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INVARIABLY SUBOPTIMAL. AN ATTEMPT TO IMPROVE THE VOTING RULES OF TREATIES OF NICE AND LISBON

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Invariably suboptimal.
An attempt to improve the voting rules of Treaties of Nice and Lisbon*

Werner Kirsch** and Jessica Langner***

Abstract

We investigate the voting rules in the Council of the European Union. It is known that both the current system, according to the Treaty of Nice, and the voting system proposed in the Lisbon treaty strongly deviate from the square root law by Penrose which under certain assumptions can be shown to be the ideal power distribution.

Since it seems easier to make corrections to the current systems than to agree upon completely different new voting rules, one may hope that adjustments of the quota in the Lisbon treaty might give rise to a system which is close to the square root law. In this paper we investigate this question. Our computations show that a mere change of quota in the treaties of Nice and Lisbon cannot bring the system substantially closer to the ideal distribution of power as given by the square root law.

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Introduction

Political power, notably voting power, can be measured through the concept of power indices (Felsenthal and Machover, 1998; Taylor and Pacelli, 2008). In particular, the Banzhaf index measures how frequently a given voter is decisive in a voting panel if all possible voting outcomes are counted with the same weight (for details see Taylor and Pacelli (2008) or our discussion below).

The Council of the European Union is a typical example where power indices can help to understand the power structure within this legislative body. In fact, with its current 27 members and the complicated decision rules voting in the Council is hard to analyze without using mathematical tools. The Council consists of one representative per member state. The members of the Council have different voting power depending on the size of the country they represent, at least for the Nice system, in a nonsystematic way. The current voting system, according to the Treaty of Nice, has three components. The first component requires a qualified majority with respect to voting weights (see table 1) assigned to the states by the treaty as a result of negotiations rather than of a plausible algorithm, the second component asks for a majority of states, the third component requires that the supporters of a proposal represent at least 62% of the Union's population.

The draft constitution contained a new voting system for the Council, the “double majority”. This system was adopted by the Reform Treaty, the Treaty of Lisbon, and has been ratified very recently. The double majority, as the name suggests, has two components. To make a proposal pass the Council members supporting it must represent both a qualified majority (55%) of the states and a qualified majority (65%) of the population of the European Union. This new decision-making rule will be applicable only in 2014, allowing three more years for transitory way of its application.

Obviously, these two voting systems are very different. In fact, they lead to very different distributions of power among the states. Thus, the question arises, how a *fair* voting system should look like.

An answer to this was given by Lionel Penrose as early as 1946 (Penrose, 1946). He found that a distribution of power among the states which is proportional to the square root of the population of the respective state is a fair distribution of power.

Penrose’s argument for the square root law is based on a concept which is now known as the Banzhaf index. The Banzhaf index is based on the assumption that the voters make up their minds independently of each other; in other words: we assume an a priori ignorance to the nature of future legislative proposals. Thus, in course of any extension new members are treated equally as old members (see Leech 2003).

If there is a strong correlation between the citizens’ voting behavior, then other indices, like the Shapley-Shubik index for example, may be more appropriate to describe that particular situation. In this case the square root law is not applicable. However, as long as the correlation between the voters is not too strong one finds that the square root law is valid (see Kirsch 2007).

A treaty of constitutional character, like the Lisbon treaty, is not tailored to describe a particular system with its specific dependence structure at a specific moment in time. Rather it should be made with an unbiased view on the bases general principles to fix the rules for a generic system in the long run. Thus the a priori assumption that voters act in an only weakly dependent way seems to be appropriate and most natural.

Consequently, in a multinational body, such as the Council of the EU, with one representative per country each member state should have a power proportional to the square root of the country's population (Felsenthal and Machover, 1998).

There are various considerations about the Council of the EU in connection with the square root law (Ade, 2005; Baldwin and Widgren, 2004; Baldwin, Berglöf, Giavazzi and Widgren, 2000; Bilbao, 2004; Bobay, 2004; Felsenthal and Machover, 2000, 2003; Hosli and Machover, 2002; Hosli and Taagepera, 2006; Kirsch, 2001; Laruelle and Valenciano, 2002; Laruelle and Widgren, 1998; Leech, 2002; Leech and Machover, 2003; Lindner, 2004; Lindner and Machover, 2004; Maaser and Napel, 2007; Moberg, 2002; Plechanovova, 2004; Sutter, 2000; Słomczynski and Życzkowski 2004, 2007a; Tiilikainen and Widgren, 2000; Widgren, 2003).

Also, in the EU context a voting system based on square root weights was proposed without any relation to Penrose's square root law by several authors (Schmitter and Torreblanca, 1997; Moberg, 1998; Beisbart, Bovens and Hartmann, 2004).

In particular, it is well known that both the voting rules of the Nice Treaty and those of the Treaty of Lisbon deviate strongly from the square root distribution of power. Consequently, those voting systems distribute the voting power unequally among the citizens of the member states (Algaba, Bilbao and Fernández, 2004; Baldwin and Widgren, 2003a, 2003b, 2004; Barberá and Jackson, 2004; Felsenthal and Machover, 2004a, 2004b; Felderer, Paterson and Silarszky, 2003; Leech, 2002; Paterson and Silarszky, 2003; Plechanovova, 2003, 2004).

In negotiations among the member states a voting system based on square root weights was proposed by governments several times: In 2003 such a system was proposed by Sweden in connection with negotiations on the Amsterdam Treaty. In 2004 the Irish made positive references to implement a voting system based on Penrose's square root law.

In 2004 two Polish scientists, Wojciech Słomczynski and Karol Życzkowski devised a voting system known as the "Jagiellonian Compromise" (Słomczynski and Życzkowski, 2004, 2007a) which results in a power distribution extremely close to the square root law. In this system each member state obtains a *voting weight* proportional to the square root of its population. This does not automatically give a distribution of *power* according to the square root law. However, Słomczynski and Życzkowski observed that this is the case with a particular choice of the *quota*, i.e. the threshold to reach a qualified majority. In fact, they found that with a quota of 61.4% the *voting power* (as measured by the Banzhaf index) agrees to a very high degree of accuracy with the square root law.

The Jagiellonian Compromise was proposed by the Polish government on the EU summit in Brussels in 2007. However, the heads of states and governments rejected this proposal in favor of the double majority. Presumably, at this late stage the summit did not want to change the voting rules completely.

One might hope that it would be much easier to convince politicians to make minor changements to the existing system than to agree upon a completely new voting rule. In particular it should be easier to make adjustments to the quota than to redistribute the voting weights.

In this paper we explore to which extent one can approximate the square root distribution of power by just changing quota in the Nice system and for the double majority. To do so, we compute the Banzhaf indices for a large variety of quota for the different components of the voting systems. These results are compared to the square root law. As a measure of deviation from the square root law we consider the total sum of the squared deviations as well as the maximum (over the states) of the deviation from the square root law.

Besides the distribution of power within the Council we also take into consideration the ability of the body to make decisions, i.e. the efficiency of the system. This value, also known as decision probability, is given by the percentage of the constellation of votes, which make a proposal pass: The higher the efficiency the easier to change the status quo, the lower the efficiency the easier to block a change. It is clear that an increase of quota will decrease the systems efficiency. While one might argue that the efficiency of the Council should not be too high to avoid domination of a big minority of states by a small majority, the efficiency must also be not too low to ensure the EU's ability to make decisions at all.

This paper is organized as follows. In section I we give a brief introduction of the voting systems towards the Treaty of Nice and the Treaty of Lisbon. In section II we introduce the theory of voting power and a fair distribution of voting weights. In this context the square root law of Penrose will be explained. The Jagiellonian Compromise, a voting system which fulfills the square root law, is mentioned in section III. With our acquired knowledge we analyze the two treaties in detail, in particular, the obvious defects concerning the distribution of voting weights, voting power and the effectiveness will be discussed in section IV. The fifth section is the main part of this paper. Here, we introduce our course of action to improve the two treaties towards the principle of equality under European citizens. We present and discuss our results and give a compromise solution for the current state of affairs. The last section of this paper contains concluding remarks.

I. Two voting systems for the Council

Since 2001 decision rules for voting in the European Union Council of Ministers are laid down in the Treaty of Nice: Each Member State of the European Union is assigned a voting weight (see table 1) which is a result of negotiation among the Member States. This value reflects to some degree the country's population. The Council adopts a proposal if the following three conditions ("triple majority") are satisfied:

1. The sum of the weights of the Member States vote in favor is at least 255 (of 345).
2. A simple majority of Member States vote in favor is required (14 of 27).
3. The Member States forming the simple majority represent at least 62% of the overall population of the European Union.

