC) methods like Metropolis-Hasting are needed for sampling the distributions. However, MCMC procedures are not viable in an agent-based model in which numerous agents update with a sample size of one.

It is noteworthy that Bayesian updating was used by Martins (2009) in a related application, that is, for modeling communication between agents with opinions distant from the own. To this end, Bayesian updating was combined with the notion of bounded confidence (Deffuant et al., 2000; Hegselmann and Krause, 2002), that is, that the impact of new information diminishes with the distance to ones own opinion and at a certain point no longer exists. However, for our case of radical innovations, we propose that opinions of the agents are not sufficiently established that they would disregard any information about the innovation.

2.3. Agent-based modeling

Agent-based modeling (ABM) is a relatively young technique, which started its major expansion in the social sciences only in the early 1990s (e.g. Holland and Miller, 1991). Essentially, it describes a complex system in a bottom-up manner (i.e., decentralized) from the perspective of its constituent units (Bonabeau, 2002). These units—that is, the so-called agents—are self-contained and uniquely identifiable, autonomous, and self-directed (i.e., agents relate individual information to their own decisions and actions), have an individual state (e.g., having adopted a particular innovation) that can vary over time, and interact with other agents (Macal and North, 2010). Therefore, employing ABM makes it straightforward to account for population heterogeneity regarding preferences (e.g., for certain product attributes), needs (with respect to, e.g., type, frequency, and quantity), individual resource constraints (e.g., budget available), specific decision-making rules (e.g., preference matching, stage-based approaches, utility maximization, or meeting certain thresholds; see Negahban and Yilmaz, 2014), and further characteristics such as the susceptibility with respect to social influence (e.g., propensity to conform with the behavior of the respective peers).

In the past two decades, ABM has been applied extensively for studying innovation diffusion (for an early application, see Janssen and Jager, 2002). Recently, Rand and Stummer (2021) provide an overview of strengths (e.g., an agent's individual attitude can be driven by the combination of the history of this agent's decisions, the agent's internal notion of the external world, the agent's observation of the reactions of other agents in response to their actions, and the agent's retained memory of past events) and criticisms (e.g., regarding the sometimes challenging parameterization, the need for a proper verification and validation, issues regarding arbitrariness and lack of causality, as well as the computational cost incurred for simulation experiments), Romero et al. (2023) run a bibliometric analysis, and Stummer (2024) points to promising fields of research (e.g., consideration of social identity, supply constraints, competitors, business model innovations, mixed methods) among which accounting for consumer uncertainty and the application to the market of smart products closely correspond to our research described in the article at hand.

With respect to consumer uncertainty, a couple of previous ABM approaches have already incorporated the notion of uncertainty in their work. For example, Jager et al. (2000) show that consumers' degree of uncertainty regarding the outcomes of behavior as well as their uncertainty tolerance can affect system behavior (e.g., as in their application case, over-harvesting from a common resource). The more recent work by Zhang et al. (2022) is related to our work in that they consider the individual agent's opinion and its uncertainty toward this opinion. Therein, agents accumulate product information through interactions with other agents and, thus, uncertainty regarding the perceived utility of the product gradually decreases until it does not constitute an adoption barrier any longer. Most recently, the ABM approach by Dawid et al. (2024) studies how uncertainty regarding the performance and safety of autonomous vehicles (AVs) can influence the success of two prototypical strategies governing the producers' timing of the release of new models and, thus, the market diffusion of the respective AVs. Their findings suggest that consumer uncertainty and the way consumers deal with this uncertainty has crucial implications not only for the speed of AV diffusion but also for the relative performance of the investigated product launch strategies. However, in contrast to our approach, Dawid et al. resort to an Bayesian updating model, an approach that is outlined in the previous section.

Regarding smart products as the application case for ABM on innovation diffusion, two such works were already mentioned in the previous paragraph: Zhang et al. (2022) investigate the diffusion process of smartwatches and Dawid et al. (2024) study the diffusion of AVs. An additional (real-world) case dealing with a smart washing machine is described by Stummer et al. (2021), who advise the combination of ABM with scenario analysis. While the latter is used for generating multiple scenarios ("pictures of the future"), ABM of innovation diffusion in the investigated scenarios offers insight into the potential outcomes of possible strategic (technological) decisions in each of the alternative futures.

3. Model formulation

3.1. Framework

In order to investigate the effect of consumer uncertainty in the presence of two diverging information sources, we keep our agent-based model as simple as possible (see Fig. 1 for a graphical overview of the model, including references to the sections in which we elaborate on the respective model elements). Accordingly, we consider only a *single new product* that is equipped with a single attribute (e.g., a specific smartness functionality) determining the product's performance and, thus, its utility. This product is radically new, and therefore, none of the consumers have prior information about the true product performance value. Still, the new product is a high-involvement product, that is, consumers ponder over the adoption decision (in contrast to, for example, some low-involvement convenience goods). The product is directly marketed by a single producer and, consequently, we disregard competition, intermediaries such as retailing, and we also do not account for pricing, changes in consumer behavior (e.g., an increase or decrease in need), or social influence. The model features two types of agents: consumer agents and advertisement agents.

Consumer agents (i) collect information through *word-of-mouth communication* with peers as well as from advertisement campaigns and accordingly form their belief (attitude, opinion) regarding the performance of the new product under consideration, (ii) occasionally decide upon the *purchase of the new product* (i.e., innovation adoption) and, in the course of this decision process, not only require a minimum utility gained from the product but also account for their individual *uncertainty*, which is driven by the extent and the coherence of information received, (iii) make *first-hand experience* with the new product after purchasing it, and (iv) provide their peers with information (if sufficiently certain regarding their own belief). Consumer agents are heterogeneous in their characteristics (e.g., with respect to the minimum utility expected or the

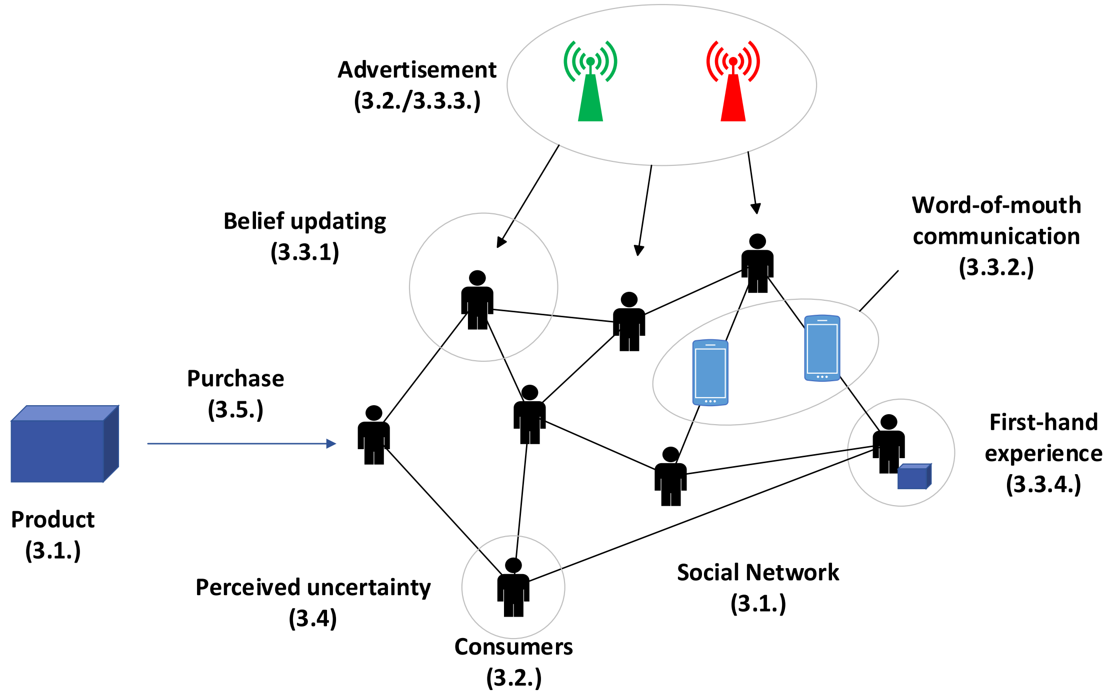


Figure 1: Model entities and dynamics.

extent of information required to be certain) and they are connected to other consumer agents through a static (scale-free) *social network* created through the algorithm proposed by Barabási and Albert (1999).

Advertisement agents execute *advertisement campaigns* by providing certain information regarding the product performance to a subset of consumer agents during a specified period of time. In our simulation experiment, we use two advertisement agents offering diverging information at the very beginning of the process, thus setting the scene for the bimodal distribution of information for the purpose of our investigation.

Regarding the treatment of time in our simulation model, we have opted for a *discrete-event approach* with a quasi-continuous timeline. Thus, we avoid consumer agents being involved in multiple communication events at the same time, which would have required an event-scheduling regime (as necessary in a discrete-time approach).

In the following, we first introduce the above-mentioned two types of agents and describe the parameters and variables associated with them (Sect. 3.2). Then, we describe how consumer agents' beliefs are updated and describe the sources of information that may play a role in this process, that is, word-of-mouth communication, being exposed to advertising, or making first-hand experiences (Sect. 3.3). Next, we explain how consumer agents form their (perceived) uncertainty (Sect. 3.4). Finally, we elaborate on the purchase decision (Sect. 3.5).

Table 1 lists the relevant variables and parameters used in the model. It is noteworthy that, for clarity purposes, we omitted auxiliary variables as well as parameters that are used only in a specific part of the model; still, we explain these variables and parameters in the course of the model description right in the place in which they appear.

Products:	
$a^\star$	True performance value of the product
Consumer agents:	
N	Population size (consumer agents $i = 1, \ldots, N$)
a_i	Consumer i's belief regarding product performance
v_i	Consumer i's number of received information points
$u_i^{\min}$	Consumer i's minimum utility, drawn from $U(\alpha^{\min}, \beta^{\min})$
$v_i^{\min}$	Consumer i's minimum extent of information, drawn from $Pois(\lambda^{\min})$
Θ_i	Consumer i's dataset of received information with v_i tuples $(a_{i,s}, w_{i,s})$
d_i, h_i	Determinants of consumer i's uncertainty
Advertisement agents:	
Q	Amount of advertisement (advertisement agents $q = 1, \ldots, Q$)
m_q^{dur}	Duration of advertisement campaign q
m_q^{int}	Intensity of advertisement campaign q
a_q^{adv}	Content of advertisement campaign q
Information flows:	
$a_{i,s}$, $w_{i,s}$	Information received and weight of channel in information event s
ϵ^{wom}, ϵ^{adv}, ϵ^{fhx}	Noise of communication channels
w^{wom}, w^{adv}, w^{fhx}	Weight of communication channels
k	Number of new connections during generating the social network
Perceived uncertainty:	
$\hat{f}_i$	Probability density function of consumer i's information (based on Θ_i)

Table 1: Most relevant variables and parameters

3.2. Agents

In our model, we account for a population of N *consumer agents* indexed by i ($i = 1, \ldots, N$). These agents are heterogeneous with respect to two individual parameters, namely, the reservation level $u_i^{\min}$ of the product performance as a precondition for possible product adoption and the minimum amount (volume) $v_i^{\min}$ of information to make an adoption decision. The values for two further parameters, namely, h_i and d_i (described in detail in Sect. 3.4), are identical for all consumer agents (for simplification purposes). Overall, the three parameters $v_i^{\min}$, h_i, and d_i determine how a consumer agent deals with uncertainty, that is, whether she is reluctant to adopt a new product due to a lack of knowledge (i.e., the extent of information is too low) or due to a lack of security (i.e., the information is too ambiguous). Accordingly, the new product needs to promise a sufficiently high performance level, and consumers need to be sufficiently certain regarding their information base before product adoption takes place.

The consumer agents store all the information retrieved in the course of a simulation run and use it throughout the simulation to form their individual beliefs (or attitudes) a_i regarding the performance level of the new product (see Sect. 3.3.1) and determining their (perceived) level of uncertainty (see Sect. 3.4). Consequently, it is possible that the beliefs and the respective behaviors of two twin agents, who otherwise have identical parameters, deviate because the agents have received differing information. For the sake of simplicity, we model consumer agents as stable in that they do not die and no new consumers are born, consumer characteristics do not change, and the social network is static (i.e., no new friends are made, and existing links to peers are not removed).

Advertisement agents form the second group of agents. Each of the Q advertisement agents indexed by q ($q = 1, \ldots, Q$) represent one advertisement campaign. They are characterized by the duration m_q^{dur}, the intensity m_q^{int}, and the content a_q^{adv} of the campaign they represent.

3.3. Information flows

3.3.1. Belief update

In our model, a consumer agent updates her belief about the product whenever she receives new information. This information can originate from different sources, that is, from peers through word-of-mouth communication (see Sect. 3.3.2), from being exposed to advertisement (see Sect. 3.3.3), or from first-hand experience after having adopted the product (see Sect. 3.3.4). The communication channels are associated with (i) communication noises ϵ^{wom}, ϵ^{adv}, ϵ^{fhx}, which represent minor disturbances in the communication process, and (ii) weights w^{wom}, w^{adv}, w^{fhx}, which represent the channels' credibility.

In the course of a simulation run, each consumer i stores v_i data pairs $(a_{i,s}, w_{i,s})$ containing information points regarding the product's performance $a_{i,s}$ and the corresponding weights $w_{i,s}$

in a tuple Θ_i, which represents the consumer's individual database. The belief at a certain point in time can therefore be calculated as a weighted mean:

$$a_i = \frac{\sum_{s=1}^{v_i} a_{i,s} \cdot w_{i,s}}{\sum_{s=1}^{v_i} w_{i,s}} \tag{1}$$

It is noteworthy that we do not account for the notion that older information might have less impact on opinion formation than newer information. However, a corresponding extension would be straightforward by adding a discount factor.

