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Wolfgang Leininger and Hamed M. Moghadam[1]

Evolutionary Stability in Asymmetric Oligopoly

A Non-Walrasian Result

Abstract

It is a very well-known result that in terms of evolutionary stability the long-run outcome of a Cournot oligopoly market with finitely many firms approaches the perfectly competitive Walrasian market outcome (Vega-Redondo, 1997). However, in this paper we show that an asymmetric structure in the cost functions of firms may change the long-run outcome. Contrary to Tanaka (1999) we show that the evolutionarily stable price in an asymmetric Cournot oligopoly needs not equal the marginal cost, it may rather equal a weighted average of (different) marginal cost. We apply a symmetrization technique in order to transform the game with asymmetric firms into a symmetric oligopoly game and then extend Schaffer's definition (1988) of a finite population ESS (FPESS) to this setup. Moreover, we show that the FPESS in this game represents a stochastically stable state of an evolutionary process of imitation with experimentation.

JEL Classification: C72, C73, D43, L13

Keywords: Cournot oligopoly; asymmetry; finite population evolutionary stable strategy; stochastic stability

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1. Introduction

In a 1999 paper (*Economic Theory*, 14, 705-715), Tanaka considers an evolutionary game theoretic model for an asymmetric oligopolistic industry composed of two groups of firms, in which the output choices of low cost firms and high cost firms jointly determine market price of a homogeneous good. His main result shows that the finite population evolutionarily stable strategies (*FPESS*) of low cost and high cost firms are equal to the respective competitive (Walrasian) outputs in the two groups. Moreover, this static outcome is also the long-run equilibrium of a dynamic evolutionary model based on imitation and stochastic mutation. Tanaka`s result is read as a generalization of a celebrated result of Vega-Redondo (1997), namely that FPESS in a *symmetric* oligopoly of identical firms selects the Walrasian outcome (and not the Cournot-Nash equilibrium), to asymmetric oligopoly.

The purpose of the present paper is to reexamine this issue and give a critical account of Tanaka`s unintuitive result. More precisely, we will argue that Tanaka applies the finite population evolutionary argument in a particular parameterized way, which amounts to an evolutionary analysis of the two groups *in separation.* Otherwise it would not be possible to apply a symmetric solution concept like *(FP)ESS* to the (full) asymmetric model. A standard procedure of evolutionary game theory (originating with Selten (1980)) when applying ESS to a multi-population model is to *symmetrize* the given asymmetric game and then apply the evolutionary solution concept to the symmetrized game. We will show that if one follows this prescription in a simple alternative model, then the marginal cost pricing result will not be confirmed.

The paper is organized as follows: in the next section we present Tanaka`s model as well as our approach to evolutionary stability in asymmetric finite populations. Section 3 contains the analysis following from our approach. Section 4 considers dynamics explicitly and derives evolutionary equilibrium in the long run. Section 5 concludes.

2. Evolutionary stability in asymmetric oligopoly

In our model we employ an alternative conceptualization of evolutionarily stable strategies to asymmetric oligopoly markets. Recall that Tanaka applies the concept of a finite population evolutionarily stable strategy (FPESS) by Schaffer (1988), in which agents in economic and social environment adhere to *relative payoff* maximizing rather than absolute payoff maximizing behavior. Tanaka (1999) does so in the following way: each group of either low cost firms or high cost firms performs its relative payoff maximization under the assumption that the other group`s behavior is *given.* The parametric treatment of the, respectively, other group`s behavior allows for the symmetric treatment of firms in each single group; a prerequisite for the use of symmetric solution concept ESS. However, this parameterization also means that a mutation of firm strategies in the evolutionary process cannot occur in both groups simultaneously; in fact, the evolution of behavior in one group is *not* influenced by the evolution of behavior in the other group as the two evolutionary processes are studied in isolation of each other. And it is this fact that leads to the conclusion that the equilibrium price resulting from FPESS of the good must

equal the marginal costs of low cost firms as well as high cost firms; accordingly, high cost firms supply a lower amount of the good than low cost firms in order to equalize marginal cost.

In contrast, we use Selten`s (1980) approach to construct a symmetric monomorphic population game out of an asymmetric multi-population game. It is assumed that firms have one of the following two roles: either they become a member of the low cost group or they become a member of the high cost group. Accordingly, a strategy for a firm assigns behavior to its potential role as a high cost firm as well as the potential role of a low cost firm. Nature assigns these roles to each of the $N = n_1 + n_2$ firms with a probability function proportioned to the number of firms in each group. Then Schaffer`s (1988) definition of finite population ESS is applied onto these extended strategies in the now symmetric game. Note that this - in contrast to Tanaka (1999) - also amounts to a kind of "production uncertainty" as a firm`s cost function might be high or low in any new play of the game.

Since a mutant strategy now may contain different behavior for *both* roles our result will differ from the one derived in Tanaka (1999). The analysis in Tanaka`s paper does not allow for the possibility of instantaneous strategy mutation for both types of firms while, with this approach, mutation is allowed to take place simultaneously for both types.

The result by Vega-Redondo (1997) that in a symmetric quantity competition game with a homogenous product imitation of successful strategies leads to the competitive equilibrium outcome, has been challenged on several grounds. Alós-Ferrer (2004, 2008) and Alós-Ferrer and Shi (2012) introduce memory capacity into the learning model and show that once firms remember more than one period, all quantities between the Walrasian (perfectly competitive) and the Cournot–Nash one become stochastically stable. Apesteguia et al. (2010) also show that Vega-Redondo's result is not robust to the slightest asymmetry in fixed costs in the theoretical part of their paper; however they cannot find any significant difference in the experimental part between symmetric and asymmetric treatments in terms of quantities.

3. The Model

Consider a game between N firms, which are divided into two groups, low cost firms and high cost firms. All firms produce a homogeneous good. The number of firms in the low cost group is n_1, and the number of firms in the high cost group is n_2 with $n_1 + n_2 = N$. Low cost firm i produces output level x_i and high cost firm i produces output level y_i. The price of the good is denoted by p.