Table 1: Voting weights according to the Treaty of Nice

Member State	*Weight*	*Member State*	*Weight*	*Member State*	*Weight*
Germany	29	Belgium	12	Finland	7
France	29	Portugal	12	Ireland	7
United Kingdom	29	Czech Republic	12	Lithuania	7
Italy	29	Hungary	12	Latvia	4
Spain	27	Sweden	10	Slovenia	4
Poland	27	Austria	10	Estonia	4
Romania	14	Bulgaria	10	Cyprus	4
Netherlands	13	Denmark	7	Luxembourg	4
Greece	12	Slovak Republic	7	Malta	3

For the qualified majority to be achieved, all three criteria have to be met simultaneously. Mathematical analyses have shown that the three voting criteria have very different impact on the voting outcome. The first condition is the most significant one, because it is the most restrictive rule. If a coalition achieves a qualified majority of voting weight, then in almost all the instances it also attains a simple majority of member states as well as the required share of the Union's population. In other words, the influence of the second and third component of Nice is very small (see Felsenthal and Machover, 2001; Kirsch 2001).

Most experts agree that Nice has major drawbacks. A first one lies in the very small decision making efficiency of the voting body. The decision making efficiency is defined as the probability that a proposal will be passed by the Council. The value of this quantity is very low (about 2.03%[1]). There already exist publications about modifying Nice such that voting power doesn't change fundamentally but its formal effectiveness increases significantly (Baldwin and Widgren, 2004; Baldwin, Berglöf, Giavazzi and Widgren, 2000). A second drawback of Nice lies in the required efficiency to enlarge the European Union. Any enlargement of the Union needs new negotiations of voting weights and thresholds due to the lack of an algorithm to assign these numbers.

From 2014 on an alternative voting system laid down in the Treaty of Lisbon should replace the current voting system according to the Treaty of Nice. The Treaty of Lisbon was signed in Lisbon in 2007 and has been ratified by the year 2009. According to the Treaty of Lisbon the Council adopts a proposal if the following two criteria ("double majority") are satisfied:

1. At least 55% of the Member States vote in favor is required (15 of 27).
2. The Member States forming the qualified majority represent at least 65% of the overall population of the European Union.

In addition, a blocking minority must include at least four Members, failing which the qualified majority shall be deemed attained. In our further considerations we disregard additional condition because it has no mathematically significant effect. (The political-psychological effect is another issue.)

The same procedure is also contained in the draft constitution of the European Convention. The voting system for the Council according to the Treaty of Lisbon is less complex than the current system of Nice because only two criteria must be satisfied. More precisely, there are no extra weights for each state like appointed in condition 1 of the Nice treaty. Voting weights according to Lisbon are applied directly proportional to the population of each individual Member State and the decision making efficiency is reasonably balanced with a value of 12.83%. Moreover, any further enlargement of the Union is easy practicable because there is an explicit procedure how to calculate the voting weights.

Summarizing, one might get the impression that the voting system according to Lisbon is "better" or "fairer" than the one according to Nice. Analyses have shown that this is not the case: A fair voting system of the European Union Council of Ministers should be based on a compromise between the two principles: "equality of Member States" and, in particular, "equality of citizens". Both the Treaty of Nice and the Treaty of Lisbon violate these two fundamental requirements. We will verify this statement due to concepts of the theory of voting power and its fair distribution.

[1] Data for calculations are used from EUROSTAT: First results of the demographic data collection for 2008 in Europe.

II. The theory of voting power

Voting systems consist of a set of voters and voting rules. The voting rules determine whether a proposal is accepted or not. Frequently, there are voting weights assigned to each voter. Additionally, a decision threshold is defined: a proposal will be passed if the sum of the weights of the members, who vote in favor, meets or exceeds the given threshold.

An important aspect of voting systems is the political power of the members which is also known as *voting power*. Voting power is a mathematical concept which quantifies the influence a voter has on election at the system. Its theory can be traced back to works of Penrose and Banzhaf (Penrose, 1946; Banzhaf, 1965) (see also Shapley and Shubik (1954) for alternative concepts). Assume a member can either vote in favor or against a proposal within a decision. Then he or she has influence on the decision if he or she can turn the voting outcome by changing his or her voting behavior (to make the proposal pass by voting in favor and to make it fail otherwise). In such a situation a member is *decisive*. This *decisiveness* is the basic idea behind voting power (Felsenthal and Machover, 1998).

There are several methods to measure the voting power of a member. These methods are developed in the theory of the indices of political power (see books (Felsenthal and Machover, 1998; Taylor and Pacelli, 2008). Power indices count in different ways in what extend an actor is decisive. One of the most popular ones is the Banzhaf index (Banzhaf, 1965). The Banzhaf index measures the a priori voting power of each member of a voting body without any previous knowledge of a possible correlation between the voters. This simulates the assumption of an a priori indifference to the nature of future legislative proposals.

The Banzhaf index is defined as follows. Assume n is the number of members in a voting system. Consider each possible coalition within a member i. These are 2^{n-1}. Then, the total Banzhaf index of i, TB_i, is equal to the number of coalitions for which i is decisive. The normalized Banzhaf index of i, NB_i, is equal to the probability that i is decisive: $NB_i = TB_i/2^{n-1}$. Finally, the percentage of influence of member i has is given by the Banzhaf index of i, $\beta_i = TB_i/\sum_{j=1}^{n} TB_j$. This quantity expresses the relative share of potential voting power of a member i in the voting body. For example see the distribution of voting power of the European Economic Community of 1958-1972 in (Taylor and Pacelli, 2008). Additionally, it is easy to see that little shifts of quota result in different voting power distributions.

In general the voting power of a member is not equal to the member's voting weight. This is due to the fact that the voting power held by a given country depends not only on its own voting weight but also on the distribution of the weights among the other Member States as well as on the quota. The members of the Council of the EU represent countries of quite different size. Thus, we are faced with the question of what is a fair distribution of voting power and how to construct it. One principle requirement of a voting system for the Council is that each citizen of the EU should have the same influence regardless of his or her home country. To incorporate the idea that the EU is not only a union of people, but also of states one might accept that citizens from small countries get somewhat more power than those from bigger countries, but it should certainly not be the other way around.

To quantify the influence of citizens we need to compute their power in an election in their home country. In fact, it is through local election that citizens influence the Council's decision in an indirect way. A citizen is decisive in an election in his or her country only if the other voters are split in two equal parts if a simple majority of votes in favor is required. The probability that this happens is approximately proportional to the inverse of the square root of the number of citizens (see for examples (Kirsch, 2004; Kirsch, Machover, Słomczynski and Życzkowski, 2004; Słomczynski and Życzkowski 2004, 2006, 2007a)). Thus, if a country has N citizens, then the influence of a citizen on a country's decision is proportional to $1/\sqrt{N}$. If we want to give all citizens the same influence on the Council's decision regardless of their home country we have to assign voting power in the Council proportional to $\sqrt{N}$. This is the square root law of Penrose. Summarizing, a fair distribution of voting power in the case of heterogeneity or indirect voting consists of the Banzhaf indices $\beta_{0i} = \sqrt{N_i}/\sum_{j=1}^{n}\sqrt{N_j}$ for each member i. Here N_i represents the population factor of each state *i*.

Note that this concept of Penrose' square root law is based on the assumption that only a majority, not all citizens, is represented in the Council, as the government of a Member State is representing only the preference of the voters who voted for parties which consequently formed the government (Felsenthal and Machover, 1998).

Accordingly, the Banzhaf index as measuring unit for a fair distribution of voting power of members in the Council seems to be a good choice (Algaba, Bilbao and Fernández, 2004; Baldwin and Widgren, 2003a, 2003b, 2004; Baldwin, Berglöf, Giavazzi and Widgren, 2003; Barberá and Jackson, 2004; Felsenthal and Machover, 2000, 2004a; Leech, 2002; Plechanovova, 2004; Słomczynski and Życzkowski, 2004, 2007a, see also our discussion of the Banzhaf assumptions in the introduction).

The EU is not only a union of citizens but also a union of states. One might argue that it is acceptable that citizens of smaller states get somewhat more power than voters from bigger states, in this way emphasizing the role of the states. But it is certainly unacceptable that citizens of big states get more power than their due share. As we shall see this is exactly the case for the Lisbon procedure.