3.3.2. Word-of-mouth communication

A common source of information for consumers is word-of-mouth communication with their peers—that is, between two consumers who are directly linked in the social network. In our model, we follow Stummer et al. (2015) and schedule the communication events using exponentially distributed inter-arrival times. During an event, the involved consumers independently decide whether they want to share their belief regarding the performance of the new products with their interlocutor. This decision is based on the consumer's certainty regarding her own belief, that is, consumer i will communicate her belief a_i only if she has already received a sufficiently high number of information points (i.e., $v_i \geq v_i^{\min}$) and if these information points are sufficiently coherent (as described in Sect. 3.4); otherwise, she abstains from communicating her belief (for the theoretical underpinning of this behavior, see, e.g., Cheatham and Tormala, 2015; Tormala and Rucker, 2018). If consumer j decides to communicate her belief a_j to consumer i, consumer i adds a new entry to her information database (and, thus, $v_i := v_i + 1$) and updates her belief a_i based on Equation (1) with received information $a_{i,v_i} = a_j + \epsilon^{\text{wom}}$, where ϵ^{wom} represents a normally distributed communication noise, and weight $w_{i,v_i} = w^{\text{wom}}$.

3.3.3. Advertisement

Advertisement constitutes another source of information for consumers. To this end, we use advertisement agents, who broadcast information to selected consumer agents. The parameter values assigned to a certain advertisement agent determine the content (i.e., the advertised level of product performance), the time period in which the campaign is executed, the intensity of the campaign (i.e., the number of advertising messages), and the coverage of the campaign (i.e., the number and maybe the type of consumers reached). For reasons of simplicity, we employ advertisement campaigns only in the initial phase of innovation diffusion, that is, when the new product is introduced into the market, as a means to provide rivaling sources of information.

Whenever an advertisement agent q reaches a consumer i, a new information event is added to the database (again, $v_i := v_i + 1$) and this consumer's belief is updated using Equation (1) with received information $a_{i,v_i} = a_q^{\text{adv}} + \epsilon^{\text{adv}}$ and weight $w_{i,v_i} = w^{\text{adv}}$.

3.3.4. First-hand experience

After a consumer agent has adopted the new product, she makes first-hand experiences. We model this experience in an analogous way to the above-mentioned information events, that is, a new entry is added to the consumer agent's data base (i.e., $v_i := v_i + 1$) and the belief is updated according to Equation (1) with respect to $a_{i,v_i} = a^\star + \epsilon^{\text{fhx}}$, where $a^\star$ stands for the true performance level of the new product under consideration, and the weight is $w_{i,v_i} = w^{\text{fhx}}$. If the observability of the new product's performance is high, w^{fhx} usually is high as well. In most such application cases, $w^{\text{fhx}} > w^{\text{wom}} > w^{\text{adv}}$ should hold true.

3.4. Perceived uncertainty

In our model, the perceived uncertainty of consumer agents hinges upon a lack of information or the incoherence of the information she received. We implement a straightforward method for dealing with the first-mentioned reason in that we check whether a minimum number v_i^{min} of information points has been received; alternatively, we could have resorted to a minimum sum of weights, which would also account for the quality of the signals, but the difference would be slim, as in the beginning, nearly all signals stem from advertisement agents with identical weights. In contrast, it is considerably more challenging to properly address the latter reason. In this respect, the uncertainty of an agent i obviously should increase when she receives diverging information, because then her belief a_i (which is the weighted mean over all $a_{i,s}$ stored in her personal database Θ_i) might no longer be the point with the highest probability density (in contrast to cases in which the distribution of information is unimodal or which feature a dominant mode that is considerably higher than other peaks) and can even have one of the lowest densities. Consequently, the agent's belief a_i may no longer be a good estimate for the true product performance $a^\star$, and the consumer becomes uncertain about the validity of her belief.

At first glance, it could be tempting to resort to the variance as the only measure for uncertainty because it can be easily updated together with the belief and it has the right basic properties. Moreover, the variance is relatively high if multiple peaks are present in the distribution, increases if these peaks move further apart, and is highest if they are similar in height.

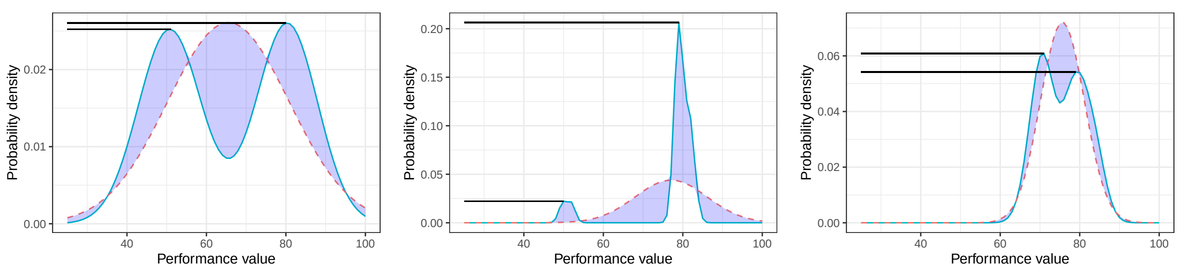
However, the variance does not account for the specific form of the distribution; that is, for example, the variance of a distribution in which 50% of values are around 50 and 50% are around 80 is roughly equal to that of a distribution with 17% around 40 and 83% around 80 (or 10% around 30 and 90% around 80). This property contradicts literature on attitudinal ambivalence according to which a higher imbalance between positive and negative information yields lower ambivalence (e.g., Priester and Petty, 1996). Therefore, the strong increase in imbalance seen in the above example should decrease the perceived uncertainty of our consumer agents. Moreover, if the variance is the only measure for uncertainty, consumer agents cannot distinguish between the case of opposing information sources, each providing its information with low variance, and the case of a single information source providing information with high variance. In the latter case, however, the consumer agents would experience no ambivalence and, therefore, should have lower uncertainty and behave differently. In Part B of the online supplement, we present exemplary simulation results that illustrate the effects of resorting to the variance as a measure of uncertainty versus employing an alternative approach, as proposed in the following.

Therein, we account for the shape of the distribution, which needs to be estimated based on the information points that consumer i received during (diverse) information events s and stored in her database. Accordingly, the estimation of the underlying distribution has to be non-parametric, and we therefore use a kernel density estimation (KDE), which is common practice in non-parametric cases for determining a probability density function (PDF). Procedurally, each piece of information $a_{i,s}$ is represented as its own distribution, and we calculate the weighted mean over the PDFs of all these distributions. In our case, the resulting PDF is given by

$$\hat{f}_i(x) = \frac{1}{W_i \cdot b_i} \cdot \sum_{s=1}^{v_i} \left(w_{i,s} \cdot \phi \left(\frac{x - a_{i,s}}{b_i} \right) \right), \tag{2}$$

where v_i is the number of information points that the consumer i received, $a_{i,s}$ (received information) and $w_{i,s}$ (corresponding weight) refer to the v_i information points in Θ_i, W_i is the sum of all weights (i.e., $W_i = \sum_{s=1}^{v_i} w_{i,s}$), b_i is the bandwidth parameter which is computed using Silverman's rule of thumb (Silverman, 2018), and $\phi(\cdot)$ is the standard normal density function. We chose the normal kernel, as it is widely used and a natural fit for our information pieces $a_{i,s}$, which include normally distributed communication noises. Experiments with other kernels yielded similar results (for a detailed description, see Part C.5 of the online supplement).

The PDF shows the shape of the distribution of information points gathered by a certain consumer. In the first step of the proposed approach, we thus can check whether $\hat{f}_i$ is unimodal and, if so, we can rule out that consumer agent i is uncertain due to having received opposing information. Figure 2 contrasts bimodal distributions (depicted by the solid blue line) with their unimodal counterparts (depicted by the dashed red line), which have the same mean value. Currently, we assume that consumers whose information distribution is unimodal are (sufficiently) certain that the mean represents the true value (given, of course, that they have received at least the minimum number of information points). While this assumption is sufficient for the purpose of our research, the variance of the (unimodal) distribution could play a role in other applications, in which a unimodal distribution also needs to be accompanied by an acceptably low variance. If so, it would be straightforward to extend the model correspondingly by introducing another (individual) threshold for the variance as an additional measure.



(a) Uncertain (b) High ratio between peaks (h_i) (c) Low distance between PDFs (d_i)

Figure 2: Visualization of parameters h_i and d_i. With $\hat{f}_i$ depicted by the blue solid line, the PDF of a normal distribution with equal mean and variance depicted by the red dashed line, and the total variation distance between the distributions represented by the blue shaded area.

Next, we check for a dominant mode by computing the ratios between the PDF value of the possibly dominant mode and the PDF values of the other peaks. If all the ratios are higher than a given individual threshold h_i, consumer agent i considers the mode to be dominant and the differing information to be negligible outliers. This case is visualized in Figure 2b, in which the black bars indicate the heights of the two peaks.

Finally, a distribution might not have a single dominant mode but still resemble the form of a normal distribution. The three most important reasons for such a form are (i) spiky PDFs (resulting from stochasticity or a low bandwidth parameter b_i), (ii) peaks that are close together, and (iii) the arrangement of multiple peaks with different heights that resemble a normal distribution. To identify these and similar cases, we calculate the total variation distance (L^1 norm divided by 2) between a consumer i's PDF and the PDF of a normal distribution with this consumer's belief a_i as the mean and the weighted variance of all information points in Θ_i as the variance. If the variation distance is smaller than a given individual threshold d_i, we consider consumer i to be sufficiently certain. An example is provided in Figure 2 in which the blue shaded area represents the total variation distance between the distributions, which is small for the case of the two close peaks in Figure 2c but not in the other two cases in Figures 2a and 2b.

In a nutshell, a consumer agent feels safe to communicate her belief to peers in word-of-mouth communication (see Sect. 3.3.2) and to possibly adopt the new product (see Sect. 3.5), if she has received sufficient information about the new product (i.e., $v_i \geq v_i^{\min}$) and if she considers this information to be coherent (i.e., the underlying distribution is unimodal, the distribution has a dominant mode, or the distribution has several peaks but its form still is similar to that of a normal distribution). This is in line with the literature on ambivalence (e.g., Luce et al., 1997) according to which consumers postpone decisions if information is insufficient or too ambiguous.

3.5. Purchase decision

Consumers decide upon the adoption of the new product whenever they receive new information about it. A consumer agent i adopts the product if she is content with the information received so far (as described in the preceding section) and if she believes that the product's performance level excels her individual reservation level. For the latter, we simply check whether $a_i \geq u_i^{\min}$ and, in doing so, implicitly assume that the utility function regarding the performance level is the identity function (i.e., $u_i(a_i) = u(a_i) = a_i$). If necessary, it would be straightforward to employ alternative utility functions accounting for multiple product attributes as well as varying preferences and behaviors among consumers. After a consumer agent has become an adopter, she receives further information about the product's performance through first-hand experience.

4. Parameterization

As parameterization was not restricted to a certain (real-world) application case, we were free to choose a reasonable set of parameters, allowing us to demonstrate the effects of opposing information sources in innovation diffusion. Still, the main effects shown in this work are not confined to a particular parameter constellation, but can be observed for a wide range of alternative parameter settings, which is illustrated through sensitivity analyses (see Sect. 5.4 as well as in Part C of the online supplement).

4.1. Products

Products are characterized by their *true performance value* $a^\star$. In the baseline scenario, it is set to $a^\star = 80$, which conforms with the information provided by the well-meaning advertisement agents, while the opposing one advertises a considerably lower performance level (of only 50).

4.2. Consumer agents

In our simulation experiments, we allow for heterogeneity of consumer agents with respect to certain characteristics (parameters). The *minimum amount of information* $v_i^{\min}$ is drawn from a Poisson distribution for each consumer i with parameter $\lambda^{\min} = 4$ (i.e., $v_i^{\min} \sim Pois(\lambda^{\min})$), with draws of 0 or 1 set to 2 because KDE requires at least two information points. Accordingly, consumers, on average, need a minimum of four information points before they can become certain of their beliefs. With respect to the parameterization of the *reservation level* (i.e., minimum utility) for the new product's performance $u_i^{\min}$, our simulation tool offers to draw either from a normal distribution with mean $\mu^{\min}$ and variance $\sigma^{\min}$ ($a_i^{\min} \sim \mathcal{N}(\mu^{\min}, \sigma^{\min})$) or from a uniform distribution with a minimum value $\alpha^{\min}$ and maximum value $\beta^{\min}$ ($a_i^{\min} \sim U(\alpha^{\min}, \beta^{\min})$). In the baseline scenario, we opted for a uniform distribution with $\alpha^{\min} = 55$ and $\beta^{\min} = 80$. These bounds were chosen because they yield reservation levels that are (clearly) higher than the negative advertisement and lower than (or equal to) the positive advertisement.

The *coherency parameters* h_i and d_i (introduced in Sect. 3.4) could in principle be consumer-specific as well, but for reasons of keeping the model simple, we model consumer agents as homogeneous in this respect. We assume that consumers should become certain once approximately 90% of their information points in Θ_i are near the modus, which is achieved by setting $h_i = 9$. Moreover, we set $d_i = 0.15$ as it allows for nicely distinguishing between a normal and a bimodal distribution already for a low number of information points (according to our experiments).

The *population size* is set to $N = 1000$ consumer agents. This seems to be sufficient, as results from simulation runs with more consumer agents (namely, 3000, 5000, or 10000) showed only marginal differences in the diffusion curve.

4.3. Advertisement agents

As we investigate the effect of opposing information sources and, in doing so, strive to keep the model as simple as possible, we set the *number of campaigns* $Q = 2$ with identical values for the duration $m_q^{\text{dur}} = m^{\text{dur}} = 90$ (days) and the intensity $m_q^{\text{int}} = m^{\text{int}} = 0.1$ (i.e., the information sources reach ten percent of the consumer agents at any given day). Regarding the *content of the campaign* a_q^{adv}, the baseline values are $a_1^{\text{adv}} = 80$ for the producer's information source and $a_2^{\text{adv}} = 50$ for the opposing information source. It should be noted that the scale for the new product's performance was chosen for presentation purposes and that it can easily be rescaled if required.