The inverse demand function is as follows

$$P(X + Y) \quad where \; X = \sum_{i=1}^{n_1} x_i \quad and \quad Y = \sum_{i=1}^{n_2} y_i$$

$P(\,.\,)$ is decreasing in its argument $X + Y$.

The cost functions for the low cost firms and for the high cost firms are respectively given by:

$$C_l(x_i) = c(x_i) \quad and \quad C_h(y_i) = \gamma \, c(y_i) \, where \, \gamma > 1.$$

$c(.)$ represents an increasing, twice differentiable and convex function.

Consequently, payoff functions for low cost firms and high cost firms are as follows,

$$\pi_i^l(x_i, X_{-i} + Y) = P(X + Y)x_i - c(x_i), \qquad i = 1,2, \dots, n_1$$

$$\pi_i^h(y_i, X + Y_{-i}) = P(X + Y)y_i - \gamma c(y_i), \qquad i = 1,2, \dots, n_2$$

Where $X_{-i} = \sum_{j \neq i}^{n_1} x_j$ and $Y_{-i} = \sum_{j \neq i}^{n_2} y_j$.

Tanaka (1999) defines FPESS for low cost firms and high cost firms, respectively, as follows:

x^* and y^* are a $FPESS$ if the following two conditions (1)and (2) hold

$$\pi_i^l(x^*, x_m, X_{-i-j}^* + Y) > \pi_j^l(x_m, X_{-j}^* + Y), \forall \, x_m \neq x^* \, and \, all \, i \neq j \, where \, x_j = x_m,$$
$$and \, x_i = x^* \, for \, all \, i \neq j \qquad (1)$$

This requirement means that no mutant strategy of a low cost firm can yield higher profits than x^* *given the total output of the high cost firms Y*. Moreover,

$$\pi_i^h(y^*, y_m, X + Y_{-i-j}^*) > \pi_j^h(y_m, X + Y_{-j}^*), \forall \, y_m \neq y^* \, and \, all \, i \neq j \, where \, y_j = y_m,$$
$$and \, y_i = y^* \, for \, all \, i \neq j \qquad (2)$$

This requirement means that no mutant strategy for a high cost firm can yield higher profits than y^* *given the total output of the low cost firms X*.

Hence Tanaka`s model set-up is not really playing the field among N firms; it is rather playing the field among the low cost firms exclusively *and* the high cost firms exclusively and then coupling both results in a consistent way.

FPESS is then derived in accordance with Schaffer (1988) as the solutions of the following two problems independently,

$$x^* = \underset{x_m}{\operatorname{argmax}} \varphi^l = \pi_j^l(x_m, X_{-j}^* + Y) - \pi_i^l(x^*, x_m, X_{-i-j}^* + Y) \qquad (3)$$

$$y^* = \underset{y_m}{\operatorname{argmax}} \varphi^h = \pi_j^h(y_m, X + Y_{-j}^*) - \pi_i^h(y^*, y_m, X + Y_{-i-j}^*) \qquad (4)$$

Values of $x^*(Y)$ and $y^*(X)$ are obtained given the total output of high cost firms and total output of low cost firms respectively. Hence a low cost firm only evaluates own payoff relative to the

payoffs of other low cost firms; and a high cost firm only evaluates own payoff relative to payoffs of other high cost firms. Effectively, this approach postulates two separate independent mutation processes in the two groups. And this accounts for the marginal cost pricing result.

In our alternative set-up the N firms contemplate behavior before they know their assigned roles of high or low cost firm, hence they condition their behavior on roles. Nature then assigns these roles to each of the N $= n_1 + n_2$ firms, so firm $i \in \{1, \dots, N\}$ is chosen by nature with probability $\frac{n_1}{N}$ as a low cost firm and with probability $\frac{n_2}{N}$ as a high cost firm. Firms then carry out their strategies based on the assigned role. This assignment process of roles takes place before any play of the game anew. From this ex-ante point of view the game played in role-contingent strategies is symmetric (see Selten (1980) and for a generalization Balkenborg (1994)). Therefore one can define the expected payoff function for each firm in this monomorphic population game as

$$E\,\pi_i\big((x_i, y_i), [\boldsymbol{x}, \boldsymbol{y}]_{-i}\big) = \frac{n_1}{N}\left[\pi_i^l(x_i, X_{-i} + Y)\right] + \frac{n_2}{N}\left[\pi_i^h(y_i, X + Y_{-i})\right] \qquad \forall\, i \in \{1, \dots, N\} \quad (5)$$

Where $[\boldsymbol{x}, \boldsymbol{y}]_{-i} = [(x_1, y_1), \dots, (x_{i-1}, y_{i-1}), (x_{i+1}, y_{i+1}), \dots, (x_N, y_N)]$

Note that a strategy for a firm in this symmetric monomorphic population game is defined as a tuple (x_i, y_i) giving a decision for the role as a high cost *and* the role of a low cost firm.

Consider now a finite population evolutionarily stable strategy of the game among these N firms. A strategy is evolutionary stable, if no mutant firm which chooses a different strategy than (x^*, y^*), say, can realize higher expected profits than the firms which employ the incumbent strategy (x^*, y^*). In other words, no mutant strategy (x, y) can invade a population of (x^*, y^*) strategists successfully.

Formally Schaffer`s definition (1988) then reads

Definition 1. *A strategy profile (x^*, y^*) is an evolutionarily stable strategy (ESS) if*

$$E\,\pi_i\big((x^*, y^*), (x_m, y_m), [\boldsymbol{x}, \boldsymbol{y}^*]_{-i-j}\big) > E\,\pi_j\big((x_m, y_m), [\boldsymbol{x}, \boldsymbol{y}^*]_{-j}\big), \qquad\qquad (6)$$

$\forall (x_m, y_m) \neq (x^*, y^*)$, and all $i \neq j$ where $(x_j, y_j) = (x_m, y_m)$, and $(x_i, y_i) = (x^*, y^*)$ *for all* $i \neq j$.