To design a system with a square root distribution of power an obvious candidate is a system with voting weight equal to the square root of the state's population (this is the second square root law according to Morris (1987) (see Felsenthal and Machover (1998)), thus proportional to $\sqrt{N_i}$. This is neither a necessary nor a sufficient condition, since for example the distribution of voting power depends on the quota (the threshold to make a proposal pass) as well. There exists an optimal quota for which the voting power of any state is proportional to its voting weight (Feix, Lepelley, Merlin and Rouet, 2007; Słomczynski and Życzkowski, 2006, 2007b; Słomczynski, Zastawniak and Życzkowski, 2006). To gain this optimal quota q_0 we use the method of least squares: That choice of q which has its least value of the sum of squared residuals σ_q is our demanded quota. Accordingly, we minimize the value of the term $\sigma_q^2 = \sum_{i=1}^{n}(\beta_{0i} - \beta_{qi})^2$ which depends on the given quota q. σ_q^2 is also called *error rate*. In addition, the value of $(\beta_{0i} - \beta_{qi})/\beta_{0i}$ expresses the relative deviation between demanded and obtained voting power. In the case of a minimal error rate voting weights and voting power equals best possibly. The less the error rate σ_{q_0} the more transparent the system.

Beyond a fair distribution of influence we should consider the effectiveness of a system. Effectiveness is equal to the decision probability the voting body passes a proposal. This quantity is also called the Coleman power of a collectivity act (Coleman, 1971). Assuming that all coalitions are equally likely its value is given by the percentage of the constellation of votes, which make a proposal pass: The higher the effectiveness the easier to change the status quo, the lower the rate the easier to block a change. So, the degree of the effectiveness depends on the given voting rules, in particular the quotas.

Voting systems based on the square root law of Penrose were proposed and discussed many times. One of the best-known proposals is the Jagiellonian Compromise.

III. The Jagiellonian Compromise

In 2004 two polish scientists, Wojciech Słomczynski and Karol Życzkowski, from the Jagiellonian University of Krakow, Poland, presented a voting system for the Council of Ministers of the European Union, the Jagiellonian Compromise (Słomczynski and Życzkowski, 2004, 2006, 2007a). They constructed a system for the Council as follows: The voting weight of each Member State is chosen according to $\sqrt{N_i}$ where N_i is the population factor of the i-th Member State. Then, an optimal quota q is calculated using the methods above. The Jagiellonian Compromise is also known as $P - q\%$ solution due to the work of Penrose. With current population data we gain an optimal quota q_0 = 61.5%[2] with a minimal error rate of 0.00005‰. Our analyses have shown that the maximal relative deviation between β_{0i} and its corresponding $\beta_{q_0 i}$ is about the less value of 0.14%. In addition, the effectiveness value is about 16.43%. For voting weights and voting power see table 2.

A number of advantages arise from the proposed voting system: First of all it is *simple*, because it is based on a single criterion, more precisely, only one condition must be satisfied. It is *neutral* by reason that it cannot a priori favor or handicap any Member State. It is *fair*, because every citizen has the same potential influence on decisions regardless from his home country. It is *transparent* in the case that voting power and voting weight are almost equal. It is *easy extendible*: any new Member State achieves a voting weight proportional to the square root of its population factor. Solely a new optimal quota must be calculated. It is *moderately efficient* because an addition of Member States does not decrease the effectiveness.

The European Union is not only a union of individuals but also a union of states. An additional requirement of a simple majority of Member States ("One State, One Vote") would cause only a moderate deviation from the ideal case (Kirsch, 2004; Kirsch, Machover, Słomczynski and Życzkowski, 2004; Kirsch, Słomczynski and Życzkowski, 2007).

Our computations show that the additional requirement of a simple majority of Member States (in the following denoted by JC+) yields to an error rate of 0.07425‰ (with q = 61.5%). The relative voting power deviation takes a maximum value of 30.64%. Only the effectiveness value almost levels off with 16.08%.

Various efforts have been made to promote this alternative voting system. For examples, in 2004 about 50 scientists supported the Jagiellonian Compromise in an open letter to the governments of the European Union Member States with the title "Scientists for a democratic Europe". Moreover, in the course of the EU summit in Brussels in 2007 the polish mission statement "The square root or death" made the problem the subject of discussions again, unfortunately, without observable success.

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[2] Słomczynski and Życzkowski (2007b) presented a simple mathematical formula to approximate such a quota q_0, in particular
$q_0 = 1/2 \times (1 + \sqrt{N_1 + \cdots + N_n}/(\sqrt{N_1} + \cdots + \sqrt{N_n}))$, which yields to q_0 = 61.57%.

Table 2: Distribution of voting weights and voting power in the Council of Ministers according to the Jagiellonian Compromise – P-61.5 solution

Member State	*Population*	*Population square root*	*Voting weight in %*	*Banzhaf index β_i in %*
Germany	82.221.808	9.067,6242	9,4108	9,3978
France	63.753.140	7.984,5563	8,2867	8,2933
United Kingdom	61.185.981	7.822,1468	8,1181	8,1254
Italy	59.618.114	7.721,2767	8,0135	8,0214
Spain	45.283.259	6.729,2837	6,9839	6,9924
Poland	38.115.641	6.173,7866	6,4074	6,4141
Romania	21.528.627	4.639,8951	4,8155	4,8175
Netherlands	16.404.282	4.050,2200	4,2035	4,2038
Greece	11.214.992	3.348,8792	3,4756	3,4746
Belgium	10.666.866	3.266,0168	3,3896	3,3885
Portugal	10.617.575	3.258,4621	3,3818	3,3807
Czech Republic	10.381.130	3.221,9761	3,3439	3,3428
Hungary	10.045.000	3.169,3848	3,2893	3,2881
Sweden	9.182.927	3.030,3345	3,1450	3,1437
Austria	8.331.930	2.886,5083	2,9957	2,9939
Bulgaria	7.640.238	2.764,098	2,8687	2,8671
Denmark	5.475.791	2.340,0408	2,4286	2,4269
Slovak Republic	5.400.998	2.324,0047	2,4119	2,4100
Finland	5.300.484	2.302,278	2,3894	2,3876
Ireland	4.419.859	2.102,3461	2,1819	2,1801
Lithuania	3.366.357	1.834,7635	1,9042	1,9025
Latvia	2.270.894	1.506,9486	1,5640	1,5623
Slovenia	2.025.866	1.423,3292	1,4772	1,4757
Estonia	1.340.935	1.157,9875	1,2018	1,2003
Cyprus	794.580	891,3922	0,9251	0,9241
Luxembourg	483.799	695,5566	0,7219	0,7210
Malta	410.584	640,7683	0,6650	0,6642
Sum	497.481.657	96353,8647	100,00	100,00

IV. Penrose versus the Treaty of Nice and the Treaty of Lisbon

With the acquired knowledge about voting power and its fair distribution we will have a second look on the two treaties for the European Union. The voting power values according to the Treaty of Nice and the Treaty of Lisbon are shown in figure 1 and in the corresponding table 3.

First of all, both systems are *not simple*, because at least two conditions must be satisfied. The votes, weights and thresholds for the Council laid down in both treaties are not scientific based chosen. They are results of negotiations among the Member States. Thus, both systems are *not objective*. In addition, they violate the square root law, because the voting power is not distributed like Penrose specified.

According to the Treaty of Nice the four biggest states, Germany, France, United Kingdom and Italy, are assigned with too little voting power in comparison with the optimal voting weights β_0. In contrast, especially Spain and Poland achieve too much power also some middle Member States and the four smallest Estonia, Cyprus, Luxembourg and Malta. Here, the maximal relative deviation in voting power is about 73.18%. The corresponding error rate has a value of 0.6052‰.

