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Keywords: Renewable energy, power-to-heat, demand response, residential heating market, regulation

JEL: Q41, Q48, O38

*This working paper is a result of the cooperation of Hamburg Institute of International Economics (HWWI) and the Research Centre "Shaping the Future" (FoKoS) of the University of Siegen. The opinions and statements expressed by the authors of this paper do not necessarily reflect the opinions or positions of the HWWI and FoKoS. Email addresses: ehrlich@hwwi.org (Lars Ehrlich), klamka@hwwi.org (Jonas Klamka), wolf@hwwi.org (André Wolf)

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The potential of decentralized Power-to-Heat as a flexibility option for the German electricity grid: a microeconomic perspective[☆]

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1. Introduction

The transition of Germany’s energy system (Energiewende) is proceeding rapidly and has recently generated considerable research interest. Nuclear power is about to phase out in 2022, while energy from renewable resources (RES-E) has to account for at least 40% of the German gross electricity consumption in 2025 (CDU/CSU SPD, 2013). In 2013, the share of electricity from renewable energy sources accounted for 25.3% of gross electricity consumption, whereas the percentage of RES-E was only about 6.2 in the year 2000 (BMWi, 2014b). This rising share of electricity from volatile renewable energy sources such as wind power and photovoltaics has created complex challenges for the power system (Schleicher-Tappeser, 2012). One fundamental challenge in the system integration of RES-E is the increasing imbalance between volatile generation patterns and the generally inelastic load, which is why different flexibility options have become an essential element to match electricity demand and supply.

Politicians are aware of the challenges ahead. Alongside the challenges of the electricity sector, the sluggish decarbonization of the heating sector is also gaining attention. In the 2013 German Federal Governments coalition agreement (CDU/CSU SPD, 2013), it is emphasized that the heating sector is of significant importance for a successful energy transition. Specifically, the exploitation of potential surplus electricity from renewables for other applications such as heating purposes is recommended. The Federal Ministry for Economic Affairs and Energy (BMWi) gives a concrete example: “If the residual load is low, electricity can be used to generate heat directly and therefore save on heating oil or gas” (BMWi, 2014a). In 2012 the oil and gas consumption for purposes of heating and hot water in single and two family houses amounted to roughly 250 TW (Destatis, 2013a,b). Ensuring that only one percent (2.5 TW) of this energy demand is provided by RES-E could be a significant step towards the use of less fossil fuels and an increasing share of renewable energy in the residential sector.

In this context, Power-to-Heat (P2H) systems have been propagated as a potential tool to enhance demand-side flexibility and to interlink power and heat markets (e.g. Böttger et al., 2015). Our paper tries to add a new perspective to the debate by investigating the capability of decentralized P2H to serve as an alternative flexibility option for the German grid. Specifically, we focus on two forms of P2H-hybrid systems. In the first one, a conventional gas condensing boiler system (including a thermal storage) is equipped with an electric heater. In the second one, the same is done with an oil-fired condensing boiler system. The appeal of these hybrid solutions is obvious both from a technical and an economic perspective. Given that gas and oil-fired heating systems currently constitute the most frequent technologies in decentralized heating in Germany, the transformation of these systems can create a channel for delegating sig-

nificant amounts of excess electricity to a useful purpose. Moreover, in contrast to pure electric heating systems, these hybrid systems do not cause a permanent increase in electricity demand. Provided they are operated based on economic principles, a demand effect will only occur in those times where the costs of electricity-based heating are comparatively low. In presence of appropriate price signals on the electricity retail markets, this would contribute to the matching of electricity demand with volatile electricity supply, thereby exerting a positive impact on grid stability. Economic incentives are likewise central for the investment decision: the potential societal gains will only be realized if heating owners can expect to recoup the costs of modifying their heating system (purchase of electric heater, communication equipment etc.) in reasonable time through lower heating expenses.

The focus of our analysis will therefore be on the economics of decentralized P2H. Based on the premise of time-flexible electricity retail prices in the future, the potential annual cost savings from switching to a hybrid system are simulated for the exemplary year 2020. The next section (2) outlines the flexibility options currently available, followed by a short literature review in section 3. Subsequently (Section 4), we describe our integrated simulation approach. In section 5, the main results of the analysis are presented and a sensitivity analysis regarding crucial model parameters is conducted. Furthermore, in section 6 the limitations of our analysis are discussed. Section 7 concludes with policy implications.