The novelty in this definition, compared to Tanaka (1999), is that it allows the mutation to occur simultaneously for both types of firms. Hence evolution of behavior for one type depends on the evolution of behavior for the other type and vice versa.

In line with Schaffer (1989), we can find a finite population evolutionarily stable strategy (ESS) as the solution of following optimization problem

$$(x^*, y^*) = \underset{x_m, y_m}{\operatorname{argmax}} \varphi, \qquad\qquad (7)$$

$$\text{where} \quad \varphi = E\,\pi_j\big((x_m,y_m),[\boldsymbol{x}^*,\boldsymbol{y}^*]_{-j}\big) - E\,\pi_i\big((x^*,y^*),(x_m,y_m),[\boldsymbol{x}^*,\boldsymbol{y}^*]_{-i-j}\big)$$

The strategy that survives in economic natural selection under playing the field conditions is the *relative*, not absolute, payoff maximizing strategy. A firm needs to beat the average of expected payoffs over all firms rather than to maximize its absolute expected payoff to be evolutionarily successful.

Hence we write expected payoff of the mutant and expected payoff of an incumbent as follows:

$$E\,\pi_j\big((x_m,y_m),[\boldsymbol{x}^*,\boldsymbol{y}^*]_{-j}\big)$$
$$= \frac{n_1}{N}\big[P(x_m,X^*_{-j}+Y^*).x_m - c(x_m)\big] + \frac{n_2}{N}\big[P(y_m,X^*+Y^*_{-j})y_m - \gamma.c(y_m)\big] \tag{8}$$

$$E\,\pi_i\big((x^*,y^*),(x_m,y_m),[\boldsymbol{x}^*,\boldsymbol{y}^*]_{-i-j}\big)$$

$$= \frac{n_1}{N}\Big\{\frac{n_1-1}{N-1}\big[P(x^*,x_m,X^*_{-i-j}+Y^*).x^* - c(x^*)\big] + \frac{n_2}{N-1}\big[P(x^*,y_m,X^*_{-i}+Y^*_{-j}).x^* - c(x^*)\big]\Big\} +$$

$$\frac{n_2}{N}\Big\{\frac{n_1}{N-1}\big[P(y^*,x_m,X^*_{-j}+Y^*_{-i})y^* - \gamma.c(y^*)\big] + \frac{n_2-1}{N-1}\big[P(y^*,y_m,X^*+Y^*_{-i-j})y^* - \gamma.c(y^*)\big]\Big\} \tag{9}$$

The first term of $E\,\pi_m(\,.\,)$ indicates the potential role of the mutant as a member of low cost group, and the second term refers to its potential role as a member of high cost group. Accordingly, for incumbents $E\,\pi_I(\,.\,)$ the calculation is slightly more complicated and consists of four terms in order to account for the mutant's role.

Note that we have the following equivalency

$$P(x_m,X^*_{-j}+Y^*) = P(x^*,x_m,X^*_{-i-j}+Y^*) = P(y^*,x_m,X^*_{-j}+Y^*_{-i}) \equiv P(x_m+(n_1-1)x^*+n_2y^*),$$
$$P(y_m,X^*+Y^*_{-j}) = P(x^*,y_m,X^*_{-i}+Y^*_{-j}) = P(y^*,y_m,X^*+Y^*_{-i-j}) \equiv P(y_m+(n_2-1)y^*+n_1x^*)$$

Consequently the finite population evolutionarily stable strategy (ESS) can be established as the solution of following optimization problem

$$(x^*,y^*) = \underset{x_m,y_m}{\operatorname{argmax}}\,\varphi, \tag{10}$$

$$\text{where} \quad \varphi = \frac{n_1}{N}\big[P(x_m,X^*_{-j}+Y^*).x_m - c(x_m)\big] + \frac{n_2}{N}\big[P(y_m,X^*+Y^*_{-j})y_m - \gamma.c(y_m)\big]$$
$$- \Big(\frac{n_1}{N}.\frac{n_1-1}{N-1}\Big)\big[P(x^*,x_m,X^*_{-i-j}+Y^*).x^* - c(x^*)\big]$$
$$- \Big(\frac{n_1}{N}.\frac{n_2}{N-1}\Big)\big[P(x^*,y_m,X^*_{-i}+Y^*_{-j}).x^* - c(x^*)\big]$$
$$- \Big(\frac{n_1}{N}.\frac{n_2}{N-1}\Big)\big[P(y^*,x_m,X^*_{-j}+Y^*_{-i})y^* - \gamma.c(y^*)\big]$$
$$- \Big(\frac{n_2}{N}.\frac{n_2-1}{N-1}\Big)\big[P(y^*,y_m,X^*+Y^*_{-i-j})y^* - \gamma.c(y^*)\big]$$

First order conditions with respect to x_m,y_m respectively are as follows:

x_m:
$$\frac{n_1}{N}\left[P(.) + P'(.)x_m - c'(x_m)\right] - \left(\frac{n_1}{N}\cdot\frac{n_1-1}{N-1}\right).P'(.)x^* - \left(\frac{n_1}{N}\cdot\frac{n_2}{N-1}\right).P'(.)y^* = 0$$

y_m:
$$\frac{n_2}{N}\left[P(.) + P'(.)y_m - \gamma.c'(y_m)\right] - \left(\frac{n_2}{N}\cdot\frac{n_2-1}{N-1}\right).P'(.)y^* - \left(\frac{n_1}{N}\cdot\frac{n_2}{N-1}\right).P'(.)x^* = 0$$

$P'(.)$ is the derivative of inverse demand function.

For the reason that the solution must be symmetric in roles and satisfies definition (1), we impose $x_m = x^*$, and $y_m = y^*$.