Figure 1: Deviations in the Banzhaf index between Penrose' optimal quota and the current voting systems for the Council of Ministers

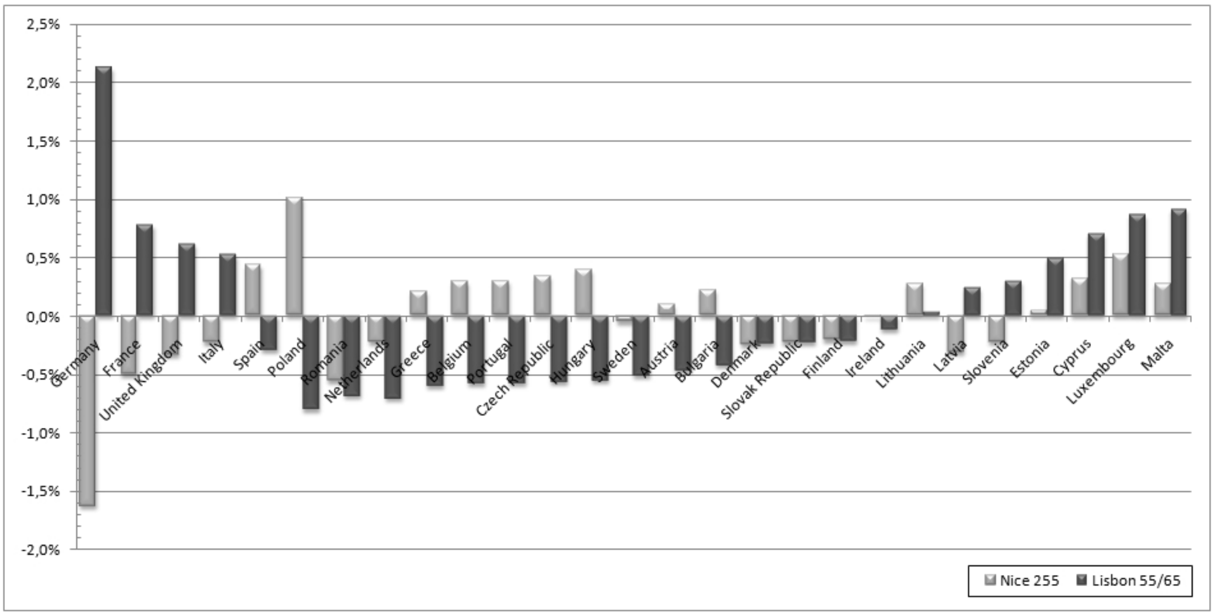

Table 3: Distribution of voting weights and voting power in the Council of Ministers

Member State	*Penrose' optimal* β_{0i} *in %*	*Treaty of Lisbon 55/65* *Population in %*	*Banzhaf index* β_i *in %*	*Treaty of Nice 255* *Votes in the Council*	*Weight in %*	*Banzhaf index* β_i *in %*
Germany	9,4108	16,53	11,5362	29	8,41	7,7828
France	8,2867	12,82	9,0667	29	8,41	7,7828
United Kingdom	8,1181	12,30	8,7322	29	8,41	7,7827
Italy	8,0135	11,98	8,5360	29	8,41	7,7827
Spain	6,9839	9,10	6,6893	27	7,83	7,4199
Poland	6,4074	7,66	5,6050	27	7,83	7,4198
Romania	4,8155	4,33	4,1306	14	4,06	4,2591
Netherlands	4,2035	3,30	3,4952	13	3,77	3,974
Greece	3,4756	2,25	2,8747	12	3,48	3,6843
Belgium	3,3896	2,14	2,8092	12	3,48	3,6843
Portugal	3,3818	2,13	2,8033	12	3,48	3,6843
Czech Republic	3,3439	2,09	2,7750	12	3,48	3,6843
Hungary	3,2893	2,02	2,7349	12	3,48	3,6843
Sweden	3,1450	1,85	2,6321	10	2,90	3,0924
Austria	2,9957	1,67	2,5302	10	2,90	3,0924
Bulgaria	2,8687	1,54	2,4478	10	2,90	3,0924
Denmark	2,4286	1,10	2,1891	7	2,03	2,1809
Slovak Republic	2,4119	1,09	2,1803	7	2,03	2,1809
Finland	2,3894	1,07	2,1681	7	2,03	2,1809
Ireland	2,1819	0,89	2,0625	7	2,03	2,1809
Lithuania	1,9042	0,68	1,9362	7	2,03	2,1809
Latvia	1,5640	0,46	1,8044	4	1,16	1,2502
Slovenia	1,4772	0,41	1,7747	4	1,16	1,2502
Estonia	1,2018	0,27	1,6920	4	1,16	1,2502
Cyprus	0,9251	0,16	1,6260	4	1,16	1,2502
Luxembourg	0,7219	0,10	1,5886	4	1,16	1,2502
Malta	0,6650	0,08	1,5796	3	0,87	0,9422
Sum	100,00	100,00	100,00	345	100,00	100,00

According to the Treaty of Lisbon especially the bigger and the smaller Member States achieve more voting power. Thus the middle States are suffered of this. Here the error rate is very high with 1.2438‰. Also the maximal relative deviation is about 137.53%. Hence, both systems are *not representative*. They do not fulfill the principle “One Person, One Vote”. Moreover, it is quite obvious that voting weight and resulting voting power deviate strongly in comparison with the deviations in obtained and demanded voting power according to the Jagiellonian Compromise. Thus, both systems are *not transparent*. According to the Treaty of Lisbon an enlargement of the European Union is easy practicable in the way that it needs no negotiations among the Member States about voting weights. Contrariwise, the Treaty of Nice always required new debates.

As denoted above the decision making efficiency according to the Treaty of Nice is very low: It is about 2.03%. The effectiveness of the Treaty of Lisbon is about 12.83% which is only somewhat smaller than in the case of the Jagiellonian Compromise with an additional requirement of a simple majority of Member States.

Summarizing, Nice has less power distortions than Lisbon but it is more complex. Unfortunately, the effectiveness of Nice hardly allows changes of the status quo. The current voting rules, weights and thresholds were fully discussed whole nights long without scientific based background. In addition, we know from the European Economic Community of 1958-1973 that little shifts of the quota yield to different voting power distributions. It would be a fortunate coincidence if the current thresholds produce the best possible error rate measured by the ideal voting weights due to Penrose.

V. Improvements

It is our goal now to try to improve, thus optimize, the current Treaty of Nice and the Treaty of Lisbon. Therefore, we fractionally modify the voting rules: The existing voting weights will be unchanged retained and only the several (up to three) thresholds will be shifted. We search for a constellation of quotas such that the resulting Banzhaf indices reach the least possible error rate. Therefore, we have programmed a Java-applet which calculates for several threshold tuples the Banzhaf index values of each Member State, the corresponding error rates, the maximal deviations between demanded and obtained voting power and the effectiveness of the voting systems.

The Treaty of Nice will be investigated with an unchanged simple majority of Member States, thus 14. The quota of the sum of voting weights (currently 255 (= 73.91%)) will be shifted from 190 (= 55.07%) up to 275 (= 79.71%) in integers. For each given quota of voting weights we shift the overall population quota (currently 62%) from 51% up to 85% in steps of 1%.

The Treaty of Lisbon will be analyzed with integer majority of Member States from 15 up to 18 (currently 15). A majority of 15 states relates to a relative majority of 55.55%, 18 of 66.66%. For each given integer majority, we shift the overall population quota (currently 65%) from 51% up to 85% in steps of 0.1%.

Beyond our boundary values the error rate significantly increases. This is due to the fact that a higher quota gives more power to smaller states (a proposal will be passed with almost unanimity) and lower quota more power to bigger states. Furthermore, we want to include the corresponding effectiveness value within our approach of improvement. It is easy to see that the decision making efficiency goes to zero with increasing quota.

In the case of the Treaty of Nice our calculations have produced the threshold tuple (14 / 263 / 80%) due to the least minimal error rate of 0.2286‰. Compared with the Jagiellonian Compromise (JC: 0.00005‰ / JC+: 0.07425‰) Nice's best possible error rate still deviates strongly from the ideal case. This is also indicated by a maximal relative deviation in voting power with 42.9% (JC: 0.14% / JC+: 30.64%). Therewith, the effectiveness is very low with 0.99% (JC: 16.43 / JC+: 16.08%).

In the case of the Treaty of Lisbon our calculations have produced the threshold tuple (17 / 77.5%) due to the least minimal error rate of 0.52118‰. Compared with Nice the best possible error rate of Lisbon is additionally 127% higher. This is also indicated by a maximal relative deviation in voting power with 135.51% (JC: 0.14% / JC+: 30.64%). Concluding, the effectiveness is very low with 2.23% (JC: 16.43% / JC+: 16.08%) thus near to Nice in its current version.