4.4. Information flows

The generation of the underlying social network is driven by parameter k, which sets the *number of new connection* associated with every consumer agent who is (sequentially) added to the network in the course of the network generation. As the stochasticity of this process can yield different networks (even for identical k's), we generated a complete network (using $k = 3$ as a common parameter value) and used this specific network in all simulation runs.

The *communication noises* ϵ are drawn from normal distributions around zero with channel-specific standard deviations σ (i.e., $\epsilon \sim \mathcal{N}(0, \sigma^2)$). The corresponding baseline values for the standard deviation are $\sigma^{\text{adv}} = 2$, $\sigma^{\text{wom}} = 1.33$, and $\sigma^{\text{fhx}} = 0.5$, which accounts for varying levels of precision associated with the three channels.

With respect to the *weights* that are given to specific information sources by consumers, we set them identically for all consumers to $w^{\text{adv}} = 0.5$, $w^{\text{wom}} = 0.75$, and $w^{\text{fhx}} = 2$. Obviously, advertisement is the least trusted channel, word-of-mouth communication comes next, and personal first-hand experience is the most trustworthy. This notion is backed up by marketing literature suggesting that advertisement has a smaller impact than inter-personal communication (e.g. Day, 1971) and by literature on attitude (belief) certainty, which states that consumers are more certain if their information comes from direct experience (e.g. Rucker et al., 2014).

Regarding the frequency of word-of-mouth communication, we set parameter $\lambda^{\text{wom}} = 1/30$, which implies that consumers, on average, communicate once every month with every consumer they are directly connected with.

5. Findings

In our research, we are concerned with the "uncertainty effect" on innovation diffusion (i.e., consumers delay adoption due to insufficient or ambiguous information received) that in particular may arise in markets in which two divergent information sources are active (e.g., a channel operated by the producer and an opposing channel operated by an adversary). To this end, we implemented the ABM approach in a simulation tool (Sect. 5.1). We use this tool, first, for simulating the behavior of a regular market featuring a classical unimodal information distribution originating from a single source of information and contrast the diffusion curves obtained by applying our new approach with the curves obtained for the common variant of Bayesian updating, for which we also implement a required minimum number of information points (Sect. 5.2). Afterwards, we move on to the special case of a market with two opposing information sources (Sect. 5.3). Finally, we summarize the results from the sensitivity analyses (Sect. 5.4).

5.1. Implementation

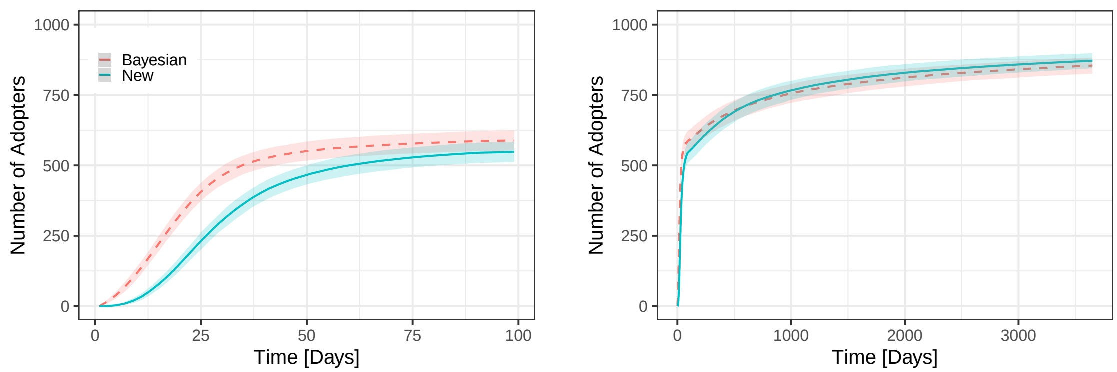
The agent-based model described in Section 3 was implemented by means of the software package Anylogic 7 (`https://www.anylogic.com/`). As the only modification, we do not recalculate the computationally expensive density function $\hat{f}_i$ for consumers who have already locked in their opinion (i.e., are sufficiently certain and have received an extraordinarily high number of information points). This modification reduces the runtime of a scenario with a time horizon of 10 years and 1000 consumer agents to approximately 5 minutes for 20 simultaneous runs of our baseline model (with different seeds) on a PC with a 12$^{\text{th}}$ Generation Intel® Core™ i7-12700 as the CPU. Run times for different parameter sets ranged from 3 to 10 minutes.

5.2. Single information source

In order to check whether our ABM approach also generates a reasonable innovation diffusion curve in a (standard) market with a single information source, we set up such a market. The corresponding advertisement agent in this market communicates a performance level that is situated in the middle of the two (opposing) agents from the baseline setting (i.e., $a^{\text{adv}} = (80 + 50)/2 = 65$) with a communication noise ϵ^{adv} that is drawn from a normal distribution that mirrors the variance associated with the two agents (i.e., $\epsilon^{\text{adv}} \sim \mathcal{N}(0, ((80 - 50)/2)^2))$. Moreover, the agent sends information with double the intensity as in the baseline model with the two advertisement agents (i.e., $m^{\text{int}} = 2 \cdot 0.1 = 0.2$).

For this specific (unimodal) setting, Figure 3 contrasts the diffusion curves (averaged over 100 simulation runs) gained by employing traditional Bayesian updating (dashed red) with the diffusion curve resulting from our approach (solid blue). In the first 100 days of the simulation, depicted in Figure 3a, the number of adopters starts to increase slightly earlier and grows faster for the Bayesian approach, but still the diffusion curves look similar and both indicate nearly the same number of adopters (588 vs. 549) after the advertisement activity ends (i.e., after 90 days). The graph for innovation diffusion over the simulation horizon of 10 years, shown in Figure 3b, confirms that the gap between the curves is nearly closed, with a difference of less than 20 adopters from day 327 onward.

This diffusion curve is special to some degree in that it is extraordinarily steep in the first part, followed by a long tail. The rapid diffusion at the beginning is induced by massive advertisement, which results in consumer agents receiving, on average, an extent of information that satisfies their minimum requirement (v_i^{min}) within the first 20 days. As another factor, 40% of consumer agents in our simulation are already content with a product performance lower than



(a) First 100 days

(b) Full 10 years

Figure 3: Number of adopters in the presence of a single source of advertisement for (a) the first 100 days and (b) the full simulation horizon of 10 years. The colored lines depict the mean over 100 simulation runs for the Bayesian updating (dashed red) and our new (solid blue) approach; the shaded areas indicate the 95% confidence intervals.

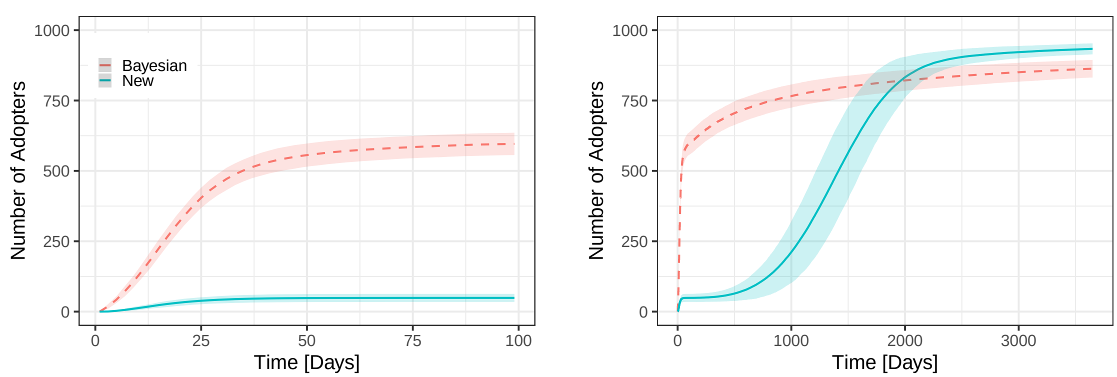
the advertised performance level, that is, their respective reservation level (minimum utility) $u_i^{\min} \leq a^{\text{adv}} = 65$; moreover, for 60% of consumer agents, $u_i^{\min} \leq 70$. The turning point of the adoption rate is reached after 25 days, and after 80 days, the adoption rates become rather low (with less than two new adopters per day), as numerous consumers with reservation levels below or close to the advertised performance level have already adopted the new product. It is noteworthy that the number of adopters after the advertisement period of 90 days is higher than just 40% because some consumer agents have beliefs exceeding the advertised information content either because of stochasticity (as the information received by them is higher than the mean value a^{adv} due to communication noise) or because they received word-of-mouth communication from adopters (who have experienced the true performance value $a^{\star} = 80$).

The remaining non-adopters typically either have a high reservation level or they become certain early on in a (wrong) belief that the performance level is low. First-hand experience made by their adopting peers and communicated through the social network over time should ultimately increase these peers' beliefs, and eventually, they would be persuaded by the adopters to give the new product a try. However, this process is slowed down by the fact that, after some time, most non-adopters also communicate their own (lower) beliefs and, thus, influence each other (plus, even might negatively affect the attitudes of adopters). Consequently, the diffusion curve features a long tail.

Such behavior nicely resembles markets for the type of products we had in mind (e.g., AI-driven smart services, crypto currencies, or a new drug helping in losing body weight). These products are strongly hyped and information is disseminated rather quickly—as opposed to a lesser-hyped market in which consumers by themselves have to seek information in a lengthy process. Also, it is difficult (and needs time) to persuade the remaining non-adopters, even after the majority of consumers have given the new product a try and made (positive) first-hand experiences. Moreover, in such a market it is very likely that the product is opposed by another group of consumers who also might have a strong opinion and, thus, another source providing diverging information arises, with an effect that is investigated in the following.

5.3. Opposing information sources

As we primarily strive for insights into market behavior in the presence of two competing advertisement agents, we simulated this setting, and, indeed, the results provided in Figure 4 illustrate the effect of suitably accounting for the two information sources. Therein, the respective diffusion curve when employing our approach is colored blue, while the diffusion underlying the classical Bayesian updating is colored red (as in the respective curves in Fig. 3); again, the mean numbers of adopters over time for 100 repetitions are depicted by a solid (dashed) line, and the 95% confidence interval is represented by shaded areas. Most notable, the number of consumer agents who adopt during the advertisement campaigns (i.e., the early adopters) decreases by 91% to only 48 adopters (see Fig. 4a). Still, the diffusion curve over the full simulation horizon features the typical S-shape (see Fig. 4b), in which the rate of diffusion strongly increases after approximately two years and levels off after four more years.



(a) First 100 days

(b) Full 10 years

Figure 4: Number of adopters over time for our baseline model for (a) the first 100 days and (b) the full simulation horizon of 10 years. The colored lines depict the mean over 100 simulation runs for the Bayesian updating (dashed red) and our new (solid blue) approach; the shaded areas indicate the 95% confidence intervals.

The obvious reason for the difference between the two diffusion curves is the ambivalence of consumer agents regarding information received through advertisement from the opposing information source. In other words, we see the "uncertainty effect" in that the consumers remain uncertain in the early stages of the simulation and postpone the adoption of the new product.

However, once a critical number of consumer agents become early adopters, they continue to convince others, and the diffusion relatively quickly takes off. In this phase, it helps that both advertisement agents have already stopped their communication activities (i.e., after 90 days according to their parameterization), and consumer agents only communicate their belief if they are certain about it (or otherwise, remain silent). Consequently, it is mostly the adopters who communicate their (favorable) beliefs, and there is almost no new (negative) information source.

It is noteworthy that the number of adopters at the end of the simulation runs in which the new approach is employed (i.e., the blue curve) is significantly higher (with $p < 0.01$) than the respective number of adopters in the runs based on traditional Bayesian learning (i.e., the red curve). The reason is that in the latter case, numerous consumers relatively early become sufficiently certain in their belief that the performance level is somewhere around 65 (which is wrong), and thus, they start to communicate this belief and, in so doing, reinforce this belief in the market. Consequently, consumers with high reservation levels will never (or at least only considerably later) become adopters. In contrast, when ambiguity is addressed more properly as in the case of the new approach, consumers who at the beginning are exposed to the diverging information remain uncertain and, thus, do not spread their belief until they have become more certain. Consequently, the correct information (mostly) provided by the adopters (who have already learned about the true performance value of 80) has more relative impact (due to a considerably smaller amount of incorrect information), and it becomes easier to increase the beliefs of agents with relatively higher reservation levels.

Lastly, a comparably large 95% confidence interval (i.e., the blue shaded area) is due to the impact of the number of early adopters and how connected these are in the social network. The variance in the number of early adopters is mainly driven by (i) the variability in parameterization of the consumer agents with respect to their individual minimum amount of information $v_i^{\min}$ and the minimum utility $u_i^{\min}$ and (ii) the variability in number of times consumer agents are impacted by each of the advertisement campaigns. Thus, the speed of market diffusion can be rather different depending on whether consumers who have certain individual characteristics (e.g., are highly connected in the social network) are early on addressed by a certain campaign.

5.4. Sensitivity analysis

We performed sensitivity analyses for all parameters listed in Table 1, except for the number of advertisement agents (i.e., we limit our analysis to either one or two such agents). In the following, we provide an overview and, essentially, confirm that the model—for a reasonable range of parameter values—behaves as expected. More detailed simulation results (together with figures illustrating the respective effects) are provided in Part C of the online supplement.