Then after rearranging, the following equations are obtained

$$P(n_1 x^* + n_2 y^*) + P'(n_1 x^* + n_2 y^*)\left[x^* - \frac{n_1-1}{N-1}.x^* - \frac{n_2}{N-1}.y^*\right] = c'(x^*) \tag{11}$$

$$P(n_1 x^* + n_2 y^*) + P'(n_1 x^* + n_2 y^*)\left[y^* - \frac{n_1}{N-1}.x^* - \frac{n_2-1}{N-1}.y^*\right] = \gamma c'(y^*) \tag{12}$$

Equations (11) and (12) can be rewritten as follows:

$$P(n_1 x^* + n_2 y^*) + P'(n_1 x^* + n_2 y^*)\left[\frac{n_2}{N-1}.(x^*-y^*)\right] = c'(x^*) \tag{13}$$

$$P(n_1 x^* + n_2 y^*) - P'(n_1 x^* + n_2 y^*)\left[\frac{n_1}{N-1}.(x^*-y^*)\right] = \gamma c'(y^*) \tag{14}$$

Two important conclusions are immediate now:

Firstly, subtracting (14) from (13) yields

$$c'(x^*) - \gamma c'(y^*) = P'(n_1 x^* + n_2 y^*)\left[\frac{N}{N-1}(x^*-y^*)\right]$$

From this it is easily seen that $x^* > y^*$ must hold: assume the opposite; i.e. that $x^* \leq y^*$. Then the right-hand side of the above expression is positive as $P'(.) < 0$. But $c(.)$ is convex and $c'(.)$ hence increasing, which yields a contradiction to $\gamma > 1$.

So FPESS output of a low cost firm is larger than that of a high cost firm.

Secondly, after multiplying both sides of equation (13) by $\frac{n_1}{N}$ and equation (14) by $\frac{n_2}{N}$ and then summing up both equations, the following expression for price is obtained

$$P(n_1 x^* + n_2 y^*) = \frac{n_1}{N}c'(x^*) + \frac{n_2}{N}\gamma c'(y^*) \tag{15}$$

It means that the price of the good in evolutionary equilibrium is equal to a convex combination of the marginal costs of low cost firms and high cost firms. The weights of marginal cost in this combination are identical with the population share of each group of firms. Note that (15)

reproduces the Walrasian result of Tanaka for either group, if $n_2 = 0$ resp. $n_1 = 0$.

Moreover, equation (15) means that the price of the good is not equal to the marginal cost for either the low cost or the high cost firms. In fact, one type of firm, namely the high cost firm sells at a price below its marginal cost. This is summarized in the following proposition.

Proposition 1 (Non-Walrasian Outcome). *In the symmetrized game of the asymmetric oligopoly market with two groups of low cost firms and high cost firms any FPESS output does not conform to Walrasian behavior; neither for the low cost firms nor the high cost firms. Rather the equilibrium price equals a weighted average of marginal costs, where the weights are given by the population shares of high and low cost firms. Low cost firms supply a higher amount of output than high cost firms.*

It is important to note that all firms – low cost firms *and* high cost firms - must make non-negative profit in FPESS equilibrium; otherwise high cost and low cost firms could not *coexist* in equilibrium. This requirement can indeed be met, if the cost function $c(.)$ is sufficiently convex.

To see this in more detail, observe that the profit function of a high cost firm in FPESS equilibrium can be written as

$$\pi_i^h = P(n_1 x^* + n_2 y^*) y^* - \gamma c(y^*) = [\frac{n_1}{N} c'(x^*) + \frac{n_2}{N} \gamma c'(y^*)] y^* - \gamma c(y^*)$$

Let the cost function be of the following convex form:

$$c(q) = q^\rho \tag{16}$$

Proposition 2. *Assume that (16) holds. Then $\rho \geq \gamma$ implies that a FPESS exists in which both types of firms make positive profits.*

Proof:
By assuming (16), we obtain profit function of high cost firm as follow:

$$\pi_i^h = y^* \left(\frac{n_1}{N} \rho(x^*)^{\rho-1} + \frac{n_2}{N} \gamma \rho(y^*)^{\rho-1} \right) - \gamma(y^*)^\rho$$

We can decompose the term $\gamma(y^*)^\rho$ as follows:

$$\pi_i^h = [y^* \left(\frac{n_1}{N} \rho(x^*)^{\rho-1} + \frac{n_2}{N} \gamma \rho(y^*)^{\rho-1} \right)] - [y^* \left(\frac{n_1}{N} \gamma(y^*)^{\rho-1} + \frac{n_2}{N} \gamma(y^*)^{\rho-1} \right)] > 0$$

Since we have $x^* > y^*$, the condition $\rho \geq \gamma > 1$ implies that $\pi_i^h > 0$.

■

Note that $\rho \geq \gamma$ is a sufficient condition for existence of a FPESS, but not a necessary one.

We now study the relation between our results and the notion of Walrasian outcomes. To this end we give formal definition of this notion in our model.

Definition 2. A *Walrasian* outcome for N firms is given by a quantity profile (x^W, y^W) and a market price P^W that satisfy the set of following conditions:

$$P^W = c'(x_i^W) = c'(x^W) \qquad\qquad \forall i = 1, \dots, n_1 \qquad\qquad (17)$$
$$P^W = \gamma c'(y_i^W) = \gamma c'(y^W) \qquad\qquad \forall i = 1, \dots, n_2 \qquad\qquad (18)$$

Proposition 3. *The following relationship between FPESS behavior (x^*, y^*) and Walrasian behavior (x^W, y^W) of individual firms holds:*

$$y^W < y^* < x^* < x^W$$

Proof.