The differences between Penrose's optimal power distribution and the Banzhaf index values of Nice in versions (14 / 255 / 62%) and (14 / 263 / 80%) are shown in figure 2 and the corresponding table 4, the differences of Lisbon in versions (55 (15) / 65%) and (62.5 (17) / 77.5%) are shown in figure 3 and table 5. The development of error rates by shifting the population quota of Nice and Lisbon in current and improved version, respectively, is shown in comparison with JC and JC+ in figure 4 and table 6. The development of the effectiveness in those systems is shown in figure 5 and table 7 (corresponding figures and tables can be found in the Appendix).

In particular, both in the Nice improvement and in the Lisbon improvement, Germany's large deviation between demanded and obtained influence is strongly decreased. According to Nice both Spain and Poland are still assigned too much voting power as specified by Penrose. According to Lisbon middle-size States are still assigned with little voting power and small and big states with too much.

Summarizing, our optimized threshold tuple produce fewer deviations in voting power measured by the least possible error rate than the current versions. Nevertheless, each quota constellation produces a *significant* deviation to Penrose's ideal case. For comparison, we have listed several threshold tuple with fixed voting weight and state quota and optimized population quota such with minimal error rate in table 11, 12 and 13 (tables can be found in the Appendix). In addition, the values of the related maximal relative deviation in voting power show that the resulted systems are neither transparent nor representative. Moreover, these improvements due to the error rates lead to a very low effectiveness. Thus, in such voting systems it would be easy to block proposals.

In addition, similar to our attempts to improve the treaties of Nice and Lisbon, we can further improve the Jagiellonian Compromise with the requirement of a simple majority of Member States. Observing the least squares we gain a new optimal quota of q_0^* = 64.7%. Here, the error rate takes its minimum value of 0.03275‰ (JC: 0.00005‰ / JC+61.5: 0.07425‰). This is only the half of the error rate value than with an unchanged quota. The maximal relative deviation is only about 11.68% (JC: 0.14% / JC+61.5: 30.64%). This is nearly one third in comparison with q = 61.5%. However, the effectiveness decreases on the lower value of 10.39% (JC: 16.43% / JC+61.5: 16.08%). In terms of as best as possible fair distribution of voting power the quota q_0^* = 64.7% should be applied. Figure 6 and the corresponding table 8 show that the voting power of the JC+ 64.7-solution is better approximated to Penrose's β_0 than the JC+ 61.5-solution.

Conclusions

Several publications have shown that both voting systems for the Council of Ministers of the European Union according to the Treaty of Nice and the Treaty of Lisbon deviate strongly from Penrose's solution of a fair distribution of voting power in such a voting body. In this article we tried to improve these treaties with respect to such a fair distribution. To do so we modified the voting rules by keeping the voting weights and only shifting the thresholds. This procedure results only in a modest improvement of the system. Even with optimal quota both systems deviate strongly from a fair distribution of power. In particular, the decision making efficiency of Nice is more or less than 2%. According to this decisions can be made with almost unanimity. So, it is easy to block proposals. On the other hand, Lisbon has a moderate effectiveness with about 12%, but the deviations in voting power are even further way from the ideal case according to Penrose. Thus, both the Treaty of Nice and the Treaty of Lisbon turn out to be invariably suboptimal.

As a consequence the voting system for the Council has to be changed in a more fundamental way than merely adjusting quota. It seems to us that the Jagiellonian Compromise with an additional requirement of a simple majority of Member States is a good basis for a new voting system.

Appendix

Figure 2: Deviations in the Banzhaf index between Penrose's optimal voting power and the Treaty of Nice in versions 255 and 263

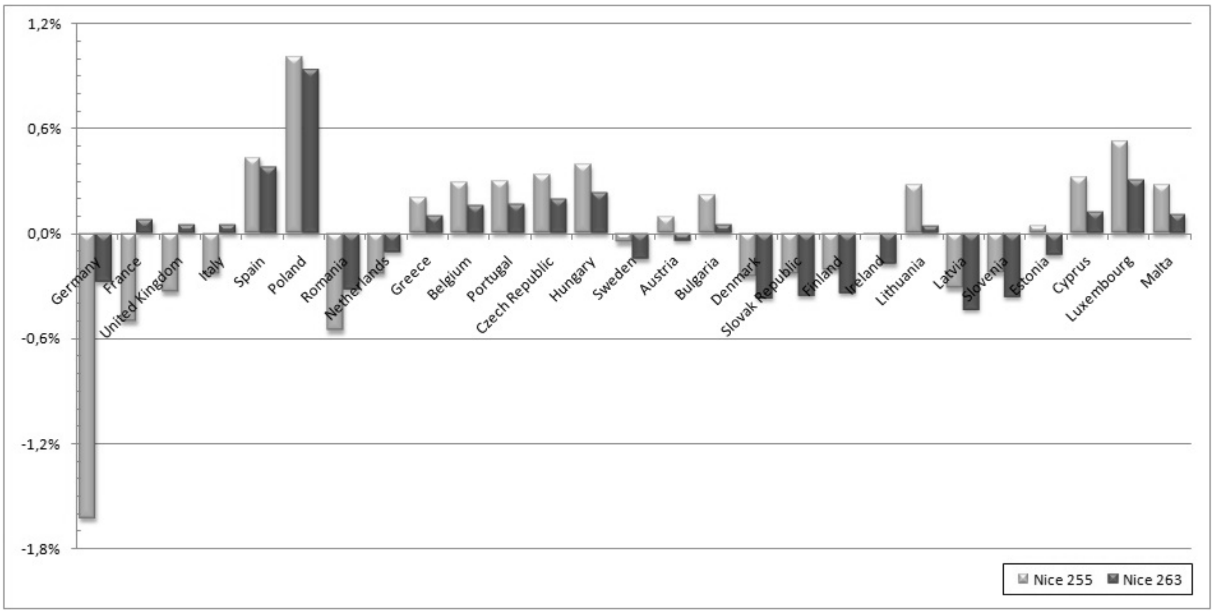

Table 4: Deviations in the Banzhaf index between Penrose's optimal voting power and the Treaty of Nice in versions 255 and 263

Member State	*Penrose' optimal* β_{0i} *in %*	*Nice 255 Banzhaf index* β_i *in %*	*Differences in* β_i *in ‰ between P. and Nice 255*	*Nice 263 Banzhaf index* β_i *in %*	*Differences in* β_i *in ‰ between P. and Nice 263*
Germany	9,4108	7,7828	-16,2793	9,1345	-2,7627
France	8,2867	7,7828	-5,0395	8,3658	0,7907
United Kingdom	8,1181	7,7827	-3,3540	8,1708	0,5269
Italy	8,0135	7,7827	-2,3074	8,0686	0,5515
Spain	6,9839	7,4199	4,3596	7,3666	3,8263
Poland	6,4074	7,4198	10,1237	7,3468	9,3940
Romania	4,8155	4,2591	-5,5639	4,4966	-3,1889
Netherlands	4,2035	3,9740	-2,2953	4,0966	-1,0693
Greece	3,4756	3,6843	2,0871	3,5808	1,0517
Belgium	3,3896	3,6843	2,9470	3,5554	1,6582
Portugal	3,3818	3,6843	3,0254	3,5533	1,7153
Czech Republic	3,3439	3,6843	3,4041	3,5425	1,9857
Hungary	3,2893	3,6843	3,9499	3,5276	2,3825
Sweden	3,1450	3,0924	-0,5256	2,9989	-1,4613
Austria	2,9957	3,0924	0,9671	2,9575	-0,3821
Bulgaria	2,8687	3,0924	2,2376	2,9254	0,5670
Denmark	2,4286	2,1809	-2,4773	2,0607	-3,6790
Slovak Republic	2,4119	2,1809	-2,3109	2,0568	-3,5511
Finland	2,3894	2,1809	-2,0854	2,0514	-3,3795
Ireland	2,1819	2,1809	-0,0104	2,0058	-1,7607
Lithuania	1,9042	2,1809	2,7666	1,9520	0,4781
Latvia	1,5640	1,2502	-3,1380	1,1305	-4,3346
Slovenia	1,4772	1,2502	-2,2701	1,1167	-3,6050
Estonia	1,2018	1,2502	0,4836	1,0795	-1,2233
Cyprus	0,9251	1,2502	3,2505	1,0488	1,2369
Luxembourg	0,7219	1,2502	5,2829	1,0316	3,0971
Malta	0,6650	0,9422	2,7720	0,7786	1,1354
Sum	100,00	100,00	0,0000	100,00	0,0000