Regarding the parameterization of the *consumer agents*, an increase of the minimum utility $u_i^{\min}$ or the minimum amount of information $v_i^{\min}$ reduces the speed of innovation diffusion for both approaches (i.e., the traditional Bayesian updating as well as our proposed approach). If the uncertainty parameter h_i (playing a role only in the new approach, and in the baseline scenario is set to $h = 9$) is increased, consumer agents can less often disregard the existence of a bimodal distribution and thus remain uncertain more often, which ultimately postpones innovation diffusion (i.e., shifts the diffusion curve to the right). The other uncertainty parameter d_i determines whether a consumer agents deems the PDF computed from the information points in Θ_i to be sufficiently close to a normal distribution with identical mean and variance. If so, we assume the PDF to be normally distributed. For low values of parameter d_i (i.e., smaller than 0.17 in our setting, which is the case for the baseline with $d_i = 0.15$), an increase only slightly shifts the diffusion curve to the left (i.e., increases diffusion speed), whereas for higher values, the shift is considerably more pronounced, and the diffusion curve quickly assumes the shape of the diffusion curve generated when applying the traditional Bayesian approach.

With respect to the *advertisement agents*, increasing the parameter for the duration of a campaign ($m^{\mathrm{dur}} = 90$ in the baseline) impedes the diffusion, as more controversial information is brought to the market by the opposing advertisement agent, which reduces the relative (and otherwise larger) impact of the true performance information gained and distributed by the adopters. Increasing the intensity of the campaigns (set to $m^{\mathrm{int}} = 0.1$ in the baseline) boosts innovation in the beginning (as it is easier for consumers to collect the minimum amount of information) but later-on impedes innovation diffusion (again, as more controversial information is available and the relative impact of the true information gained through first-hand experience is reduced). We also ran simulations with alternative parameters for the advertised content by varying the parameter for the second information source (set to $a_2^{\mathrm{adv}} = 50$ in the baseline). It should be noted that for these analyses, we accordingly adapted the lower bound of our distribution of minimum utilities $u_i^{\min}$ to be equal to the modified advertisement content a_2^{adv} ($\alpha^{\min} = a_2^{\mathrm{adv}}$) as a means to isolate the effect of the distance between the advertisement contents from the effects of a change in their mean value. While we do not see an effect in case of Bayesian updating, the increase of parameter a_2^{adv} (i.e., the negative word-of-mouth information becomes less diverging from the positive word-of-mouth information) accelerates innovation diffusion for the newly proposed approach, because consumer agents more easily become certain. The latter effect is particularly pronounced for values larger than 60, given that the producer's information source advertises $a_1^{\text{adv}} = 80$.

As to the *information flows*, a higher value for the network parameter (set to $k = 3$ in the baseline) has nearly no effect in the Bayesian approach, but shifts the diffusion curve to the left (i.e., furthers diffusion speed) for the new approach, because word-of-mouth communication becomes more active, and in the communication events, more favorable information is exchanged, given that only consumers who have already become sufficiently certain engage in communication and those typically are adopters who have already experienced the high product performance. The relative variation of the weights w^{wom}, w^{adv}, and w^{fhx} to each other shows that for higher weights, the number of adopters increases for first-hand experience, decreases for advertisement, and is unchanged for word-of-mouth. When applying the new approach, a high weight for advertisement delays innovation diffusion (as it overall weakens information provided by the adopters who have a favorable belief regarding the product's performance), and a high weight for word-of-mouth accelerates diffusion (as it helps to propel information from the adopters).

The parameter for true performance of the *product* $a^{\star}$ shows an interesting effect. Unsurprisingly, a higher value results in a higher number of adopters in the long run (for both cases). However, when applying the new approach, a true performance value between the advertised values (i.e., $a^{\star} < 80$) could serve as a bridge between the two PDFs, making consumers certain more easily, and, thus, furthering early innovation diffusion.

6. Discussion

In the following, we highlight the main theoretical and managerial implications, and propose possible avenues for further research regarding these issues. Additional ideas for enriching the specific agent-based model approach are provided in the concluding Section 7.

6.1. Theoretical perspective

As the most profound learning, researchers shall not resort to standard Bayes learning unless they are confident that the prerequisites—that is, a normal distribution of information—are met. Although this should be a matter of course, the convenience of sticking with the usual approach might still be tempting. However, our work demonstrates the possibly large error that may occur if not properly accounting for the "uncertainty effect" in the presence of opposing information sources.

A research agenda for deepening the theoretical contribution in this stream of research, first and foremost, should comprise the (empirically substantiated) identification of consumers' strategies of dealing with (i.e., "navigating") uncertainty when deciding upon the adoption of new products—and, indeed, colleagues at the Center for Uncertainty Studies (CeUS, 2024) are already working on such topics. In an intermediate step, a more profound parameterization of consumer agents in this respect could be derived from extant literature: based on the seminal work by Rogers (2003), for example, the minimum amount of information points of the 2.5% innovators (and the 13.5% early adopters) are supposedly smaller than for the 34% consumers of the late majority or the 16% laggards. Next, consumers of the same type (e.g., innovators) or with similar beliefs could be connected in the social network with higher probability (e.g., in a homophily-driven network, such as used by Backs et al., 2019). The latter would further group formation with possibly interesting effects (e.g., a saddle as described by Chandrasekaran and Tellis, 2011). Finally, it would be worthwhile to implement additional (or, alternative) measures regarding consumers' uncertainty. For example, we currently do not account for variance but assume that consumers become certain once they have evidence that the underlying distribution is unimodal, even though the variance of this distribution is high.

6.2. Managerial perspective

For practitioners, a market simulation as described in the work at hand can be useful in assessing the threat of an adversary company sending contradictory information by employing a negative word-of-mouth campaign. The simulation tool can also be used as a sandbox for testing corresponding countermeasures.

Obviously, in order to be applied in a real-world case, the underlying model needs to be adapted to the specific market (for a general discussion on setting up an ABM approach for innovation diffusion, see Rand and Stummer 2021; for an example on proceeding with respect to parameterization, see Stummer et al. 2015). First, the opposing information source, run by certain adversaries, may not necessarily start operating right with the market introduction of the innovation; its effort (and content) might not be constant over time (but be suitably adapted with respect to market behavior); and it might not end strictly after 90 days. Therefore, the challenge is to more realistically model advertising campaigns. Next, it could be interesting to explicitly introduce a competitor who plans to bring its own product into market and, thus, aims to delay the market success of the first mover, or strives to establish a different ecosystem (e.g., VHS vs. Video 2000 or Apple vs. Android). Then, technological progress over time might be considered. Finally, consumers could disregard certain new information due to a confirmation bias and they could behave more actively in searching for information about the new product; in this regard, the process of innovation adoption with its several phases (e.g., according to Rogers, 2003) could be accounted for.

7. Conclusions

The market success of radically new products can considerably be affected by consumers' increased uncertainty due to diverging information received from opposing sources. In our research, we demonstrated this effect by means of a computational simulation experiment.

To this end, we employed a rather straightforward agent-based model of innovation diffusion. While this approach serves its purpose, the agent-based model might still be refined in several directions, all of which path the way for further research. First, the model might take into account several products and additional decision criteria (e.g., more product attributes, including price) and, thus, use more complex (individual) utility functions. Second, the information in a consumer agent's database might be discounted (i.e., the weight of older information decreases over time), which can result in only a subset of newer information being relevant for the consumer's perceived uncertainty. Information also might be weighted according to whom it stems from (i.e., trustworthiness comes into play), and it might make a difference whether a consumer received all her information from a single peer or from several peers. As to the latter, consumer agents might remember only the last information from each peer and disregard earlier information, which would be more realistic, and it can be implemented easily in an ABM approach (as opposed to a model based on Bayesian learning). Third, rethinking a decision to adopt the new product should only take place after a substantial amount of new information has been received (and not automatically whenever a single information point has been added to the agent's database). Fourth, producer's might advertise the new product with a positive bias (for initial insights, see the sensitivity analysis regarding the true performance value in Part C.1 of the online supplement). If so, such a strategy might ignite disappointment and, thus, a strong negative response on the side of consumers. Finally, the model could include the effect of social normative influence, which constitutes a particular strength of ABM of social systems (like a market).

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Supplement

The Uncertainty Effect in Innovation Diffusion: An Agent-Based Market Model with Opposing Information Sources

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This supplement is composed of the following parts:

Part A: Pseudocode of the agent-based simulation

A.1 Word-of-mouth communication

A.2 Marketing

A.3 First-hand experience

A.4 Purchase

A.5 Information processing

A.6 Perceived uncertainty

Part B: The variance as an alternative measure of uncertainty

Part C: Results from further sensitivity analyses

C.1 Products

C.2 Consumer agents

C.3 Advertisement agents

C.4 Information flows

C.5 Remaining parameters

Part A: Pseudocode of the agent-based simulation

A.1 Word-of-mouth communication

Algorithm 1: Word-of-mouth communication

---

function *wom(i,j)*
- **if** $certain_i = 1$ **then**
  - $\tilde{a}_j \leftarrow \mathcal{N}(a_i, \sigma^{\text{wom}})$;
  - $processInformation(j, \tilde{a}_j, w^{\text{wom}})$;
- **end**
- **if** $certain_j = 1$ **then**
  - $\tilde{a}_i \leftarrow a_j + \epsilon^{\text{wom}}$ with $\epsilon^{\text{wom}} \sim \mathcal{N}(0, \sigma^{\text{wom}})$;
  - $processInformation(i, \tilde{a}_i, w^{\text{wom}})$;
- **end**

end

Parameters:

σ^{wom}	Variance of communication noise for word-of-mouth communication
w^{wom}	Weight of word-of-mouth communication

Variables:

a_i	Information communicated by consumer i
$\tilde{a}_i$	Information received by consumer i
$certain_i \in \{0, 1\}$	1 iff consumer i is certain

Indices:

i, j	Indices of consumers

Functions:

$processInformation(i, value, weight)$	Function that processes the information *value* (here: $\tilde{a}_i$) received by consumer i assuming a *weight* (here: w^{fhx}); function is further specified in Algorithm 5

A.2 Marketing

Algorithm 2: Marketing

```
function marketing(m^int, a^adv)
    for i ∈ [1, N] do
        if random(0,1) ≤ m^int then
            ã_i ← 𝒩(a^adv, σ^adv);
            processInformation(i, ã_i, w^adv);
        end
    end
end
```

Parameters:

σ^{adv}	Variance of communication noise for advertisement
w^{adv}	Weight of advertisement
N	Population size

Variables:

a^{adv}	Information distributed by the advertisement measure
$\tilde{a}_i$	Information received by consumer i

Indices:

i	Index of consumer

Functions:

$\mathit{processInformation}(i, \mathit{value}, \mathit{weight})$	Function that processes the information *value* (here: $\tilde{a}_i$) received by consumer i assigning a *weight* (here: w^{fhx}); function is further specified in Algorithm 5

A.3 First-hand experience

Algorithm 3: First-hand experience

---

function *fhx(i)*

  $\tilde{a}_i \leftarrow \mathcal{N}(a^\star, \sigma^{\text{fhx}})$;

  $processInformation(i, \tilde{a}, w^{\text{fhx}})$;

end

Parameters:

σ^{fhx}	Variance of communication noise for first-hand experience
w^{fhx}	Weight of first-hand experience

Variables:

$a^\star$	True performance value of the product
$\tilde{a}_i$	Information about the performance value received by consumer i

Indices:

i	Index of consumer

Functions:

$processInformation(i, value, weight)$	Function that processes the information *value* (here: $\tilde{a}_i$) received by consumer i assigning a *weight* (here: w^{fhx}); function is further specified in Algorithm 5

A.4 Purchase

Algorithm 4: Purchase decision

```
function purchase(i)
    if certain_i = 1 and a_i ≥ u_i^min then
        adopt_i ← 1;
    end
end
```

Parameters:

$u_i^{\min}$	Consumer i's minimum utility

Variables:

a_i	Consumer i's belief about the performance value
$certain_i \in \{0, 1\}$	1 iff consumer i is certain
$adopt_i \in \{0, 1\}$	1 iff consumer i buys the product

Indices:

i	Index of consumer

A.5 Information processing

Algorithm 5: Information processing

---

function *processInformation*($i, value, weight$)

  $a_i \leftarrow \frac{a_i \cdot W_i + value \cdot weight}{W_i + weight}$;

  $W_i \leftarrow W_i + weight$;

  $v_i \leftarrow v_i + 1$;

  $\Theta_i \leftarrow \Theta_i \oplus ((value, weight))$;

  if $v_i \geq 2$ **then**

    $checkUncertainty(i)$;

  end

end

Variables:

a_i	Consumer i's belief about the performance value
v_i	Number of information points received by consumer i
Θ_i	Consumer i's individual database containing pairs of information and weight
W_i	Sum of the weights of all received information points

Indices:

i	Index of consumer

Functions:

$checkUncertainty(i)$	Function that checks for consumer i whether she should be certain; function is further specified in Algorithm 6

A.6 Perceived uncertainty

Algorithm 6: Perceived uncertainty

```
function checkUncertainty(i)
    certain_i ← 0;
    if v_i ≥ v_i^min then
        f̂_i(x) ← getKDE(i);
        M ← getMaxima(f̂_i(x));
        mod ← argmax_{m∈M} f̂_i(m);
        if size(M) = 1 then
            certain_i ← 1;
        end
        if TVDistance(f̂_i, N(a_i, σ_i)) ≤ d_i then
            certain_i ← 1;
        end
        sum ← 0;
        for m ∈ M do
            if f̂_i(mod)/f̂_i(m) ≥ h_i or m = mod then
                sum ← sum + 1;
            end
        end
        if sum = size(M) then
            certain_i ← 1;
        end
    end
end
```

Parameters:

$v_i^{\min}$	Consumer i's minimum amount of information
h_i, d_i	Determinants of consumer i's uncertainty

Variables:

v_i	Number of information points received by consumer i
a_i	Consumer i's belief about the performance value
$certain_i \in \{0, 1\}$	1 iff consumer i is certain
$\hat{f}_i(x)$	PDF of information received by consumer i (kernel density estimation)
M	Set of maxima (here for PDF $\hat{f}_i$)
mod	Global maximum in M
σ_i	Standard deviation of information received by consumer i