2. Available flexibility options

The current electricity systems in Europe and Germany originate from the period when electricity was generated almost entirely by thermal power plants and fluctuations were predominantly caused by the demand side, which were easier to predict and to balance. Due to the steadily increasing share of RES-E in the generation mix, this is changing and situations become more frequent in which generation potential exceeds demand. The transmission grid and in particular the distribution grid are put under increasing pressure in times of high in-feed from RES-E. As a result, demand for flexibility to maintain grid stability is rising (Lise et al., 2013). Currently, supply and demand is primarily matched with dispatchable power plants and via interconnectors with adjacent areas (IEA, International Energy Agency, 2011).

Alternatively, a widely discussed way to enhance the flexibility of the electricity system is decoupling power production from power consumption through energy storage facilities. Currently, the only large-scale and cost-efficient storage technology is pumped-storage hydro power (PSH). Power is stored as potential energy by pumping water from a lower to an upper reservoir. In times of high electricity demand, this potential energy could be used to spin a turbine which

is attached to a generator. PSH is primarily used for satisfying peak-demand and delivering system services (e.g. frequency control). Worldwide PSH accounts for about 99% (140 GW) of the installed capacity of grid-connected energy storage (IEA, International Energy Agency, 2014). However, the further deployment of PSH capacity is limited by spatial and environmental constraints, particularly in Germany. Grid expansion and improved congestion management are other options to stabilize the electricity grid. Through increased cross-border transmission capacities, flexibility options of adjacent markets can be used and weather dependent feed-in of RES-E will be smoothed. The curtailment of RES-E is another solution, albeit an unpopular one. Due to the near zero marginal generation costs of RES-E, it is favourable to utilize most of this potential. However, situations become more frequent in which RES-E must be curtailed to prevent a congestion of the distribution grid. Against this background, Klinge Jacobsen and Schröder (2012) investigate the optimal curtailment level. Besides, there is a broad consensus that the upcoming challenges regarding system security and integration of RES-E could not be met by efforts on the supply side and the grid management alone. It will become crucial to raise flexibility potentials on the demand side as well. Final users must be actively involved in measures to stabilize the grid by raising or reducing their demand depending on current generation. In principle, such a load control could be achieved directly by the respective distribution system operator (DSO) or indirectly through individual decisions of the single end-users. Either way a new information and communication technology (ICT) needs to be established to enable such a load control.

In light of the challenges of the electricity sector, residential heating is also politically identified as a key component of the energy transition. In 2012, residential heating accounted for almost 23% of Germanys final energy consumption (Destatis, 2013b; AGEb, 2014). In order to develop the German heating market towards the use of more RES-E, a number of laws have been implemented by the German government to legally support these efforts. The Energy Saving Ordinance (EnEV) proposes certain requirements for the primary energy demand of new buildings. The Renewable Energy Heating Act (EEWärmeG) has set the goal to increase the share of RES in residential heating and cooling provision to 14% by 2020. Furthermore, the German CHP-Act (KWKG) seeks to expand the use of combined heat and power generation. Regarding the transport sector the German Federal Government announced the target of getting one million electric vehicles on German streets in 2020. Hence, there are various starting points with the common purpose to improve the utilization and integration of RES. Next, we present a short review of the related literature.

3. Literature review

Wang et al. (2011) analyse the potential of electric vehicles as a demand response instrument against the backdrop of an integration of large amounts of wind energy in Illinois in 2020. They come to the conclusion that it is technically feasible and economically reasonable. Jargstorf and Wickert (2013) contribute to the same field of research and analyse the ability of a pool of electric vehicles to provide negative secondary reserve power in the German balancing market. However, they conclude that the potential revenues of offering ancillary service are not sufficiently large to incentivize the purchase of electric vehicles. There is also a field of research that is focused on the economic potential of electrical load management by the heating sector. This literature investigates the possibilities of heat pumps (HP) and (micro-) CHP to provide flexible consumption capacity in times of imbalance in the electricity market (see Fehrenbach et al. (2014) for a review of these literature). In sum, authors find rather limited economic potential of HP and mCHP systems to serve for large scale load management in the near future. Böttger et al. (2015) analyse the potential of electric boilers (P2H) in German district heating grids. They address the provision of system services (negative secondary control power) and come to the conclusion that this can be provided in a cost-effective manner. Hao et al. (2015) and Mathieu et al. (2015) examine the potential of residential Thermostatically Controlled Loads (air conditioning, water heaters, heat pumps) in California to serve as providers of system services and estimate cost and revenues for these devices. Both papers come to the conclusion that the technical potential is high and probable revenues are in some extent sufficiently large, but differ between the considered technologies.