Figure 3: Deviations in the Banzhaf index between Penrose's optimal voting power and the Treaty of Lisbon in versions 55/65 and 62.5/77.5

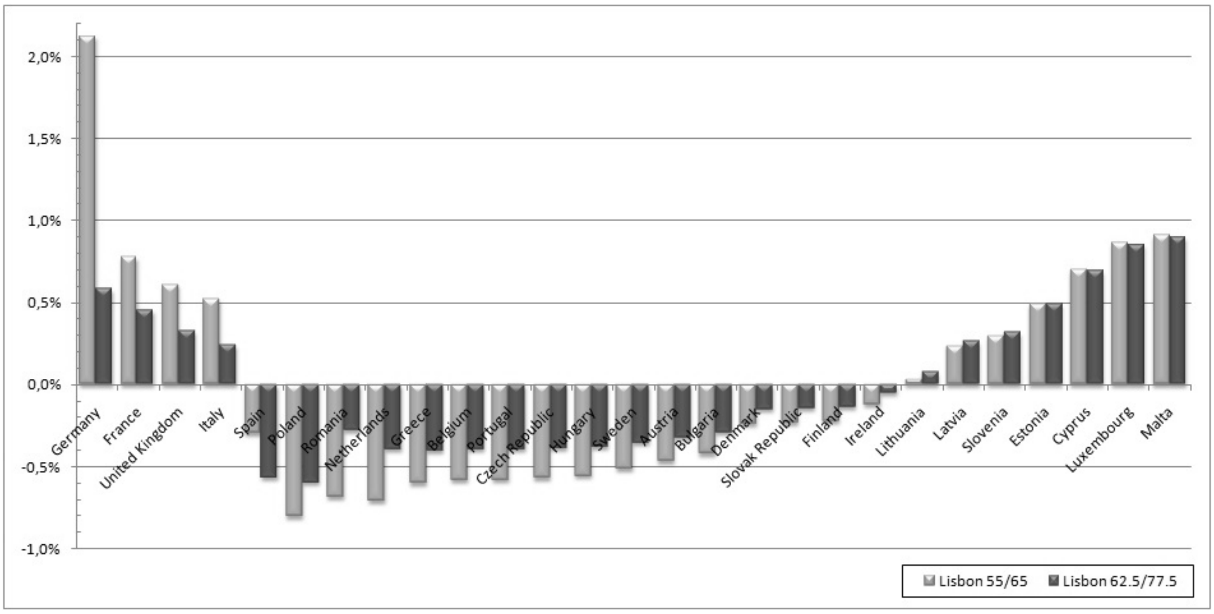

Table 5: Deviations in the Banzhaf index between Penrose's optimal voting power and the Treaty of Lisbon in versions 55/65 and 62.5/77.5

Member State	*Penrose' optimal β_{0i} in %*	*Lisbon 55/65 β_i in %*	*Differences in β_i between P. and L.55/65 in ‰*	*Lisbon 62.5/77.5 β_i in %*	*Differences in β_i between P. and L.62.5/77.5 in ‰*
Germany	9,4108	11,5362	21,2545	9,9951	5,8431
France	8,2867	9,0667	7,7997	8,7436	4,5688
United Kingdom	8,1181	8,7322	6,1403	8,4470	3,2882
Italy	8,0135	8,5360	5,2253	8,2558	2,4233
Spain	6,9839	6,6893	-2,9467	6,4190	-5,6488
Poland	6,4074	5,6050	-8,0237	5,8117	-5,9570
Romania	4,8155	4,1306	-6,8492	4,5381	-2,7733
Netherlands	4,2035	3,4952	-7,0833	3,8104	-3,9308
Greece	3,4756	2,8747	-6,0085	3,0730	-4,0263
Belgium	3,3896	2,8092	-5,8036	2,9969	-3,9273
Portugal	3,3818	2,8033	-5,7842	2,9899	-3,9183
Czech Republic	3,3439	2,7750	-5,6887	2,9574	-3,8654
Hungary	3,2893	2,7349	-5,5445	2,9113	-3,7805
Sweden	3,1450	2,6321	-5,1287	2,7915	-3,5350
Austria	2,9957	2,5302	-4,6551	2,6751	-3,2068
Bulgaria	2,8687	2,4478	-4,2087	2,5786	-2,9010
Denmark	2,4286	2,1891	-2,3946	2,2788	-1,4983
Slovak Republic	2,4119	2,1803	-2,3166	2,2684	-1,4353
Finland	2,3894	2,1681	-2,2126	2,2545	-1,3488
Ireland	2,1819	2,0625	-1,1938	2,1322	-0,4973
Lithuania	1,9042	1,9362	0,3200	1,9843	0,8016
Latvia	1,5640	1,8044	2,4041	1,8303	2,6634
Slovenia	1,4772	1,7747	2,9755	1,7959	3,1866
Estonia	1,2018	1,6920	4,9019	1,6976	4,9579
Cyprus	0,9251	1,6260	7,0083	1,6212	6,9608
Luxembourg	0,7219	1,5886	8,6671	1,5764	8,5453
Malta	0,6650	1,5796	9,1461	1,5662	9,0114
Sum	100,00	100,00	0,0000	100,00	0,0000

Figure 4: Least Squares – Error rates of the Jagiellonian Compromise, the Treaty of Nice and the Treaty of Lisbon

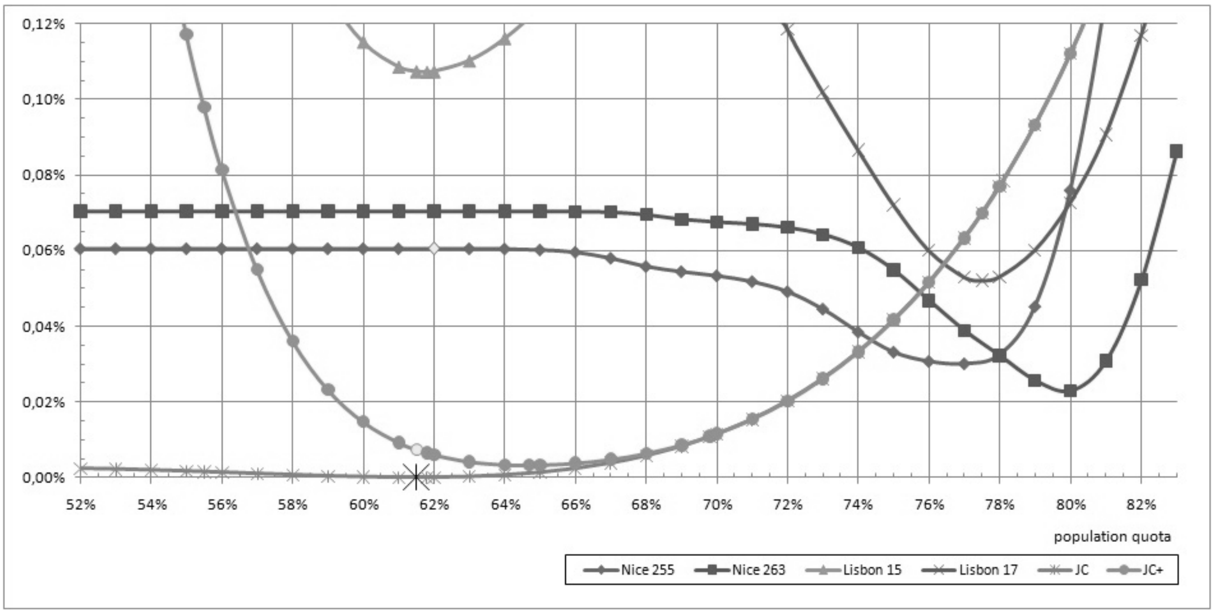

Figure 5: Effectiveness – Decision probabilities of the Jagiellonian Compromise, the Treaty of Nice and the Treaty of Lisbon