Indices:

i	Index of consumer

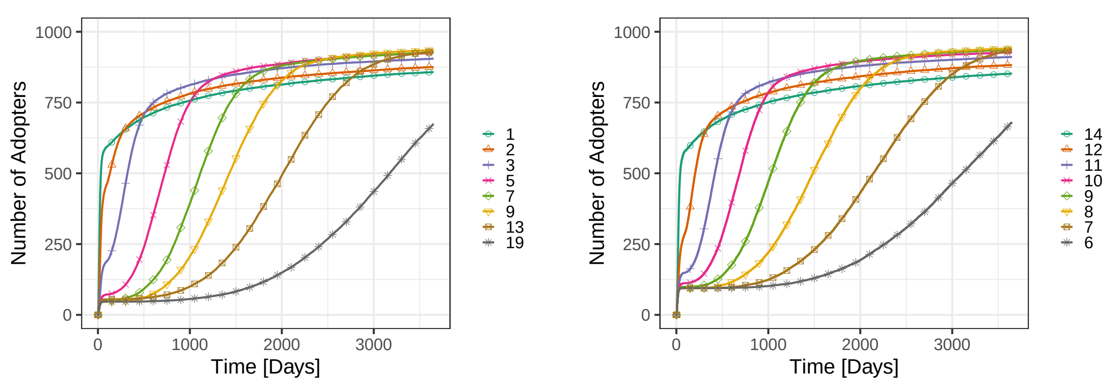
Functions:

$getKDE(i)$	Function that computes the PDF of information received by consumer i using kernel density estimation (see Equation 1 in the main paper)
$getMaxima(f)$	Function that return a set with all maxima of a function f (here: $\hat{f}_i$)
$\text{argmax}_{x \in X} f(\cdot)$	Function that returns the element $x^\star \in X$ (here: set M) that maximizes function f (here: $\hat{f}_i$)
$TVDistance(f, g)$	Function that computes the total variation distance between two PDFs f and g (here: $\hat{f}_i$ and $\mathcal{N}(a_i, \sigma_i)$)
$size(X)$	Function that returns the number of elements in a set X (here: M)

Part B: The variance as an alternative measure of uncertainty

As mentioned in the main paper (in Sect. 3.4), the variance of the information might be considered as an alternative measure for a consumer agent's uncertainty. Doing so would have the advantage that Bayesian updating can be used for updating the variance σ_i^2 alongside the belief (mean) a_i of consumer i. In the following, we analyze the difference in market behavior between applying the approach introduced in our work (referred to as "our uncertainty measure" based on parameters d_i and h_i) and using the variance as a proxy for uncertainty. To this end, we ran additional computational experiments in which consumer i becomes certain if her standard deviation σ_i is below a certain threshold for the standard deviation $\sigma^{\max}$. All simulation outcomes presented in the figures below are averaged over 20 simulation runs.

In a *first set of experiments*, we demonstrate that for the baseline scenario, varying (a) the uncertainty parameter h_i, which is the more dominant parameter in our approach (as compared to d_i), and (b) the above-mentioned uncertainty threshold $\sigma^{\max}$ can generate similar diffusion curves (see Fig. 1). Furthermore, it is noteworthy that, in both approaches, certain parameter values—that is, $h_i \leq 1$ and $\sigma^{\max} \geq 14$, respectively—can reproduce the adoption curve resulting from applying (basic) Bayesian updating with known variance.



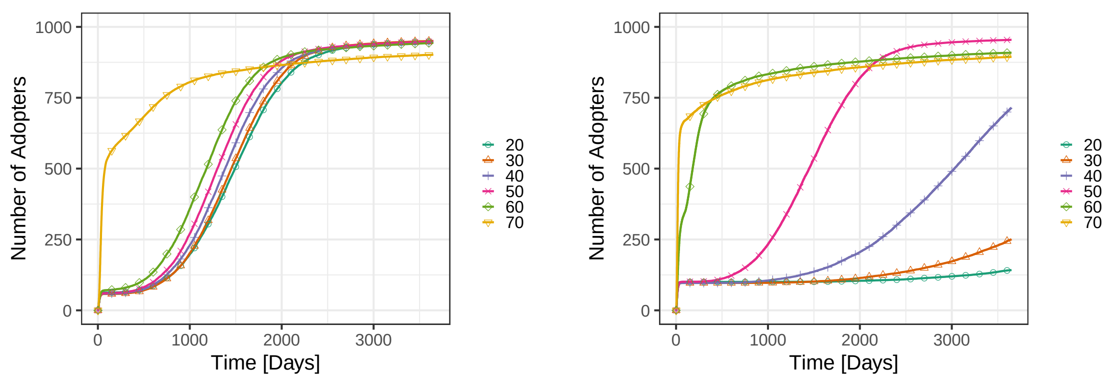
(a) Our uncertainty measure

(b) Variance as a measure of uncertainty

Figure 1: Number of adopters over time for various (a) values for uncertainty parameter h_i and (b) variance cutoffs for uncertainty $\sigma^{\max}$.

From Figure 1, we can already visually identify parameter value $\sigma^{\max} = 8$ as a reasonable counterpart for parameter value $h_i = 9$ used in the main paper. This is confirmed by calculating the mean squared errors for the deviation between the two diffusion curves: while the mean squared errors are 831 for $\sigma^{\max} = 8$, they are 45,407 for $\sigma^{\max} = 7$ and 21,078 for $\sigma^{\max} = 9$. In the following, we therefore use $\sigma^{\max} = 8$ to analyze the differences between the two uncertainty measures in more detail.

In the *second set of experiments*, we investigate the effect of varying the gap between the contents of the two advertisement agents a_q^{adv} (in the baseline scenario: $a_1^{\text{adv}} = 80$ and $a_2^{\text{adv}} = 50$) by adapting the advertisement content a_2^{adv} of the opposing information source. For the uncertainty measure used in our work (see Fig. 2a), the impact on the diffusion curves is small for most changes in a_2^{adv} (for more details and a thorough discussion, see Part C.3). In contrast, when using the variance as an uncertainty measure, the impact of adapting the advertisement content is considerably stronger (see Fig. 2b).



(a) Our uncertainty measure

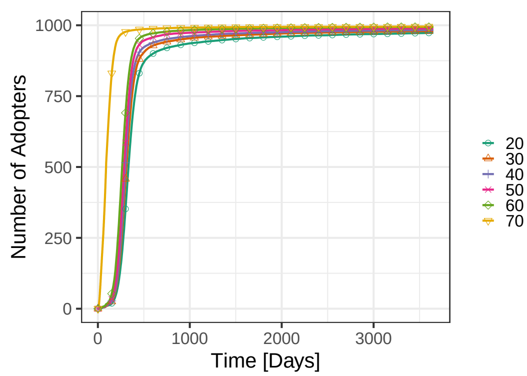
(b) Variance as a measure of uncertainty

Figure 2: Number of adopters over time for different lower advertisement contents a_2^{adv} using (a) our uncertainty measure or (b) variance as a measure for uncertainty.

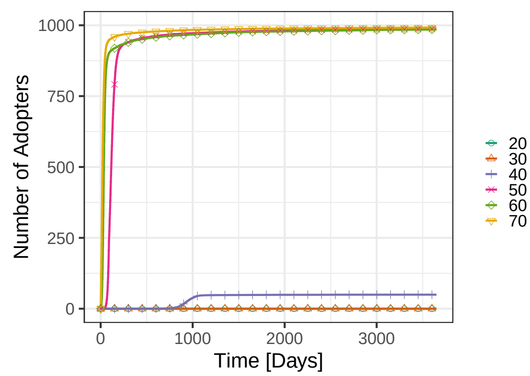
In the latter case (i.e., for using the variance as a measure of uncertainty), it is noteworthy that the (green) diffusion curve resulting from $a_2^{\text{adv}} = 60$—that is, if the original gap of $80 - 50 = 30$ is closed by 33% and, thus, the new gap is just $80 - 60 = 20$—is very close to the diffusion curve in the case of basic Bayesian updating (which is almost identical to the yellow curve). Otherwise, if the gap is increased by 33% by setting $a_2^{\text{adv}} = 40$, the rate of diffusion clearly decelerates and the respective (i.e., the purple) curve is shifted to the right. The reason for this effect is that the variance as a measure for uncertainty considers only a single factor and, in particular, does not account for the shape of the underlying information distribution (i.e., whether it is unimodal or, as in our case, bimodal).

In a nutshell, resorting to the variance as a measure for uncertainty can generate strong effects from relatively small changes in the parameterization of the information sources. The respective impact can be unrealistically high, as, for example, a consumer who is uncertain when receiving information from two opposing information sources with $a_1^{\text{adv}} = 80$ and $a_2^{\text{adv}} = 50$, would still be uncertain—although to a somewhat lesser degree—when receiving information from $a_2^{\text{adv}} = 60$. When applying our approach, in contrast, the same change in parameterization results in a considerably less strong impact on the diffusion curve, which we deem more reasonable.

In the *third set of experiments*, we investigate the effect of a single outlier. In the respective setting, all consumers start with a single information point a_2^{adv} from the opposing information source and afterwards receive only advertisement $a_1^{\text{adv}} = 80$ from the producer's information source. The simulation results obtained for different values of the outlier a_2^{adv} are depicted in Figure 3.



(a) Our uncertainty measure



(b) Variance as a measure of uncertainty

Figure 3: Number of adopters over time for different single starting information points a_2^{adv} from the opposing information source using (a) our uncertainty measure or (b) variance as a measure for uncertainty.

In both approaches, a high value of the outlier a_2^{adv} has nearly no effect on innovation diffusion, as such an outlier is still close to the (extensive) information provided by the producer. For our approach (visualized in Fig. 3a), even for considerably lower values for the outlier, the impact on innovation diffusion remains small, although the effect becomes more pronounced for lower values of a_2^{adv}.

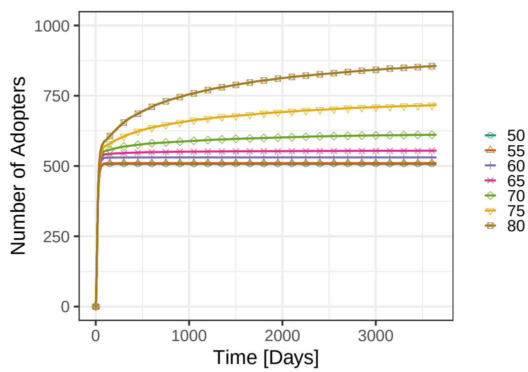
When resorting to the variance as a measure for uncertainty, a similar behavior can be seen for values of $a_2^{\text{adv}} \geq 50$ (as shown in Fig. 3b). However, for lower values, we observe only a single simulation run (for $a_2^{\text{adv}} = 40$) in which the new product succeeds on the market, while in all the remaining runs, no adoption occurs. This result can be attributed to the strong impact of outliers on the variance, which requires numerous information points from the producer's information source to compensate for the outlier: for an outlier with the value $a_2^{\text{adv}} = 40$, approximately 25 information points with $a_1^{\text{adv}} = 80$ are needed to lower the standard deviation below 8; for $a_2^{\text{adv}} = 30$ and $a_2^{\text{adv}} = 20$, the number of necessary information points increases even to 40 and 58, respectively. However, given our parameters for the advertisement, more than 40 information points are never received by a single consumer agent, while more than 20 information points can only be gathered in rare instances—and even if some consumer agents succeed in this respect, diffusion would still drastically slow down. Overall, we feel that such a strong impact of a single information point is not realistic.

Part C: Results from further sensitivity analyses

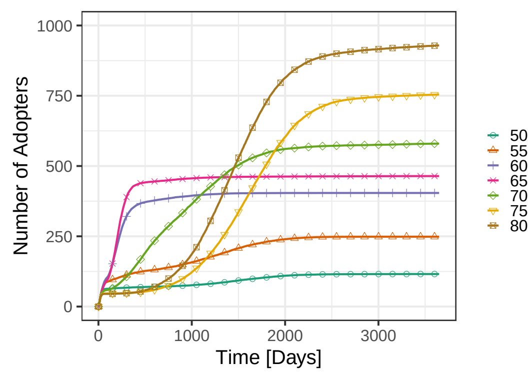
In the following, we provide results from additional sensitivity analyses in detail. To this end, we computed 20 simulation runs for each parameter value under investigation and depict the mean number of adopters averaged over these runs as colored lines in the figures.

C.1 Products

The only parameter regarding product characteristics is the true performance value $a^\star$, which is communicated to the consumers via first-hand experience. In our sensitivity analysis, we vary this value in the range between the advertised content provided by the two opposing information sources (i.e., $a_1^{\text{adv}} = 80$, $a_2^{\text{adv}} = 50$). The mean number of adopters over time is depicted in Figure 4.



(a) Bayesian updating



(b) Our approach

Figure 4: Number of adopters over time for different values for the product's true performance $a^\star$ using (a) Bayesian updating or (b) our approach.

For the Bayesian approach (with simulation results presented in Fig. 4a), lowering the true performance value $a^\star$ has only a marginal impact on the first wave of adoption, during which around 50% of the market potential can be exploited. Thereafter, the diffusion curve visibly flattens out with decreasing performance value $a^\star$, and remain nearly steady for parameter values $a^\star \leq 65$.

In contrast, the diffusion curves that are obtained by applying our approach (illustrated in Fig. 4b) show an interesting pattern. While at the simulation horizon, the market shares—as could be expected—are always larger in runs with a higher true performance value $a^\star$, in the short term, this order is not maintained, as in runs with values of $a^\star \leq 70$ the number of early adopters is higher than in runs with values of $a^\star \geq 75$. Moreover, the shape of the diffusion curves change: For low values $a^\star \in \{50, 55\}$, we observe a noticeable, although limited, increase in the number of early adopters and a flat S-shaped diffusion afterwards. Setting the true performance values closer to the average of the advertised contents (i.e., for $a^\star \in \{60, 65\}$) results in diffusion curves that resemble delayed versions of the curves resulting from Bayesian updating (i.e., many early adopters, flat afterwards). For simulation runs in which the true performance value is set to $a^\star = 70$, the variance of simulation outcomes is high among the twenty runs, with individual diffusion curves from these runs showing either of the two above-mentioned forms; it should be noted that this diversity in the shape of the diffusion curves from single runs with parameter $a^\star = 70$ is not represented in Figure 4b, as the graph only provides the averaged number of adopters over the twenty runs.