This relationship is equivalent with the following relationship $x^* < x^W$ and $y^W < y^*$ (since we already know that $y^* < x^*$). A proof by contradiction is employed. Let´s assume that these inequalities do not hold. So we must have one the following three cases.
   i) $x^* < x^W$ and $y^* < y^W$
   ii) $x^* > x^W$ and $y^* > y^W$
   iii) $x^* > x^W$ and $y^* < y^W$

Next we derive a useful condition for x^* and y^* in our solution of (13) and (14), which we employ in this proof.
So we have from earlier

$$P(n_1 x^* + n_2 y^*) + P'(n_1 x^* + n_2 y^*)\left[\frac{n_2}{N-1} \cdot (x^* - y^*)\right] = c'(x^*) \qquad\qquad (13)$$

$$P(n_1 x^* + n_2 y^*) - P'(n_1 x^* + n_2 y^*)\left[\frac{n_1}{N-1} \cdot (x^* - y^*)\right] = \gamma c'(y^*) \qquad\qquad (14)$$

Then, knowing that $\frac{n_i}{N-1} \cdot (x^* - y^*) > 0$, equations (13) and (14) imply that

$$c'(x^*) < P(n_1 x^* + n_2 y^*), \qquad P(n_1 x^* + n_2 y^*) < \gamma c'(y^*)$$

By substituting equation (15)

$$c'(x^*) < \frac{n_1}{N} c'(x^*) + \frac{n_2}{N} \gamma c'(y^*), \qquad \frac{n_1}{N} c'(x^*) + \frac{n_2}{N} \gamma c'(y^*) < \gamma c'(y^*)$$

And this implies that the values for x^* and y^* in our solution of (13) and (14) satisfies

$$\frac{c'(x^*)}{c'(y^*)} < \gamma \qquad\qquad (19)$$

If case i) were to apply, then we must have

$c'(x^*) < c'(x^W)$
$\gamma c'(y^*) < \gamma c'(y^W)$
$P(X^* + Y^*) > P(X^w + Y^w)$ as c' (.) is increasing and P(.) is decreasing,

Furthermore, we have from equilibrium conditions

$P(X^w + Y^w) = \gamma c'(y^W)$
$P(X^* + Y^*) < \gamma c'(y^*)$

As a consequence, $P(X^* + Y^*) > P(X^w + Y^w)$ leads to $\gamma c'(y^*) > \gamma c'(y^W)$ and this is a contradiction.

If case ii) were to apply, with the same reasoning, we obtain

$c'(x^*) > c'(x^W)$
$\gamma c'(y^*) > \gamma c'(y^W)$
$P(X^* + Y^*) < P(X^w + Y^w)$

But knowing that $P(X^w + Y^w) = c'(x^W)$ and $P(X^* + Y^*) > c'(x^*)$ the first implication cannot hold.

If case iii) were to apply, we obtain by convexity of the cost function, that

$c'(x^*) > c'(x^W)$
$\gamma c'(y^*) < \gamma c'(y^W)$

Moreover, by $P(X^w + Y^w) = c'(x^W) = \gamma c'(y^W)$, these two inequalities yield the following inequality

$\gamma c'(y^*) < c'(x^*)$ which contradicts (19). ∎

Hence our average marginal cost pricing result moves firms` outputs in evolutionarily stable equilibrium closer to each other compared to the Walrasian result of Tanaka (1999). Total output of high cost firms increases, while total output of low cost firms decreases. Due to the convexity of the cost function total output over both groups is reduced and *all* firms make higher profits.

We illustrate this in a numerical example :

Let the number of firms in the low cost group and high cost group, respectively, be given by an integer $n \in [1,10]$ and $m \in [1,10]$. We use a linear demand function of the form $p = 10000 - (\sum_{i=1}^{n} x_i + \sum_{i=1}^{m} y_i) = 10000 - (X + Y)$. We specify the cost function to $c(q) = q^2$, *i.e.* $\rho = 2$. Consequently, $C_l(x_i) = c(x_i) = x_i^2$ *and* $C_h(y_i) = \gamma . y_i^2$ where $\gamma \in (1, 2]$.

Then we obtain total output in equilibrium as follows:

FPESS equilibrium:

$$X^* = \frac{10000 * n(m + n - 2\gamma + 2m\gamma + 2n\gamma)}{-2m + 3m^2 + 2n + 4mn + n^2 - 4\gamma + 6m\gamma + 2n\gamma + 2mn\gamma + 2n^2\gamma}$$

$$Y^* = \frac{10000 * (-2m + 3m^2 + 3mn)}{-2m + 3m^2 + 2n + 4mn + n^2 - 4\gamma + 6m\gamma + 2n\gamma + 2mn\gamma + 2n^2\gamma}$$

$$X^* + Y^* = \frac{10000 * (-2m + 3m^2 + 4mn + n^2 - 2n\gamma + 2mn\gamma + 2n^2\gamma)}{-2m + 3m^2 + 2n + 4mn + n^2 - 4\gamma + 6m\gamma + 2n\gamma + 2mn\gamma + 2n^2\gamma}$$

Walrasian equilibrium:

$$X^W = \frac{10000 * n\gamma}{m + 2\gamma + n\gamma}$$
$$Y^W = \frac{10000 * m}{m + 2\gamma + n\gamma}$$
$$X^W + Y^W = \frac{10000 * (m + n\gamma)}{m + 2\gamma + n\gamma}$$

We can plot the above functions with n as the single variable for fixed values of m (and γ). For example, the following graphs illustrate the result where $\gamma = 2$ and $m = 5$.[1]
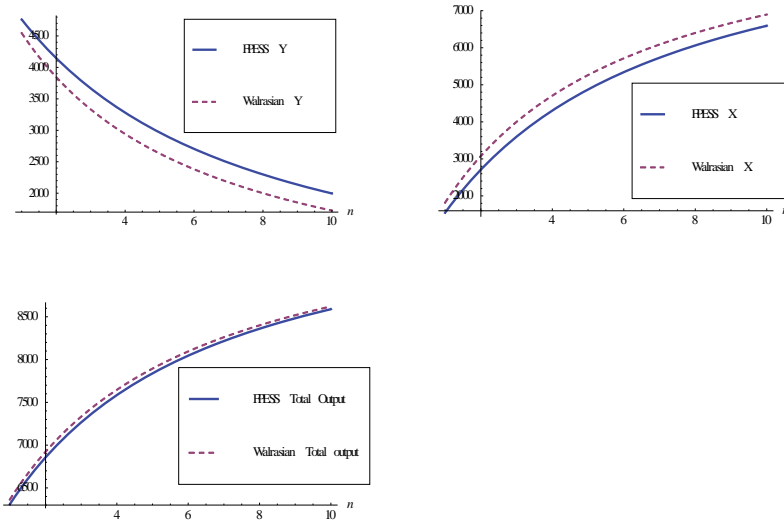