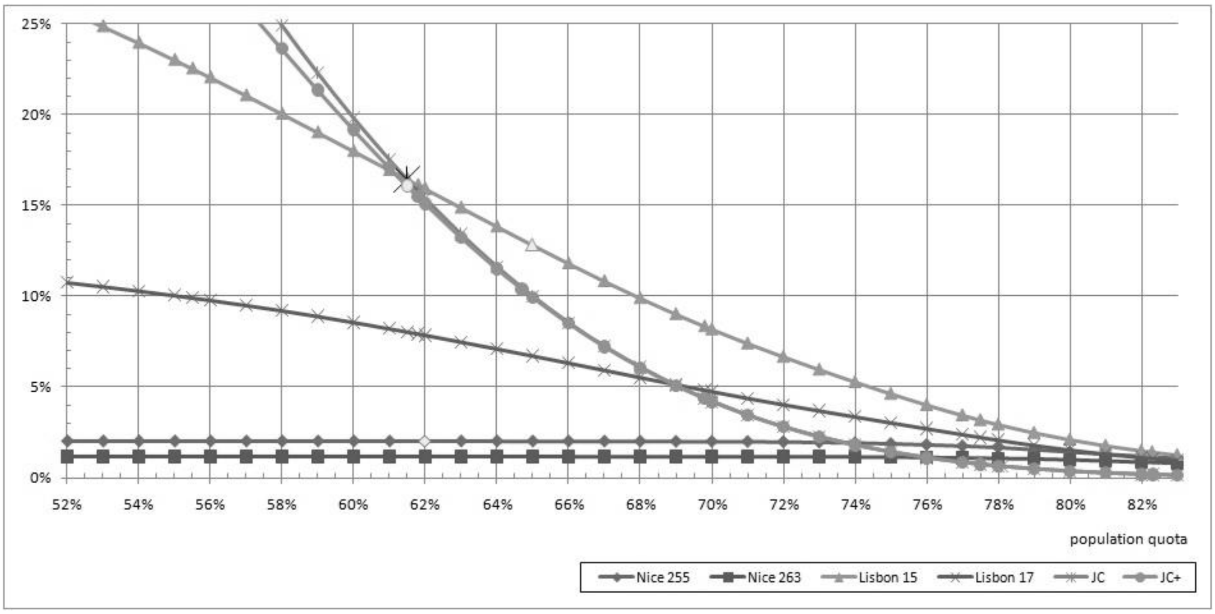

Table 6: Least Squares – Error rates of the Jagiellonian Compromise, the Treaty of Nice and the Treaty of Lisbon

Population quota in %	*JC in ‰*	*JC+ in ‰*	*Nice 14/255 in ‰*	*Nice 14/263 in ‰*	*Lisbon 15 in ‰*	*Lisbon 17 in ‰*
51,0	0,0246	3,7739	0,6053	0,7034	4,4853	11,1212
52,0	0,0236	2,9351	0,6053	0,7034	3,9337	10,5721
53,0	0,0218	2,2228	0,6053	0,7034	3,4165	10,0106
54,0	0,0195	1,6370	0,6053	0,7034	2,9336	9,4438
55,0	0,0166	1,1711	0,6053	0,7034	2,4890	8,8730
55,5	0,0150	0,9796	0,6053	0,7034	2,2834	8,5862
56,0	0,0134	0,8135	0,6053	0,7034	2,0907	8,2984
57,0	0,0101	0,5493	0,6053	0,7034	1,7499	7,7204
58,0	0,0068	0,3614	0,6053	0,7034	1,4773	7,1403
59,0	0,0039	0,2326	0,6052	0,7034	1,2781	6,5633
60,0	0,0016	0,1472	0,6052	0,7034	1,1502	5,9998
61,0	0,0002	0,0928	0,6052	0,7034	1,0856	5,4602
61,5	0,00005	0,0743	0,6052	0,7034	1,0738	5,2010
61,8	0,0001	0,0652	0,6052	0,7034	1,0722	5,0489
62,0	0,0003	0,0601	0,6052	0,7034	1,0735	4,9492
63,0	0,0021	0,0420	0,6051	0,7034	1,1014	4,4663
64,0	0,0061	0,0340	0,6047	0,7034	1,1595	4,0072
64,7	0,0106	0,0327	-	-	1,2158	3,6962
65,0	0,0130	0,0331	0,6020	0,7034	1,2438	3,5647
66,0	0,0234	0,0380	0,5960	0,7029	1,3569	3,1337
67,0	0,0378	0,0480	0,5802	0,7012	1,5062	2,7165
68,0	0,0571	0,0637	0,5583	0,6945	1,7018	2,3217
69,0	0,0823	0,0861	0,5446	0,6824	1,9503	1,9597
69,8	0,1071	0,1092	-	-	2,1811	1,7030
70,0	0,1142	0,1160	0,5337	0,6750	2,2416	1,6442
71,0	0,1537	0,1543	0,5184	0,6701	2,5418	1,3863
72,0	0,2024	0,2025	0,4916	0,6606	2,8041	1,1846
73,0	0,2615	0,2615	0,4459	0,6424	2,9967	1,0188
74,0	0,3322	0,3322	0,3853	0,6077	3,1255	0,8661
75,0	0,4162	0,4162	0,3323	0,5480	3,2255	0,7211
76,0	0,5158	0,5158	0,3078	0,4677	3,3447	0,6003
77,0	0,6334	0,6334	0,3016	0,3881	3,5214	0,5303
77,5	0,6990	0,6990	-	-	3,6413	0,5212
78,0	0,7706	0,7706	0,3254	0,3218	3,7857	0,5309
78,1	0,7853	0,7853	-	-	3,8183	0,5351
79,0	0,9317	0,9317	0,4523	0,2560	4,1650	0,6014
80,0	1,1199	1,1199	0,7590	0,2286	4,6773	0,7268
81,0	1,3393	1,3393	1,2718	0,3081	5,2987	0,9067
82,0	1,5936	1,5936	2,0076	0,5219	5,9039	1,1673
82,3	1,6767	1,6767	-	-	6,0535	1,2638
83,0	1,8881	1,8881	2,9604	0,8624	6,2830	1,5094

Table 7: Effectiveness – Decision probabilities of the Jagiellonian Compromise, the Treaty of Nice and the Treaty of Lisbon

Population quota in %	*JC in %*	*JC+ in %*	*Nice 14/255 in %*	*Nice 14/263 in %*	*Lisbon 15 in %*	*Lisbon17 in %*
51,0	46,6403	38,8799	2,0256	1,1687	26,5973	10,9617
52,0	43,3031	36,9620	2,0256	1,1687	25,7505	10,7548
53,0	40,0111	34,9123	2,0256	1,1687	24,8721	10,5310
54,0	36,7852	32,7566	2,0256	1,1687	23,9631	10,2936
55,0	33,6456	30,5219	2,0256	1,1687	23,0254	10,0430
55,5	32,1142	29,3835	2,0256	1,1687	22,5462	9,9127
56,0	30,6111	28,2359	2,0256	1,1687	22,0603	9,7789
57,0	27,6982	25,9280	2,0256	1,1687	21,0693	9,5011
58,0	24,9215	23,6295	2,0256	1,1687	20,0569	9,2073
59,0	22,2929	21,3709	2,0256	1,1687	19,0314	8,8950
60,0	19,8220	19,1805	2,0256	1,1687	17,9997	8,5635
61,0	17,5167	17,0832	2,0256	1,1687	16,9644	8,2161
61,5	16,4271	16,0750	2,0256	1,1687	16,4458	8,0374
61,8	15,7940	15,4845	2,0256	1,1687	16,1346	7,9287
62,0	15,3806	15,0973	2,0256	1,1687	15,9271	7,8558
63,0	13,4162	13,2372	2,0256	1,1687	14,8894	7,4848
64,0	11,6235	11,5142	2,0255	1,1687	13,8541	7,1051
64,7	10,4692	10,3937	-	-	13,1333	6,8353
65,0	9,9996	9,9354	2,0251	1,1687	12,8261	6,7188
66,0	8,5399	8,5038	2,0238	1,1686	11,8163	6,3264
67,0	7,2386	7,2194	2,0202	1,1685	10,8386	5,9287
68,0	6,0877	6,0783	2,0145	1,1677	9,9047	5,5301
69,0	5,0784	5,0744	2,0106	1,1664	9,0221	5,1365
69,8	4,3661	4,3643	-	-	8,3545	4,8289
70,0	4,2006	4,1992	2,0064	1,1655	8,1929	4,7533
71,0	3,4440	3,4437	1,9989	1,1648	7,4139	4,3835
72,0	2,7981	2,7981	1,9852	1,1631	6,6764	4,0293
73,0	2,2514	2,2514	1,9656	1,1599	5,9708	3,6895
74,0	1,7933	1,7933	1,9346	1,1543	5,2922	3,3589
75,0	1,4135	1,4135	1,8873	1,1437	4,6417	3,0321
76,0	1,1018	1,1018	1,8282	1,1247	4,0264	2,7068
77,0	0,8487	0,8487	1,7621	1,0985	3,4572	2,3857
77,5	0,7415	0,7415	-	-	3,1935	2,2288
78,0	0,6456	0,6456	1,6775	1,0713	2,9450	2,0760
78,1	0,6278	0,6278	-	-	2,8973	2,0460
79,0	0,4848	0,4848	1,5625	1,0375	2,4965	1,7873
80,0	0,3590	0,3590	1,4285	0,9876	2,1119	1,5293
81,0	0,2617	0,2617	1,3074	0,9200	1,7855	1,3067
82,0	0,1877	0,1877	1,1957	0,8577	1,5080	1,1189
82,3	0,1694	0,1694	-	-	1,4326	1,0685
83,0	0,1324	0,1324	1,0831	0,8045	1,2681	0,9606