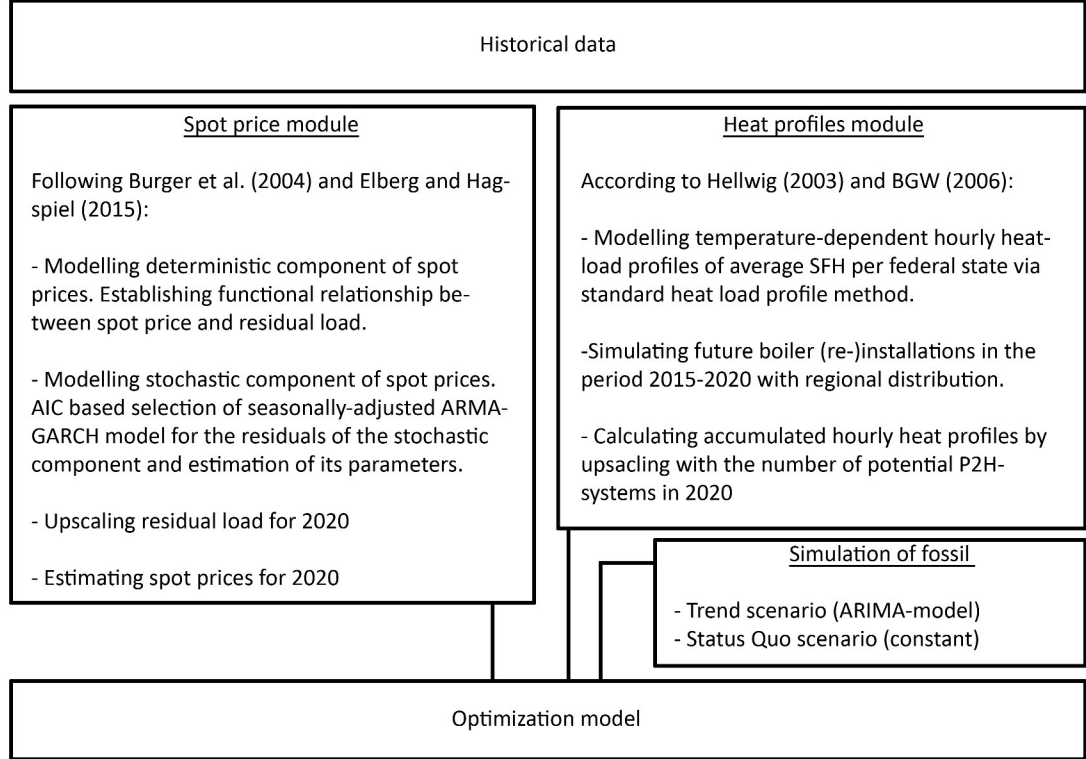
Regarding P2H most attempts focused mainly on identifying its potential for district heating purposes and/or as a part of virtual power plants providing ancillary services. Since only 5.4% of the German building stock (Destatis, 2011) is connected to a district heating network, decentralized P2H seems worth a consideration. Furthermore, the revenue potential of the balancing market is limited and there is the political will to involve the private heating sector to a greater extent. Nevertheless, this can only be achieved if financial incentives for individual heating owners are sufficiently large.

4. Methods

For this reason, the purpose of our analysis is to investigate the economic potential of decentralized P2H as an alternative flexibility option to stabilize the German grid. To this end, we use a combination of three simulations (Figure 1). First, a stochastic simulation of hourly

electricity wholesale prices on the EPEX day-ahead market in 2020 is conducted. Secondly, a forecast of the stock of gas and oil-fired condensing boiler systems in Germany in 2020 is made, which is used to derive characteristic average head load profiles. The results are then fed into a dynamic optimization model to simulate the economically optimal heating patterns of our P2H-hybrid systems over the course of a year. To the best of our knowledge, this approach is so far new in this research area.