Fig.1. Comparison between equilibrium outputs

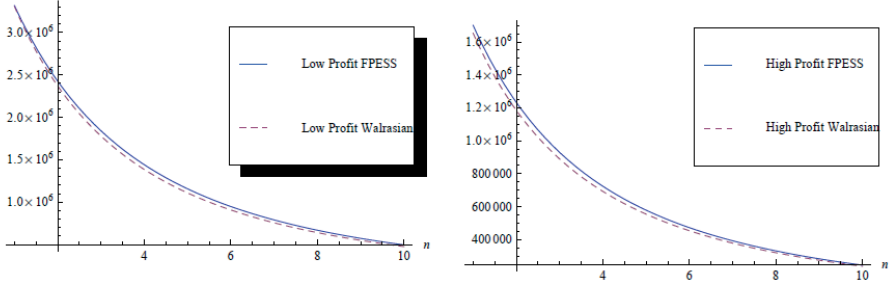[1] We obtain similar figures for other parameter choices as well.

Fig.2. Comparison between equilibrium profits

Our main result states that evolutionary forces in an oligopoly market with two types of firms determine the price in FPESS as equal to *average* marginal cost. It is not difficult to show that this *weighted average marginal cost pricing* rule of FPESS generalizes to oligopolistic competition between J groups of firms with different cost functions. However, it is then more elaborate to show existence of FPESS as the positive profit conditions become more stringent.

Proposition 4. *In the symmetrized game of the asymmetric oligopoly market with K groups of firms that differ w.r.t. cost functions the FPESS price equals a weighted average of marginal costs, where the weights are given by the population shares of the K types of firms.*

$$P\left(\sum_{k=1}^{K}\sum_{i=1}^{n_k} x_{ik}^*\right) = \sum_{k=1}^{K}\frac{n_k}{N} c_k'(x_k^*) \tag{20}$$

Proof.

Let´s define profit function as follow:

$$\pi_{ik}\left(x_{ik}, \left(\sum_{k=1}^{K} X_k\right)_{-(ik)}\right) = P\left(\sum_{k=1}^{K}\sum_{i=1}^{n_k} x_{ik}\right) x_{ik} - c_k(x_{ik}), \qquad i = 1,2,\dots,n_k, k = 1,\dots,K$$

$$where \sum_{k=1}^{K} n_k = N$$

Moreover expected payoff of each firm in our symmetrized version of game is like:

$$E\,\pi_i\left((x_{i1},\dots,x_{iK}),[\boldsymbol{x_1},\dots,\boldsymbol{x_K}]_{-i}\right) = \sum_{k=1}^{K}\frac{n_k}{N}\pi_{ik}\left(x_{ik},(\sum_{k=1}^{K} X_k)_{-(ik)}\right) \qquad \forall i = 1,\dots,N$$

Then also it requires writing expected payoff of mutant and incumbent in the general case:

$$E\,\pi_j\left((x_{j1}^m,\dots,x_{jK}^m),[\boldsymbol{x_1^*},\dots,\boldsymbol{x_K^*}]_{-j}\right) = \sum_{k=1}^{K}\frac{n_k}{N}\left[P\left(x_{jk}^m,(\sum_{k=1}^{K} X_k^*)_{-(jk)}\right) x_{jk}^m - c_k(x_{jk}^m)\right] +$$

$$E\,\pi_i\left((x_{i1}^*,\dots,x_{iK}^*),(x_{j1}^m,\dots,x_{jK}^m),[x_{i1}^*,\dots,x_{iK}^*]_{-i-j}\right)=\sum_{k=1}^{K}\frac{n_k}{N}\left(\frac{n_k-1}{N-1}\left[P\left(x_{ik}^*,x_{jk}^m,(\sum_{k=1}^{K}X_k^*)_{-(ik)-(jk)}\right)x_{ik}^*-\right.\right.$$
$$\left.\left.c_k(x_{ik}^*)\right]+\sum_{l\neq k}\frac{n_l}{N-1}\left[P\left(x_{il}^*,x_{jk}^m,(\sum_{k=1}^{K}X_k^*)_{-(il)-(jk)}\right)x_{il}^*-c_l(x_{il}^*)\right]\right)$$

Then similar to optimization problem in (7), we drive the first order conditions with respect to x_{jk}^m:

$$x_{jk}^m::\qquad\frac{n_k}{N}\left[P(.)+P'(.)x_k^m-c_k'(x_k^m)\right]-\left(\frac{n_k}{N}\cdot\frac{n_k-1}{N-1}\right)P'(.)x_k^*-\frac{n_k}{N}\sum_{l\neq k}^{K}\frac{n_l}{N-1}P'(.)x_l^*=0$$

After imposing $x_k^m=x_k^*$ we obtain:

$$\frac{n_k}{N}P\left(\sum_{k=1}^{K}X_k^*\right)+\frac{n_k}{N}P'\left(\sum_{k=1}^{K}X_k^*\right)\left[x_k^*-\frac{n_k-1}{N-1}x_k^*-\sum_{l\neq k}^{K}\frac{n_l}{N-1}x_l^*\right]=\frac{n_k}{N}c_k'(x_k^*)\qquad\forall k=1,\dots,K$$

It can also be written as follow:

$$\frac{n_k}{N}P\left(\sum_{k=1}^{K}X_k^*\right)+\frac{n_k}{N}P'\left(\sum_{k=1}^{K}X_k^*\right)\left[\sum_{l\neq k}^{K}\frac{n_l}{N-1}(x_k^*-x_l^*)\right]=\frac{n_k}{N}c_k'(x_k^*)\qquad\forall\,k=1,\dots,K$$

By summing over all equations, we obtain (by observing that $\sum_{k=1}^{K}\sum_{l\neq k}^{K}\frac{n_k}{N}\frac{n_l}{N-1}(x_k^*-x_l^*)=0$)

$$P\left(\sum_{k=1}^{K}\sum_{i=1}^{n_k}x_{ik}^*\right)=\sum_{k=1}^{K}\frac{n_k}{N}c_k'(x_k^*)$$

Finally, note that for the symmetric case in which all these marginal costs are equal, the well-known *marginal cost pricing* result of Vega-Redondo (1997) results.