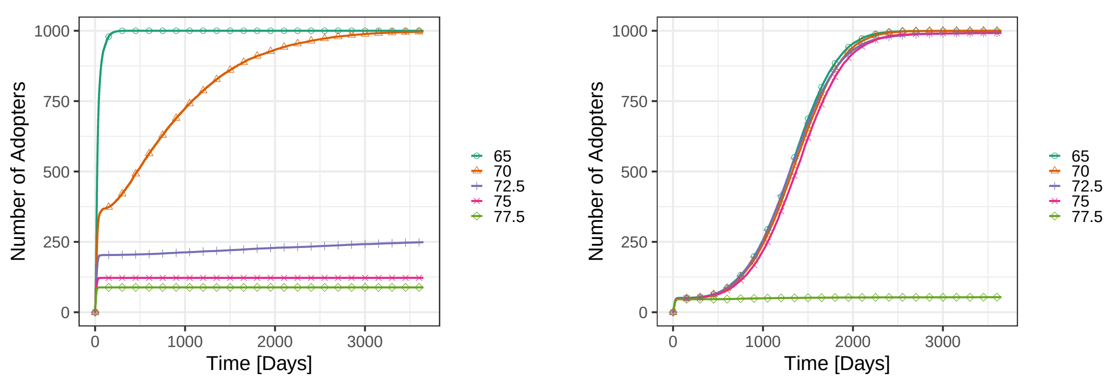
The reason for the relatively high number of early-phase adopters in settings with a low value for the true performance is that these innovators—all of whom received only the producer's advertisement promising a product performance of approximately 80 and, thus, had a favorable opinion when adopting the new product—afterwards learn about the considerably lower true performance values and accordingly adapt their belief to a value that is somewhere in between the information peaks fueled by the two advertisement agents. This adaptation in belief turns out to be beneficial in the course of early on convincing other consumers, because the word-of-mouth communication of the early adopters nicely bridges the gap between the two extreme values broadcasted by the opposing information sources and, thus, reduces uncertainty, which then induces further adoption decisions. Obviously, this effect is short-lived for low values for the true performance (i.e., for $a^\star \in \{50, 55\}$), as adopters experience the low true value and inform their peers. For a true performance value in the middle between the two information peaks (i.e., $a^\star \in \{60, 65\}$), the effect lasts longer, as consumers quickly learn about the true value and stick with it; however, only consumers in runs with a minimum utility $u_i^{\min} \leq 60$ or $u_i^{\min} \leq 65$, respectively, adopt the product which is why the diffusion curve already flattens out in a relatively early phase. Interestingly, innovation diffusion in the early phase is, to some degree, impeded by high values for the true performance (i.e., $a^\star \in \{75, 80\}$), because first-hand experience reinforces the consumer agents' notion of two diverging opinions prevalent in the market, which maintains a high level of uncertainty, and only after some time, one opinion—namely, the notion that performance is high—prevails.

It is noteworthy that the favorable impact of low values for the true performance is counterintuitive, and indeed, it is rooted in a simplification of our model, as we do not account for consumers' disappointment after they have learned that the true value is considerably lower than their belief at the point in time when they adopted the new product. It would be straightforward to extend our model in this respect, that is, introducing negative word-of-mouth as the consumers' reaction to this disappointment. However, such an extension would only indirectly contribute to answering the key question of our research, which is concerned with the uncertainty effect in the presence of two opposing information sources. Still, the impact of a true value that is considerably lower than the advertised value—for example, if the producer vastly exaggerates the alleged performance of the new product and, consequently, provokes disappointment on the consumers' side—raises another valid (and relevant) question, which we intend to address in future research.

C.2 Consumer agents

In our model, consumer agents i are characterized by the required minimum utility $u_i^{\min}$, the minimum number of information points $v_i^{\min}$ sought for by them to satisfy their need for having obtained a sufficiently large information base, and the parameters h_i and d_i, respectively, used in our approach for measuring uncertainty (which is assumed to be low if the distribution of the received information resembles a normal distribution). In the following, we analyze the impact of these parameters on the outcomes of our market simulation. In addition, we ran a sensitivity analysis for the population size N.

First, we assume that all consumers have the *same threshold for the minimum utility* $u_i^{\min}$, and run simulation experiments in which this threshold is varied. The diffusion curves, when applying either the Bayesian updating or our approach, are depicted in Figure 5.



(a) Bayesian updating

(b) Our approach

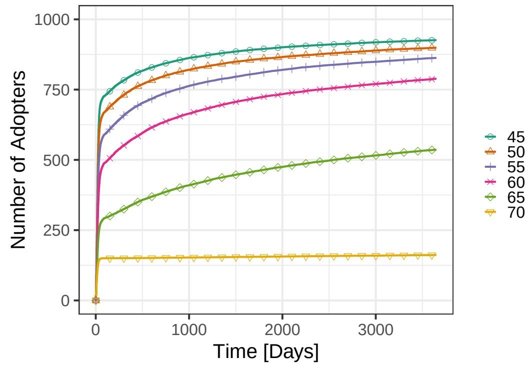
Figure 5: Number of adopters over time for variations of identical minimum utilities $u^{\min}$ for all consumers using (a) Bayesian updating or (b) our approach.

When resorting to Bayesian updating, this parameter apparently has a major effect on the number of early adopters and the speed of diffusion afterwards (see Fig. 5a). If the threshold for the minimum utility is $u_i^{\min} = 65$ (i.e., it is in the middle between the information broadcasted by the two sources) or even below (which is not included in the above graph), adoption is very quick, and nearly all consumers have adopted the new product after only a short while. However, the simulation results change remarkably with an increase in the parameter for the required minimum utility, as such an increase entails a substantial slowdown of innovation diffusion; and for parameter settings $u^{\min} \geq 75$, the diffusion even completely halts after the early adoption took place. Obviously, this market behavior is due to the relatively low number of consumer agents who become convinced just from being exposed to the information sources that product performance would be this high. Although these few adopters learn about the (high) true performance value and afterwards engage in word-of-mouth communication, they have a hard time competing with the information that is communicated by the non-adopters, particularly so, as consumers must be convinced that the performance value is higher than 70.

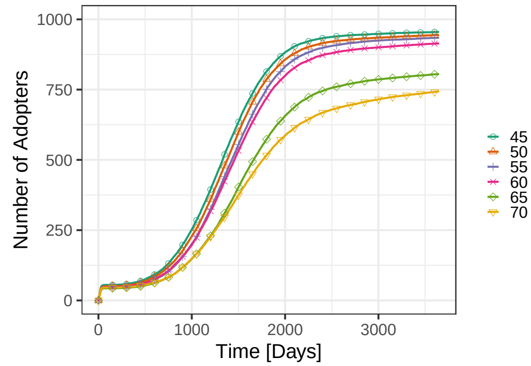
When applying our approach, the picture is different (see Fig. 5b), as our approach is less sensitive with respect to the parameter for the minimum utility, and, thus, for nearly all variations of this parameter, it produces a nice S-curve of innovation diffusion. The reason is that in our approach, in contrast to the Bayesian approach, only consumer agents who are certain communicate their beliefs to others. Therefore, after the advertisement ended in period $t = 90$, (almost) only the adopters were certain and, hence, only high beliefs were communicated. The only exception in this set of simulation runs is the case with an extraordinarily high parameter value $u^{\min} = 77.5$, for which—analogous to the Bayesian case—diffusion does not continue after the very first phase because only few consumer agents become certain just from the advertisement that this high performance value indeed is correct, and the boost of experiencing the true performance value of $a^{\star} = 80$ does not increase their belief sufficiently strong to have a major impact on persuading peers by means of word-of-mouth communication (as those peers have the same high minimum utility requirement and have already collecting plenty of information which partly was provided by the opposing information source).

Second, we investigate the case of *heterogeneous minimum utilities* $u_i^{\min}$ drawn from a uniform distribution. To this end, we vary one bound of this distribution (i.e., either the upper or the lower bound) at a time, with the other bound remaining at the baseline value of $\alpha^{\min} = 55$ or $\beta^{\min} = 80$, respectively. The simulation outcomes for different lower and upper bounds are provided in Figure 6.

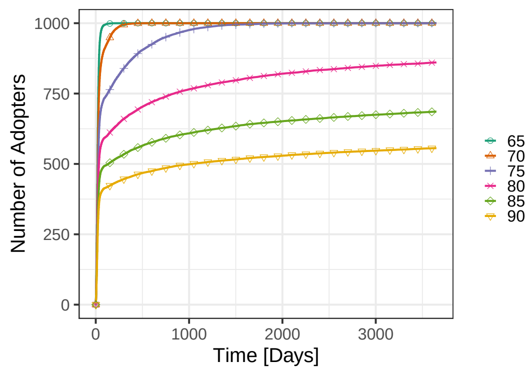
As expected, when the lower (upper) bound for the minimum utility is increased, we see a



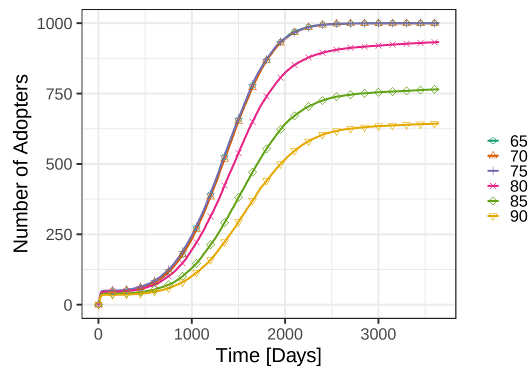
(a) Lower bound in Bayesian updating



(b) Lower bound in our approach



(c) Upper bound in Bayesian updating

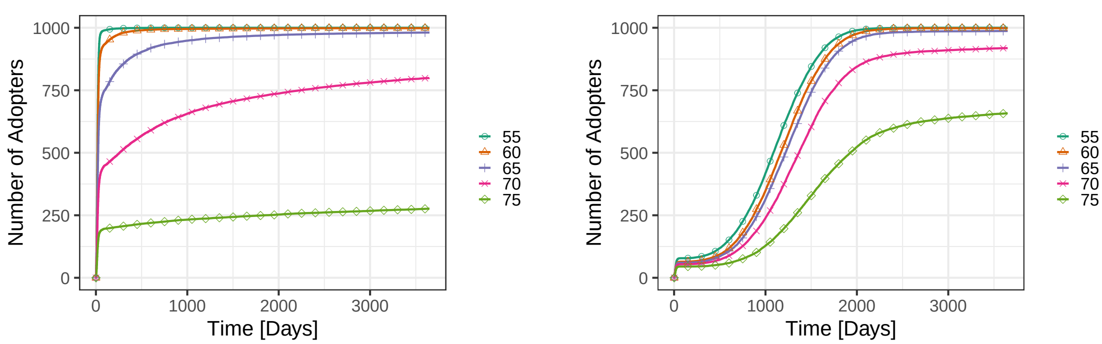


(d) Upper bound in our approach

Figure 6: Number of adopters over time for different lower or upper bounds of a uniform distribution from which (heterogeneous) minimum utilities $u_i^{\min}$ are drawn; lower bounds are varied for (a) and (b); upper values are varied for (c) and (d); simulation experiments are performed for using Bayesian updating and for our approach.

downward shift in the diffusion curves for both approaches, which is reasonable market behavior. If lower and upper bounds become close (i.e., the lower bound is considerably higher or the upper bound is considerably lower than in the baseline setting), the outcome resembles the case of homogeneous minimum utilities discussed above. It is noteworthy that we also consider bounds that are lower (higher) than the information provided by the two information sources. For the upper bound at $\beta^{\min} = 90$, for example, this setting implies that some consumer agents are never content with the product performance and, therefore, they will not adopt the new product; accordingly, the respective diffusion curve is rather low (see Fig. 6d). On the other end of the spectrum, a lower bound of $\alpha^{\min} = 45$ creates a minimum utility for some consumer agents that is below all information about the product's performance; accordingly, they will adopt the new product as soon as they become sufficiently certain of the received information and, therefore, the respective diffusion curve is relatively high (see Fig. 6b). As a side note, we acknowledge that the above analysis brought our attention to a special constellation, namely, that a consumer agent received a sufficient number of information points, with all these information points below her minimum utility, but still the consumer agent does not adopt the new product due to the ambiguity of the information points. In reality, a consumer most probably would adopt anyway, which is not covered by a model but could be easily accounted for if necessary for some future application.

Moreover, we also run experiments in which the minimum utilities $u_i^{\min}$ are drawn from a normal distribution with a constant standard deviation $\sigma = 6$ (as, after checking for alternative values for σ, this setting results in a variation that is similar to the range covered by the uniform distribution) and different mean values. The respective simulation outcomes presented in Figure 7 show the familiar pattern of shifting the diffusion curves in the expected direction, which indicates that the shape of the diffusion curve is not an artifact of using a uniform distribution (as we did in the experiments discussed in the main paper).