Figure 6: Deviations in the Banzhaf index between Penrose's optimal voting power and JC+ in versions 61.5 and 64.7

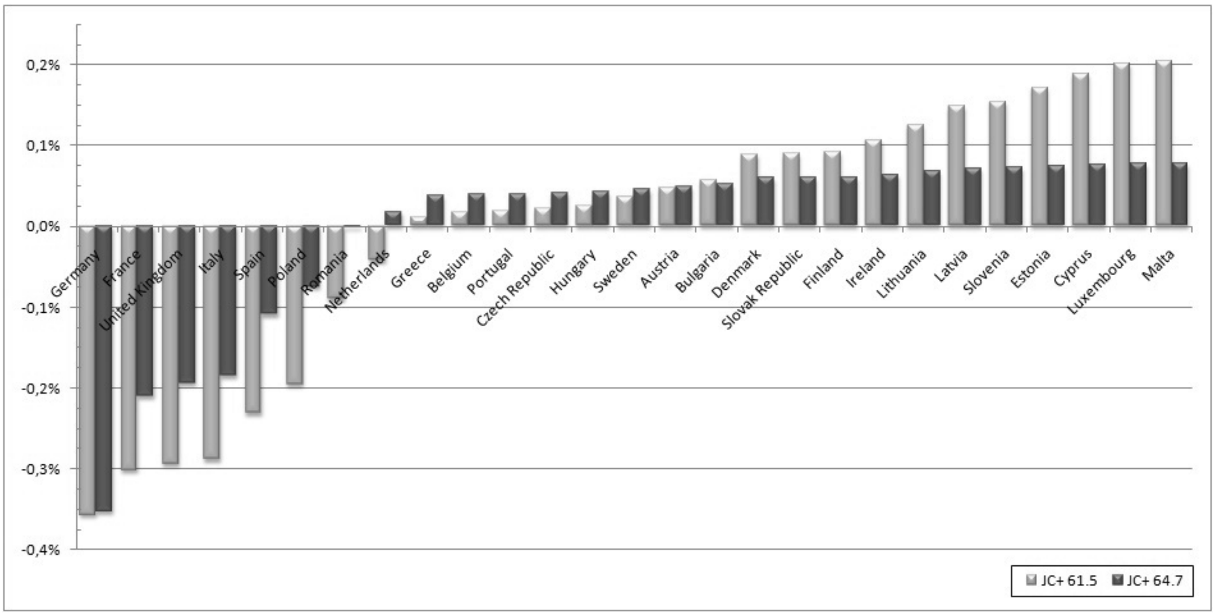

Table 8: Deviations in the Banzhaf index between Penrose's optimal voting power and JC+ in versions 61.5 and 64.7

Member State	*Penrose' optimal β_i in %*	*JC+ 61.5 Banzhaf index β_i in %*	*Differences in β_i in ‰ between P. and JC+ 61.5*	*JC+ 64.7 Banzhaf index β_i in %*	*Differences in β_i in ‰ between P. and JC+ 64.7*
Germany	9,4108	9,0537	-3,5701	9,0580	-3,5271
France	8,2867	7,9849	-3,0183	8,0762	-2,1054
United Kingdom	8,1181	7,8240	-2,9412	7,9240	-1,9417
Italy	8,0135	7,7248	-2,8865	7,8283	-1,8517
Spain	6,9839	6,7525	-2,3140	6,8743	-1,0960
Poland	6,4074	6,2115	-1,9590	6,3321	-0,7535
Romania	4,8155	4,7246	-0,9088	4,8138	-0,0167
Netherlands	4,2035	4,1601	-0,4335	4,2214	0,1787
Greece	3,4756	3,4874	0,1175	3,5135	0,3786
Belgium	3,3896	3,4077	0,1814	3,4298	0,4021
Portugal	3,3818	3,4005	0,1872	3,4221	0,4029
Czech Republic	3,3439	3,3655	0,2156	3,3850	0,4112
Hungary	3,2893	3,3149	0,2554	3,3319	0,4256
Sweden	3,1450	3,1815	0,3647	3,1907	0,4570
Austria	2,9957	3,0428	0,4709	3,0449	0,4921
Bulgaria	2,8687	2,9254	0,5673	2,9204	0,5172
Denmark	2,4286	2,5173	0,8873	2,4886	0,6005
Slovak Republic	2,4119	2,5017	0,8973	2,4721	0,6012
Finland	2,3894	2,4808	0,9142	2,4500	0,6061
Ireland	2,1819	2,2880	1,0611	2,2458	0,6388
Lithuania	1,9042	2,0296	1,2543	1,9720	0,6779
Latvia	1,5640	1,7120	1,4800	1,6355	0,7157
Slovenia	1,4772	1,6310	1,5379	1,5496	0,7237
Estonia	1,2018	1,3731	1,7127	1,2765	0,7466
Cyprus	0,9251	1,1135	1,8841	1,0017	0,7663
Luxembourg	0,7219	0,9223	2,0044	0,7992	0,7730
Malta	0,6650	0,8688	2,0379	0,7427	0,7769
Sum	100,00	100,00	0,0000	100,00	0,0000

Table 9: Optimal threshold values for the Jagiellonian Compromise

Quota of States	*Quota of Population*	*Sum of square residuals in ‰*	*maximal relative deviation in %*	*Effectiveness in %*
-	0,615	0,00005	0,14	16,43
14	0,615	0,07425	30,64	16,08
14	0,647	0,03275	11,68	10,39
optimal values		0,00005	0,14	16,43

Table 10: Optimal threshold values for the Treaty of Lisbon

Quota of States	*Quota of Population*	*Sum of square residuals in‰*	*maximal relative deviation in %*	*Effectiveness in %*
15	0,618	1,07222	180,26	16,13
15	0,650	1,24384	137,53	12,83
15	0,675	1,59748	106,62	10,36
16	0,698	0,83352	161,88	6,74
17	0,775	0,52118	135,51	2,23
18	0,823	0,75088	163,24	0,78
optimal values		0,52118	116,39	28,33

Table 11: Optimal threshold values for the Treaty of Nice

Quota of States	*Quota of Weights*	*Quota of Population*	*Sum of square residuals in ‰*	*maximal relative deviation in %*	*Effectiveness in %*
14	190	0,54	0,5192	142,08	27,74
14	195	0,55	0,3966	118,83	25,27
14	200	0,56	0,3388	95,61	22,11
14	205	0,58	0,3367	83,65	19,47
14	210	0,59	0,3646	75,48	16,85
14	215	0,60	0,3913	68,49	14,33
14	220	0,61	0,4099	63,33	11,97
14	225	0,63	0,4096	57,66	9,65
14	230	0,64	0,4117	58,76	7,79
14	235	0,66	0,4143	58,91	6,11
14	240	0,67	0,4188	61,38	4,77
14	245	0,72	0,4077	49,53	3,45
14	250	0,74	0,3469	49,47	2,55
14	255	0,62	0,6052	73,18	2,03
14	255	0,77	0,3016	45,76	1,76
14	258	0,62	0,6373	74,58	1,66
14	258	0,78	0,2620	46,73	1,44
14	259	0,79	0,2515	40,34	1,30
14	260	0,79	0,2391	43,76	1,23
14	261	0,79	0,2373	47,85	1,17
14	262	0,80	0,2372	39,38	1,04
14	263	0,80	0,2286	42,90	0,99
14	264	0,80	0,2318	47,58	0,93
14	265	0,80	0,2445	51,09	0,88
14	270	0,82	0,2762	49,75	0,58
14	275	0,84	0,3587	61,37	0,38
optimal values			0,2286	39,3825	27,74

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