Figure 1: Schematic model description



4.1. Simulation of electricity spot market in 2020

Electricity prices exhibit basically four characteristics which make the modelling a complex challenge. The prices follow daily, weekly and seasonal patterns. They show high volatility and sometimes extreme spikes and jumps. However, the prices tend to revert to the mean over time (Burger et al., 2004; Bierbrauer et al., 2007). Since the year 2000, there is a growing strand of literature on electricity price modelling and forecasting (Weron, 2014).¹ Hence, it is not

¹Weron (2014) also presents a comprehensive overview on electricity price forecasting methods.

unambiguous which modelling approach performs best as the different methods aim at various research questions and cannot directly be compared (Keles et al., 2012).

Demand (load) and electricity prices are highly correlated. High in-feed from RES-E depreciate the spot prices, because of the near-zero marginal generation costs of wind and solar power which can be interpreted as a rightward shift of the supply curve (so called merit-order effect). Very low hourly spot prices indicate a possible oversupply that may occur in the specific hour. In such a situation P2H heating systems could prospectively serve as an additional load. To analyze this case, we need to approximate the price and volatility effects of (additional) RES-E. Hence, as a first part of our integrated simulation approach we simulate hourly electricity spot prices in 2020 against the backdrop of increasing RES-E capacities. To this end we use the concept of residual load which allows the modelling of the effects of fluctuating in-feed from RES-E on spot prices. However, there is no unique definition of residual load in the literature (Schill, 2014). In this work, we define residual load as the difference between actual power consumption and weather-dependent in-feed from wind power and photovoltaics. Thus it expresses how much power must be provided by controllable power plants to balance supply and demand. We adjust the stochastic simulation approaches of Burger et al. (2004) and Elberg and Hagspiel (2015) for our purposes. The spot price model consists of two components, a deterministic and a stochastic one.

$$P_t = \overbrace{g_t(D_t - W_t - S_t)}^{\text{deterministic part}} + \underbrace{Z_t}_{\text{stochastic part}}$$

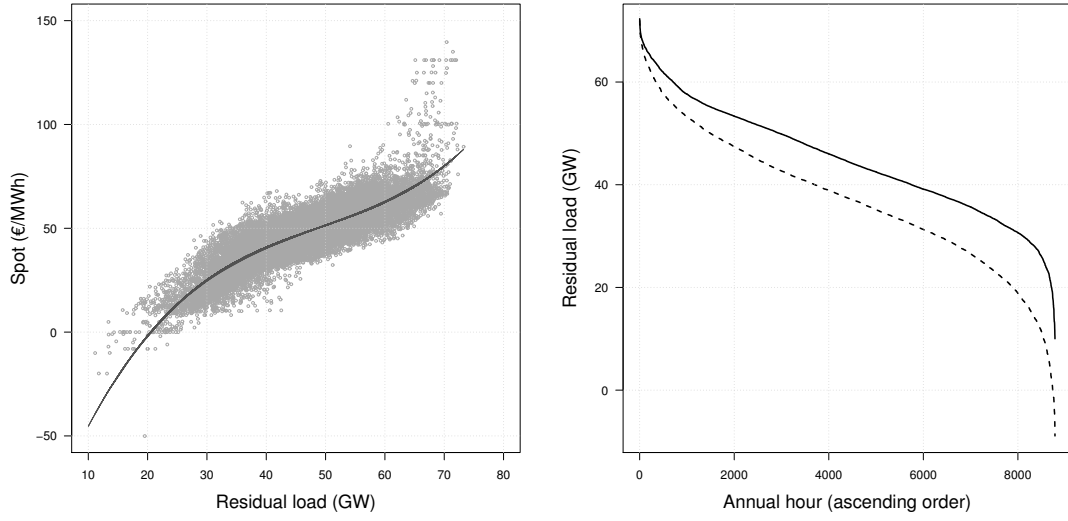
First, the functional relationship g_t of spot prices and residual load is estimated. For this deterministic part of the model a polynomial of degree 3 fits well (adjusted R-squared 0.74). The polynomial could be seen as a biased approximation of the supply curve (so called merit-order) except for wind power and photovoltaics (Figure 2, left side). Secondly, a stochastic component Z_t is added. For this purpose, we fit a seasonally-adjusted ARMA-GARCH-model to the residuals of the polynomial regression. This is intended to reflect the non-structural components which influence the spot price such as unplanned power plant outages or speculation.