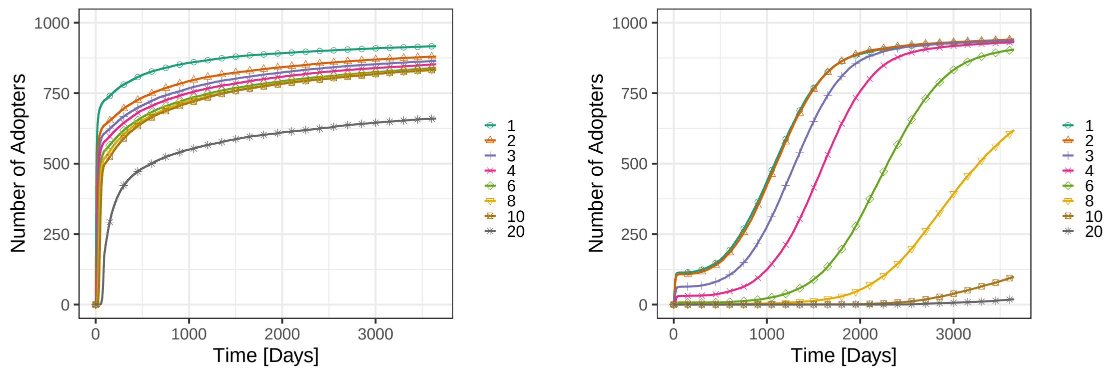
(a) Bayesian updating

(b) Our approach

Figure 7: Number of adopters over time for different mean values of a normal distribution from which minimum utilities $a^{\min}$ are drawn when using (a) Bayesian updating or (b) our approach.

Third, we assume that all consumers have the *same threshold for the minimum amount of information* $v_i^{\min}$. The simulation results for different values of this parameter are shown in Figure 8.

Unsurprisingly, an increase in the minimum amount of information $v^{\min}$ slows down the diffusion for both the Bayesian (Fig. 8a) and our approach (Fig. 8b). This effect is stronger for our approach, as with a higher threshold $v^{\min}$, it gets more unlikely to have obtained (almost) only information points from the producer's source of information as a precondition to be sufficiently certain for adopting the new product. It is noteworthy that for values $v_i^{\min} \geq 8$, the variance between the simulation runs markedly increases in a wide range of outcomes from some runs without adoption until the simulation horizon and others with a rather quick diffusion (which is not shown in the figure, as we only provide averaged results).

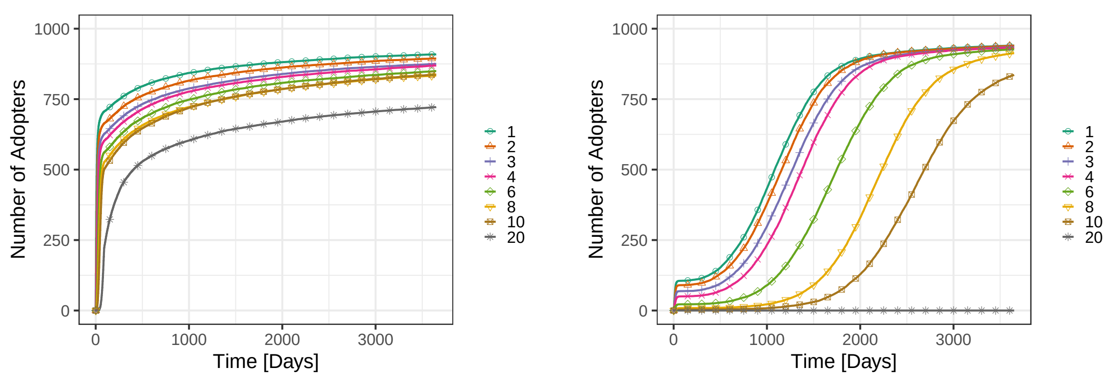


(a) Bayesian updating

(b) Our approach

Figure 8: Number of adopters over time for variations of identical minimum amounts of information $v_i^{\min}$ for all consumers using (a) Bayesian updating or (b) our approach.

Fourth, we also investigate the case of *individual thresholds for the minimum amount of information*. To this end, the threshold for a specific consumer agent i is drawn from a Poisson distribution with parameter $\lambda^{\min}$, which, hence, is the mean over all individual minimum amounts of information $v_i^{\min}$. Analogous to the above-described case of homogeneous minimum amounts of information, increasing parameter $\lambda^{\min}$ slows down the diffusion of the innovation. However, the effect is weaker than it was in the homogeneous case. Again, the variance between individual simulation runs becomes considerably high for values of $\lambda^{\min} \geq 10$.



(a) Bayesian updating

(b) Our approach

Figure 9: Number of adopters over time for different parameters $\lambda^{\min}$ of the Poisson distribution underlying the minimum amounts of information $v_i^{\min}$ using (a) Bayesian updating or (b) our approach.

Fifth, we vary *uncertainty parameter* h_i, which determines how high the ratio between the modus and other peaks of the information distribution $\hat{f}_i$ has to be for consumer i to disregard the remaining ambivalence and become certain. If $h_i = 1$, consumer i disregards all ambivalence and is always certain, which corresponds to the behavior of a consumer in the Bayesian approach. The outcomes of the simulation runs for alternative values for parameter h—which, as in our main paper, is used for all consumers (i.e., $h_i = h$)—are depicted in Figure 10.

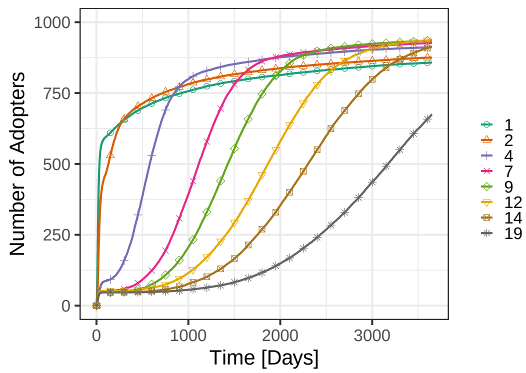


Figure 10: Number of adopters over time for different values for the uncertainty parameter h.

An increase in parameter h slows down the diffusion, because it becomes harder for consumers to disregard ambivalence and become certain. Interestingly, the figure also indicates that for a high parameter value for h (i.e., $h \geq 7$), the number of early adopters is (nearly) constant. The reason is that in such a setting, it becomes unlikely or even impossible for consumers to become certain based on the h-criterion, as in most cases of early-stage adoption, the respective consumer agents became certain due to information provided by the producer and, therefore, h does not play a major role (as the information distributions are unimodal in most cases).

Sixth, the *uncertainty parameter* d_i is investigated. This parameter determines how low the total variation distance between the information distribution $\hat{f}_i$ and a normal distribution $\mathcal{N}(a_i, \sigma_i^2)$ with belief a_i as the mean and the variance of information σ_i^2 as the variance has to be for consumer i to regard $\hat{f}_i$ as sufficiently close to a normal distribution and, thus, to become certain. If $d_i = 1$, consumer i accepts every distribution as sufficiently normal and is always certain, which resembles the case of Bayesian updating. The simulation results provided in Figure 11 show that lowering parameter d—which is then used to parameterize all consumers in an identical way (i.e., $d_i = d$)—slows down the diffusion at the beginning. Still, the number of adopters is higher after approximately $t = 2000$. The reason is the group of non-adopters who are certain and, thus, communicate their low beliefs. This group is larger in runs with a larger parameter value d (in contrast to runs with a low value for d, in which it is considerably more difficult for consumer agents to become certain), and the non-adopters' word-of-mouth communication delays product adoption of consumers with a high minimum utility threshold. Ultimately, the latter group is reached, but it takes quite some while.

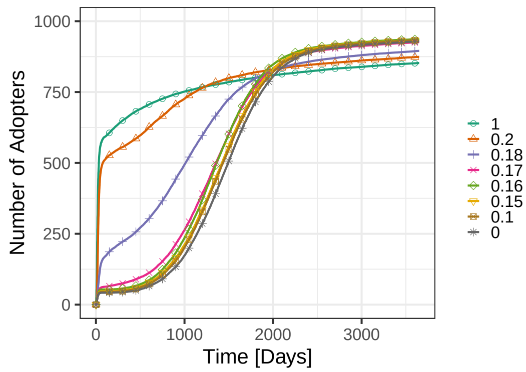
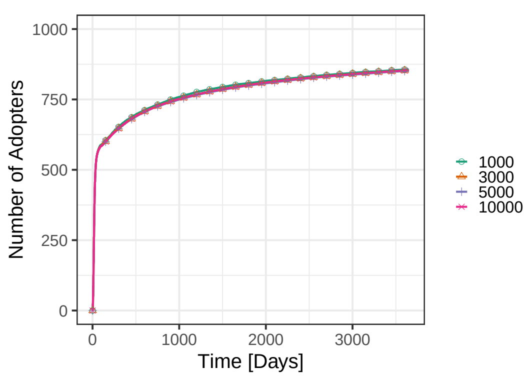
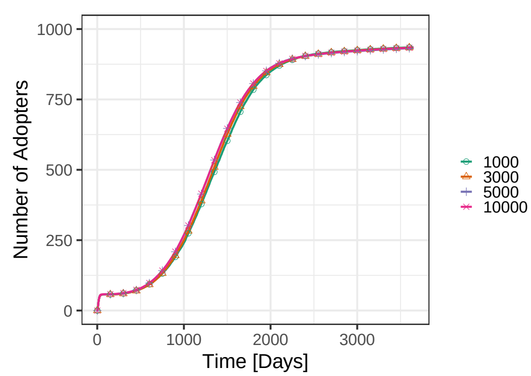


Figure 11: Number of adopters over time for different values for the uncertainty parameters d.

Seventh, the *population size* was varied to check whether the chosen value $N = 1000$ is sufficient. As can be learned from the results presented in Figure 12, for both approaches, values of $N > 1000$ do not have a noticeable effect on the diffusion curves. Still, it should be noted that the variation between the simulation runs (which is not represented in the graph) decreases with increasing population size. The reason is that larger populations are more similar to each other with regard to heterogeneous consumer parameters and, furthermore, the stochasticity effects of the advertisement are smaller for a higher population size (given the higher number of draws).



(a) Bayesian updating



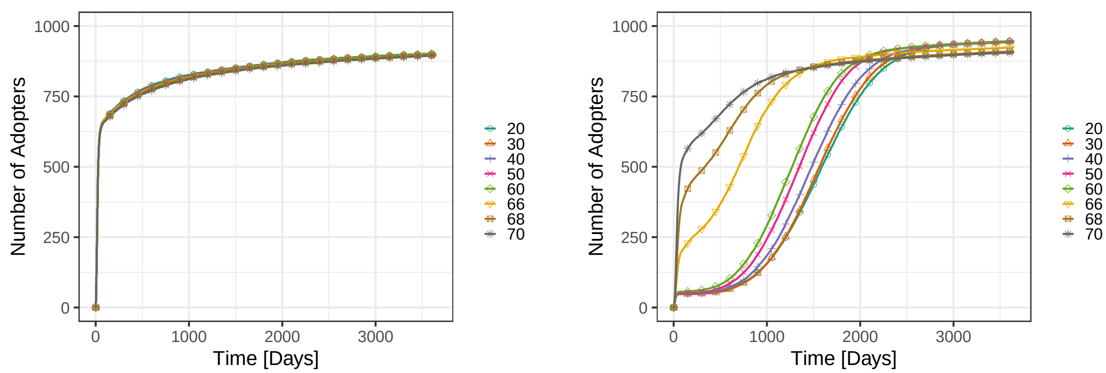
(b) Our approach

Figure 12: Number of adopters over time for different population sizes N using (a) Bayesian updating or (b) our approach.

C.3 Advertisement agents

In our model, advertisement agents q are characterized by the contents a_q^{adv} broadcasted by them, the duration of an advertisement campaign m^{dur}, and the advertising intensity m^{int}. In the following, we analyze the impact of these parameters on the outcomes of our market simulation.

First, we explore the effects of varying the *contents of the advertisement campaign* a_q^{adv}. As we are only interested in the difference between the contents broadcasted by the two opposing information sources, we only vary the content of the agent with the lower value (a_2^{adv}) and keep the higher value constant ($a_1^{\text{adv}} = 80$). Still, and for the sake of comparability, we also adapt the minimum utilities $u_i^{\min}$ of our consumer agents, that is, we set $\alpha^{\min} = a_2^{\text{adv}}$. The reason is that we strive to isolate the effects of varying the distance between the contents from the effects of varying the mean of the contents. The simulation results for both approaches are provided in Figure 13.



(a) Bayesian updating

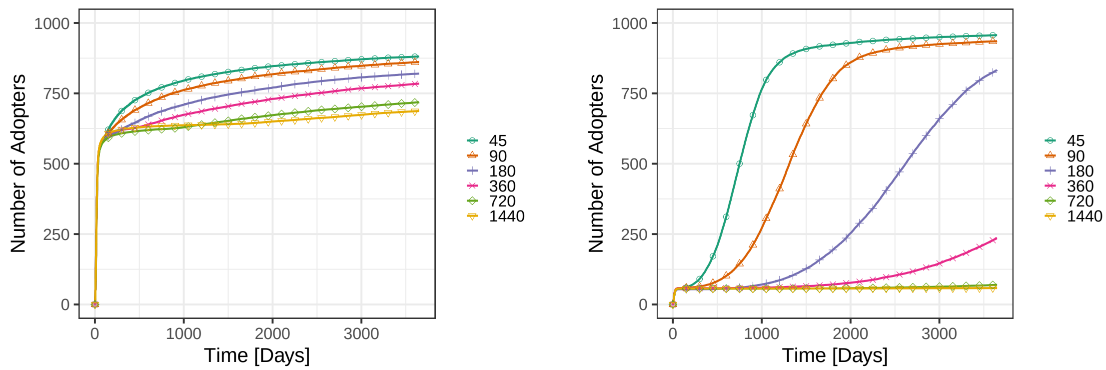
(b) Our approach

Figure 13: Number of adopters over time for different advertisement contents from the opposing information source a_2^{adv} using (a) Bayesian updating or (b) our approach.

For the Bayesian approach, we find no difference in the diffusion curves when varying the content of advertisement a_2^{adv} (see Fig. 13a). In contrast, the simulation results for our approach (see Fig. 13b) suggest that the innovation diffusion decelerates with a larger gap between the contents of both advertisement campaigns. This effect originates from the increasing gap between the information sources and, thus, the broader range of consumer agents' possible beliefs. Consequently, it becomes harder for adopters to convince other consumer agents, as their beliefs are further apart from each other and also from the producer's information source, which creates broader and lower peaks in the information distributions. While the effect is gradual for

$20 \leq a_2^{\text{adv}} \leq 60$, it is more pronounced for $60 \leq a_2^{\text{adv}} \leq 70$. The reason is that for $a_2^{\text{adv}} \geq 60$, the information from both sources appears similar enough (after communication noise is added) to be mistaken as originating from a single source. Consequently, more consumer agents become certain and adopt.