4. Stochastically stable states and long-run equilibrium

A stochastically stable state can be viewed as a refinement of the solution concept FPESS. One of the limitations of the latter as a static notion of equilibrium is that it need not capture the notion of long-run stability for an evolutionary process, if evolution is also exposed to stochastic influences. An FPESS is also stochastically stable if under vanishing stochastic influence the probability that the population is following it does not go to zero. We now show this property for the unique FPESS in our model with respect to an evolutionary process, which has been studied extensively in the literature.

We follow Vega-Redondo (1997) and other contributions to evolutionary oligopoly theory (e.g. Schenk-Hoppé (2000), Tanaka (2000)) who analyze a certain type of imitation dynamics between firms subject to a stochastic influence, called experimentation: at each stage of the process there is a positive probability for any firm to change its strategy and adopt ("imitate") the most successful strategy of the ones chosen at the stage before. I.e. if all firms had chosen the same strategy in the previous stage, no adjustments would occur. Yet, there is also the stochastic influence of "experimentation": each firm will change its strategy to an arbitrary other one with positive probability ε. In a suitably defined dynamic model – usually with finitely many states – such a process always converges to a stationary distribution of strategies. One is then interested in the strategies that form the support of the limit of these stationary distributions if the noisy experimentation probability ε approaches zero; i.e. those states are then stable against experimentation.

Like the above mentioned contributions to evolutionary oligopoly theory we can use a result by Kandori and Rob (1995), which states that only monomorphic states; i.e. symmetric strategy vectors, can occur in this limit, which, moreover, are characterized by a "best reachability condition" and a "worst leaving condition". The former means that the number of experiments, i.e. stochastic shocks, needed for the process to reach them must be the minimal feasible one. Those states intuitively have the best chances to occur again and again as they are reached most easily. The latter means that the number of experiments needed for the process to leave them is higher than in other states. Those states, which are most easily reached and most difficultly left, are most frequently observed, which is the meaning of stochastic stability.

Obviously, the minimal number of experiments is one. However, this may not be the minimal feasible one for reachability of a certain state given the process. To check, one can operationalize the one-experiment requirement in the following way: suppose there is a strategy, which can invade successfully *any* monomorphically adopted; i.e. symmetric other strategy vector (recall, that there are only symmetric states in the limit). Then *one* experiment of one firm with this strategy would suffice to move the process to this invading strategy and the minimality requirement of "best reachability" would be met. Such a strategy is called a Globally Surviving Strategy GSS (see Tanaka (2000)). Its relation to the notion of FPESS is the following:

While a FPESS, if adopted by all firms, cannot be invaded by *any* single invader different from it (see (6)), a GSS is defined as a most effective (single) invader: it can successfully invade *any* symmetrically adopted other strategy. This, in turn, is equivalent to the requirement that a GSS, if adopted by all firms, cannot be invaded by (N-1) invaders with *any* commonly adopted other strategy. Hence, while a FPESS is stable against any single invader strategy, a GSS is stable against any (N-1) identical invader strategies that occur simultaneously. (One can refer to this property as (N-1)-stability whereas FPESS demands 1-stability.) Consequently, if a GSS exists it is the only candidate for FPESS, because any other strategy, if adopted by all firms, could be invaded by it as a single invader. Hence any state different from a GSS can be left with a single experiment of a single firm, which experiments with the GSS strategy. If, in contrast, the GSS itself is also an ESS, then it is the only strategy that *cannot* be left through *any* single experiment. Consequently, the "worst leaving" requirement is fulfilled as well.

Hence, if one can show - as in the model of Tanaka (2000) - that a unique GSS, which also is the unique FPESS exists, then this FPESS must be stochastically stable. (Leininger (2006) examines *global* stability of an FPESS, which is an even stronger property as it demands m-stability for any m with $1 \leq m \leq n - 1$, not just $m = 1$ and $m = n - 1$ as is required here).

We now show that this stochastic stability argument goes through in our model; i.e. we show that there is a unique GSS, which coincides with our FPESS.

If a single mutant firm j chooses (x^*, y^*) while the other firms choose (x, y), then the expected profit of this firm is

$$
E \, \pi_j \left((x^*, y^*), \overbrace{(x, y), \ldots, (x, y)}^{n-1} \right)
$$
$$
= \frac{n_1}{N} \, [P(x^* + (n_1 - 1)x + n_2 y) . x^* - c(x^*)]
$$
$$
+ \frac{n_2}{N} \, [P(n_1 x + y^* + (n_2 - 1)y) . y^* - \gamma c(y^*)] \tag{21}
$$

And expected profits of the other firms $i \neq j$ are

$$
E \, \pi_i (\, (x^*, y^*), (x, y), \ldots, (x, y)) = \frac{n_1}{N} \{ \frac{n_1 - 1}{N - 1} \, [P(x^* + (n_1 - 1)x + n_2 y) . x - c(x)] +
$$
$$
\frac{n_2}{N - 1} [P(n_1 x + y^* + (n_2 - 1)y) . x - c(x)] \, \}
$$
$$
+ \frac{n_2}{N} \{ \frac{n_1}{N - 1} \, [P(x^* + (n_1 - 1)x + n_2 y) . y - \gamma c(y)] +
$$
$$
\frac{n_2 - 1}{N - 1} [P(n_1 x + y^* + (n_2 - 1)y) . y - \gamma c(y)] \} \tag{22}
$$