The model is calibrated and estimated with historical data of the reference year 2012. For the formation of the residual load we use hourly values of the day-ahead prognosis of wind and solar power. These are made available by the EEX Transparency Platform (2014). The hourly load values for Germany in 2012 are provided by ENTSO-E (2014). Hourly day-ahead prices are provided by the European Power Exchange (EPEX).²

²Hourly prices that exceed three times the standard deviation of the respective month are replaced with the value three times the standard deviation as proposed in Ketterer (2014).

For the seasonally-adjusted residuals of the polynomial we conduct an augmented Dickey-Fuller test, which rejects the hypothesis of a unit root at the 99% level. Moreover, the Lagrange Multiplier test for autoregressive conditional heteroscedasticity indicates to incorporate a GARCH component in our simulation. Based on the Akaike Information Criterion (AIC), we choose an ARMA(1,1)-GARCH(1,1)-model. A formal description of the model and the estimates for the parameters are presented in the Appendix A1.

Figure 2: Left side: spot price versus residual load and polynomial fit. Right side: residual load duration curves for 2012 (solid) and 2020 (dashed)



The calibrated model is used to derive electricity prices for 2020. For the formation of the residual load for 2020 we again use the hourly load values of 2012. The wind and solar power feed-in data for 2020 is simulated by upscaling hourly values for the actual wind and solar power feed-in of 2012 according to the described expansion paths in the amendments of the Renewable Energy Act (2014). In figure 2 (right side) the resulting residual load duration curves of 2020 (dashed) and 2012 (solid) are compared. One can see the downward shift due to the increased RES-E capacities for 2020. The simulation yields even a few hours for 2020 where the residual load is negative, in other words, hours in which the production from RES-E exceeds the actual demand.

By using this residual load values we obtain a time series of hourly spot prices for 2020. The production patterns of wind and photovoltaics correspond to the weather conditions of the year 2012. Our procedure of upscaling the 2012 in-feed from RES-E rests on two basic assumptions.

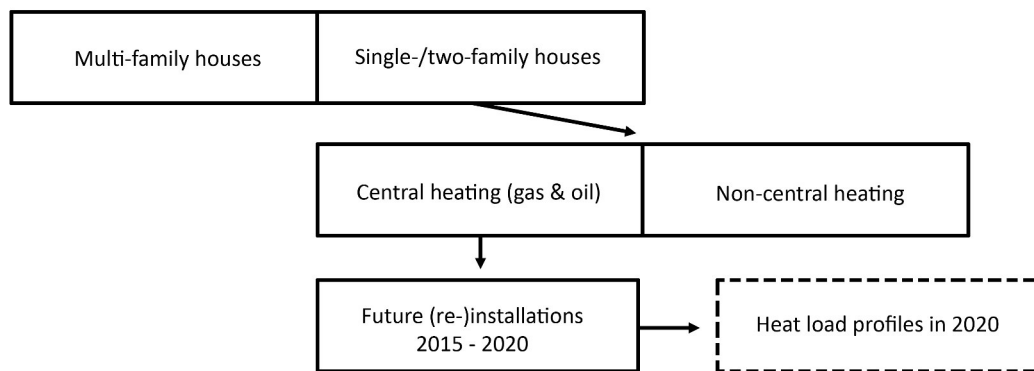
The first one is that the pattern of weather conditions observed in 2012 is not unrealistic for 2020. In our view, this is justifiable as climate conditions will probably not have changed significantly by then. The more critical assumption behind upscaling is that the additional RES-E capacities in 2020 will exhibit the same generation patterns as the existing ones.

4.2. Simulation of the German heat profiles in 2020

The demand for heating is investigated by calculating heat load profiles in a bottom-up analysis. Heat load profiles are generated for potential P2H-system users in 2020 only. In this regard, only single and two family buildings (SFH) with a fossil fuel fired central heating system are seen as potential P2H households. Moreover, only buildings with future boiler (re-) installations [2015-2020] are considered as possible households with a P2H-system.