Second, we explore the effect of varying the *advertisement duration* m^{dur} which is identical for both advertisement agents. Figure 14 shows that increasing the duration m^{dur} has no effect on the number of early adopters (at least for $m^{\text{dur}} \geq 45$) but it slows down the diffusion afterwards.



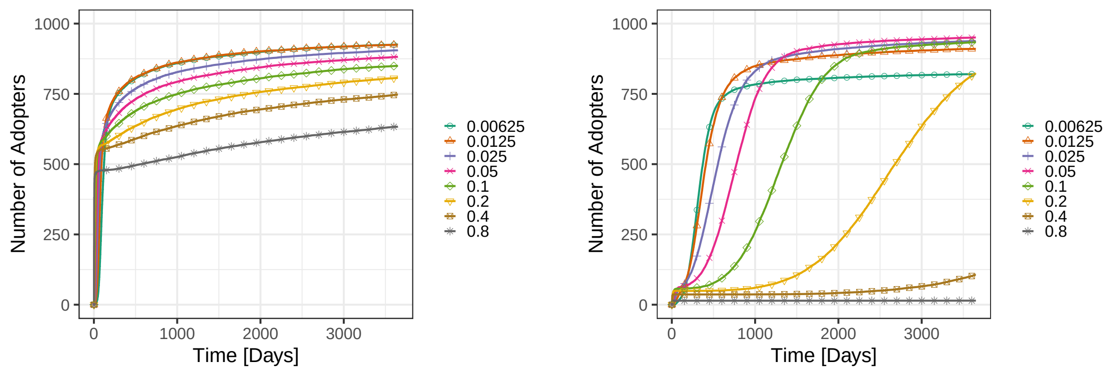
(a) Bayesian updating

(b) Our approach

Figure 14: Number of adopters over time for different advertisement durations m^{dur} using (a) Bayesian updating or (b) our approach.

When applying our approach, for high values $m^{\text{dur}} \geq 720$, the above effect is sufficiently strong that nearly no consumer adopts after the first 100 days and before the end of our simulation horizon—that is, we observe, on average, less than 14 adopters for $m^{\text{dur}} = 720$ and less than two adopters for $m^{\text{dur}} = 1440$. The reason for this market behavior is that a longer duration of the advertisement campaigns results in more ambivalent information, as the mean of the advertised information $(a_1^{\text{adv}} + a_2^{\text{adv}})/2 = (80 + 50)/2 = 65$ is lower than the true performance value $a^\star = 80$ and, hence, the simulation runs yield lower beliefs (for both approaches) and a more persistent uncertainty (our approach).

Third, when varying the *advertisement intensity* m^{int} (and in each instance, identically for both advertisement agents), a similar effect as above can be observed, namely, that higher intensity decelerates innovation diffusion, that is, shifts the diffusion curve to the right. Corresponding simulation results are shown in Figure 15. It is noteworthy that for low parameter values for the advertisement intensity, the variance between simulation runs increases, with the effect of nearly no adoption occurring in certain runs (which, however, is not depicted in the graph). In contrast to the advertisement duration, increasing the advertisement intensity m^{int} also has an impact on the early adopters, as the diffusion curve is shifted to the left (i.e., the early adopters adopt even earlier).



(a) Bayesian updating

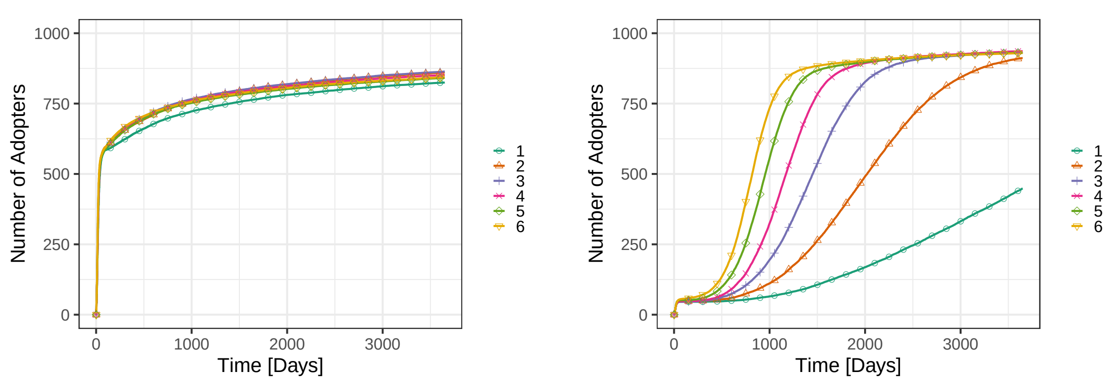
(b) Our approach

Figure 15: Number of adopters over time for different advertisement intensities m^{int} using (a) Bayesian updating or (b) our approach.

C.4 Information flows

Regarding information flows, we investigate effects that can arise either from the density of our social network or from the weights of our information channels.

During the generation of the *social network*, parameter k determines the number of existing agents to which each newly added consumer agent is connected. The impact of varying this parameter on the shape of the diffusion curve is illustrated in Figure 16.



(a) Bayesian updating

(b) Our approach

Figure 16: Number of adopters over time for values of the network parameter k using (a) Bayesian updating or (b) our approach.

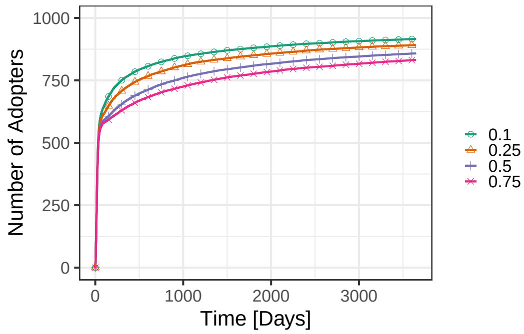
For the Bayesian approach (see Fig. 16a), a noticeable, even though small, difference to the diffusion curve obtained for $k = 3$ (as in the baseline parameterization) can be found only for $k = 1$. The reason is that the major effect from differences in network density comes from information diffusion in the early stages (i.e., contributing to reaching $v_i \geq v_i^{\text{min}}$). However, in our market setting, the advertisement agents already provide consumer agents with plenty of information, and afterwards, the beliefs of all consumers are relatively similar, which limits the network effect (i.e., how connected they are to each other); still, a slight effect of settings for k in the very early phase can be found also for the Bayesian updating.

In contrast, for our approach (see Fig. 16b), an increase of the parameter k, and hence an increase in connections in the network, considerably accelerates innovation diffusion (i.e., diffusion starts earlier and the diffusion curve has a greater slope). The reason is that in our approach (almost) only the adopters engage in word-of-mouth communication and that the (few) adopters can reach more consumers faster if they are more intensely connected in the network.

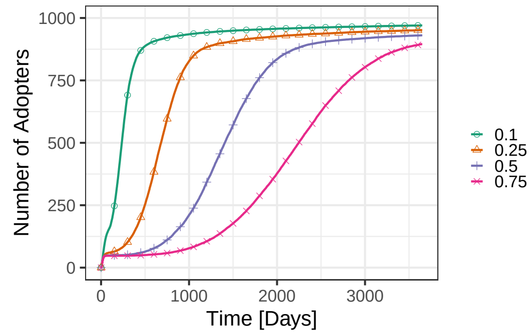
In our simulation experiments, we always used the same network (i.e., consumer agents always have the same neighbors). To test whether different networks (with the same value for parameter k) may have an effect, we computed 100 simulation runs with different networks (all with $k = 3$). When averaging the outcomes of these runs and comparing the resulting diffusion curve with the diffusion curve from our baseline model, the difference was negligible. It should be noted that nevertheless it makes a difference whether to use a specific network for all runs (as we did for our baseline model) or to generate a new network for each of the runs. In the latter case, the standard deviation of the number of adopters in each period (and in each run) increases on average by 13% for the Bayesian and by 20% for our approach, as compared to the standard deviation for the baseline model.

Next, we analyze the possible effects of the *weights of the communication channels*. The values of the weights are only relevant in relation to each other and, thus, the order of the weights as described in the main paper is kept intact—that is, $w^{\text{fhx}} \geq w^{\text{wom}} \geq w^{\text{adv}}$. Therefore, we only need to vary two weights, as varying the third weight is the same as varying the other two at the same time. Consequently, only the results for the weights of advertisement w^{adv} and first-hand experience w^{fhx} are provided in Figure 17.

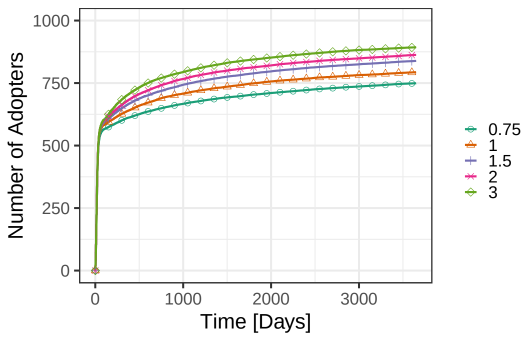
For the Bayesian approach, the diffusion curves shift downwards if more weight is assigned to the advertisement channel (see Fig. 17a), and they shift upwards if more weight is assigned to the first-hand experience (see Fig. 17c). When varying the weight for word-of-mouth communication



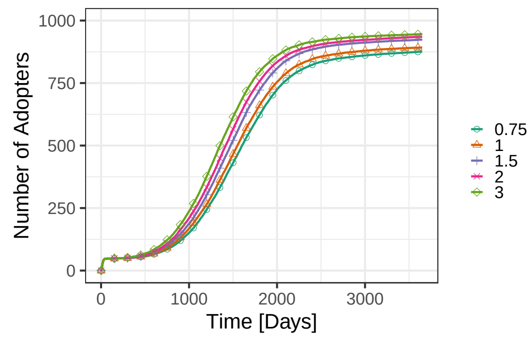
(a) Bayesian updating: ADV



(b) Our approach: ADV



(c) Bayesian updating: FHX



(d) Our approach: FHX

Figure 17: Number of adopters over time for different weights for the information channels; the weight of advertisement (ADV) w^{adv} is varied for (a) and (b); the weight for for-hand experience (FHX) w^{fhx} is varied for (c) and (d); simulation experiments are performed for Bayesian updating and for our approach.

w^{wom} there is also a shift downwards in the diffusion curve later in the simulation, as the effect of weaker first-hand experience dominates at later times.

Analogously, for our approach, there is an increase in the number of adopters and a slightly accelerated innovation diffusion when increasing the weight of first-hand experience w^{fhx} (see Fig. 17d). Also, an increase in the weight of advertisement w^{adv} yields a decrease in the number of adopters. However, this decrease is accompanied by a stronger acceleration of innovation diffusion. The effect of increasing the weight for word-of-mouth w^{wom} is a combination of both, which results in the diffusion curve being shifted to the left and down, as advertisement and first-hand experience both become weaker in comparison to word-of-mouth communication.

The reason for this market behavior is that a stronger weight for advertisement results in lower beliefs and higher uncertainty, as the opposing advertisement agents are the sources of ambivalent information that in the mean (i.e., $(a_1^{\mathrm{adv}} + a_2^{\mathrm{adv}})/2 = (80 + 50)/2 = 65$) is lower than the true performance value $a^\star = 80$. Stronger first-hand experience, however, increases the mean belief of adopters, which makes it possible for them to convince consumers more easily.

C.5 Remaining parameters

Alternative *kernel functions* that could have been used for our kernel density estimation are the Epanechnikov (parabolic), the biweight (quartic), and the triweight function. We compared these three alternatives to the normal kernel functions that we used in the baseline parameterization (see Fig. 18). The respective diffusion curves are not identical, but differences are minimal. Accordingly, our results seem to not rely on our chosen kernel function.

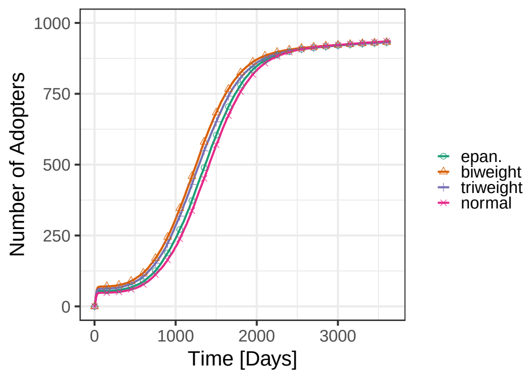


Figure 18: Number of adopters over time for different kernel functions.

Finally, we address our modification aimed at reducing computing time, which was mentioned in Section 5.1 of the main paper. In this modification, a certain parameter defines the level of information being considered extraordinarily high. In Figure 19, we display diffusion curves for several variations of this parameter, which demonstrate that it does not have an effect on the simulation outcome. Not using the modification at all (see "unres" in the graph's legend) only shifts the diffusion curve slightly to the left. In a nutshell, the above-mentioned modification considerably decreased computing time without having an impact on the simulation results.

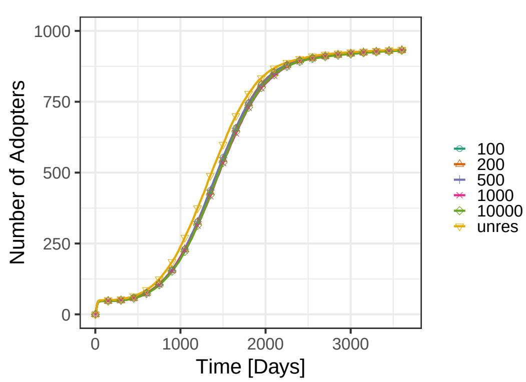


Figure 19: Number of adopters over time for different parameter values of our modification for computational efficiency.

Impressum

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