The strategy (x^*, y^*) is a GSS, if it can invade any symmetric (x, y) state; i.e. if the following condition holds

$$
E \, \pi_j (\, (x^*, y^*), (x, y), \ldots, (x, y)) - E \, \pi_i (\, (x^*, y^*), (x, y), \ldots, (x, y)) > 0 \quad \forall \, i \neq j \, and \, \forall (x, y) \neq (x^*, y^*)
$$

Define

$$
\psi = E \, \pi_i (\, (x^*, y^*), (x, y), \ldots, (x, y)) - E \, \pi_j (\, (x^*, y^*), (x, y), \ldots, (x, y)) \tag{23}
$$

A necessary and sufficient condition for (x^*, y^*) to be a unique GSS is that it is a solution of the following problem

$$
(x^*, y^*) = \underset{x, y}{\text{argmax}} \, \psi
$$

Substitution of (21) and (22) into ψ and differentiating it with respect to x and y , we obtain the following first order conditions

FOC with respect to x:

$$\frac{n_1}{N}\left\{\frac{n_1-1}{N-1}[(n_1-1)xP'(.)+P(.)-c'(x)]+\frac{n_2}{N-1}[n_1xP'(.)+P(.)-c'(x)]\right\}$$
$$+\frac{n_2}{N}\left\{\frac{n_1}{N-1}[(n_1-1)yP'(.)]+\frac{n_2-1}{N-1}[n_1yP'(.)]\right\}$$
$$-\frac{n_1}{N}[(n_1-1)x^*P'(.)]-\frac{n_2}{N}[n_1y^*P'(.)]=0 \tag{24}$$

FOC with respect to y:

$$\frac{n_1}{N}\left\{\frac{n_1-1}{N-1}[n_2xP'(.)]+\frac{n_2}{N-1}[(n_2-1)xP'(.)]\right\}$$
$$+\frac{n_2}{N}\left\{\frac{n_1}{N-1}[n_2yP'(.)+P(.)-\gamma c'(y)]+\frac{n_2-1}{N-1}[(n_2-1)yP'(.)+P(.)-\gamma c'(y)]\right\}$$
$$-\frac{n_1}{N}[n_2x^*P'(.)]-\frac{n_2}{N}[(n_2-1)y^*P'(.)]=0 \tag{25}$$

Symmetry imposes $x = x^*$ and $y = y^*$ which simplifies equations (24) and (25) to

$$\frac{n_1}{N}P(.)+\left(\frac{n_1}{N}\cdot\frac{n_1-1}{N-1}\right).(n_1-1)x^*P'(.)+\left(\frac{n_1}{N}\cdot\frac{n_2}{N-1}\right).n_1x^*P'(.)+\left(\frac{n_1}{N}\cdot\frac{n_2}{N-1}\right).(n_1-1)y^*P'(.)$$
$$+\left(\frac{n_2}{N}\cdot\frac{n_2-1}{N-1}\right).n_1y^*P'(.)$$
$$=\frac{n_1}{N}c'(x^*)+\frac{n_1}{N}[(n_1-1)x^*P'(.)]+\frac{n_2}{N}[n_1y^*P'(.)] \tag{24'}$$

$$\frac{n_2}{N}P(.)+\left(\frac{n_1}{N}\cdot\frac{n_1-1}{N-1}\right).n_2x^*P'(.)+\left(\frac{n_1}{N}\cdot\frac{n_2}{N-1}\right).(n_2-1)x^*P'(.)+\left(\frac{n_1}{N}\cdot\frac{n_2}{N-1}\right).n_2y^*P'(.)$$
$$+\left(\frac{n_2}{N}\cdot\frac{n_2-1}{N-1}\right).(n_2-1)y^*P'(.)$$
$$=\frac{n_2}{N}\gamma c'(y^*)+\frac{n_1}{N}[n_2x^*P'(.)]+\frac{n_2}{N}[(n_2-1)y^*P'(.)] \tag{25'}$$

Then by adding the two equations term by term and collecting terms we derive

$$P(X^*+Y^*)=\frac{n_1}{N}c'(x^*)+\frac{n_2}{N}\gamma c'(y^*)$$

This is the same condition as equation (15) in the previous section, which characterizes the FPESS output. It means that the maximization problem of φ has the same solution as the maximization problem of ψ. Hence if an FPESS exists it must also be (N-1)-stable.

We record

Proposition 5. *The unique FPESS of our symmetrized oligopoly game is also a GSS and hence represents a stochastically stable state w.r.t. imitation dynamics with experimentation.*

Our average marginal cost pricing result is thereby shown to apply in the long-run as well.

5. Conclusion

In this study we reexamined the issue of evolutionary stability in asymmetric Cournot oligopoly with a homogenous product. Tanaka (1999) has shown that a celebrated result by Vega-Redondo (1997) for symmetric oligopoly, namely that the long-run evolutionary outcome of such a market equals the competitive Walrasian outcome (and not the Cournot-Nash outcome), can be extended to asymmetric oligopoly. We took issue with this extension and provided an alternative analysis of an asymmetric oligopoly game, which does not lead to marginal cost pricing and the competitive outcome in the long-run.

We symmetrized the asymmetric evolutionary game in a particular way and then applied Schaffer`s (1988) definition of FPESS as a solution concept to the symmetrized game. As a first result, marginal cost pricing becomes incompatible with FPESS, both for the low cost firms as well as the high cost firms. Instead, a form of weighted *average* cost pricing results in equilibrium, where the weights are given by "population shares" of the high and low cost firms. One reason for diminished evolutionary pressure towards competitive pricing in our model is that we allow for *simultaneous* mutations in the behavior of a low cost and a high cost firm, while the analysis in Tanaka (1999) does not. Tanaka`s evolutionary analysis treats the total number of firms effectively as two *separate* populations, each composed of identical firms, which do *not* interact with each other evolutionarily. This comes close to performing Vega-Redondo`s symmetric analysis "doubly" and makes survival of both types of firms less surprising.

In contrast, in our model firms with different costs *do* interact evolutionarily with each other and – perhaps surprisingly - both types of firms can survive in the long run *despite* the persistent differences in production cost over the population. This, however, is not compatible anymore with marginal cost pricing.

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