In this study we use the standard heat load profile method (BGW, 2006) to forecast the energy

Figure 3: Procedure to generate heat load profiles



demand for space heating (SH) and domestic hot water (DHW) in individual SFH on an hourly basis. The total energy consumption for SH and DHW of buildings with potential P2H is then obtained by upscaling. The standard heat load profile method is a common technique used by municipal energy suppliers forecasting gas load curves of non-measured end-users ($< 1.5 \text{ GW/a}$)³ to provide the accurate amount of gas. This engineering tool was developed by the Technical University Munich (Hellwig, 2003) on behalf of the BGW (Federal Association of the German Gas and Water) and VKU (Association of Local Utilities Association) and allows predicting the temperature-dependent heating energy demand of households with respect to several factors. In

³cf. §24 Abs. 1 Gasnetzzugangsverordnung (GasNZV)

this study we adapt this method (BDEW, 2013) to our purposes by extending it to households using oil as an energy source. We assume that the heat load of an end-user is only depending on the individual heating need and not on the energy carrier being used. In this way, we obtain detailed load profiles for households using fossil fuel as a primary source for space heating. For this purpose, a sigmoid-function is used to calculate the heat load profile based on numerous parameters and specific reference values. The key factors affecting the energy demand for space heating are primarily the outside temperature, regional climatic parameters like wind conditions and further regional particularities as building and population structure. The target function looks as follows:

$$Q_D = h(T_D) \left[\frac{Q_N}{\sum_i^N h(T_{D_i})} \right] \quad \text{with} \quad h(T_D) = \left[\frac{A}{1 + \left(\frac{B}{T-40} \right)^C} + D' \right]$$

where Q_D is the target value and denotes the predicted energy consumption on day D ⁴. The sigmoid coefficients A , B , C and D' are specific regional parameters representing the building structures and regional climatic characteristics of the German federal states with differentiation in wind-layers. In this study the coefficients for a “normal wind-layer” are used (BGW, 2006). T denotes the temperature applied to the specific day. In order to take the thermal storage capacity of buildings into account, the geometric series of the average outside temperature of days D , $D-1$, $D-2$ and $D-3$ is calculated. For this purpose, the online data basis of the German National Meteorological Service⁵ is used to obtain time series for the daily average outside temperature from reference weather stations in each federal state in 2012. Finally, Q_N denotes the customer value (in kWh). In practice this value would be the total amount of energy of the previous period for the specific household. Since these data are not available we constructed a theoretical energy consumption average of an individual building for space heating and hot water per annum. This value is heavily dependent on the constructional heat insulation and the used heating system of the building, implying that the assumptions for the building configuration are crucial to the analysis. There is no way around some generalising assumptions, since one of the striking characteristics of the heating market is its diversity and complexity. The number of possible combinations of heating systems and the configuration of the building stock is enormous and many variables need to be taken into consideration. Consequently, a closer look at the assumptions made in defining the reference building used in this study seems necessary. To begin with, we focus on single-family and two-family houses (SFH) only. We do this for two reasons. First, SFH make up about 82.6% of the national housing stock (Destatis, 2013b) and

⁴Detailed values are given in appendix A2

⁵WebWerdis/DWD: <http://www.dwd.de/webwerdis>

are the dominating building type in rural/decentralised regions. Secondly, multi-family houses (MFH) are hard to handle in terms of an average size, since this category covers the range from three to over 40 dwellings per unit. Furthermore, only SFH are likely to have enough space in the basement for a P2H-system. Following that, we use an average single-and two-family house with a living space of 141 m^2 for our analysis (Destatis, 2013a). We assume the specific final energy use per m^2 living area to be $134 \text{ kWh/m}^2\text{a}$ (Adolf et al., 2013). This is the assumed specific end-use of energy for heating and hot water in a refurbished building according to the Heat Insulation Ordinance (WSchVO95) with an installed condensing oil or gas boiler. We think of this combination as an adequate description of an average SFH. Combining these two values, the resulting Q_N is 18.894 kWh end-energy use for an average SFH for space heating and hot water per annum.

Furthermore, only future installations of boilers are seen as potential P2H-systems in our analysis. In consequence, it is assumed that all gas and oil boilers that are installed in SFH from 2015 to 2020 are P2H compatible. For this purpose we extrapolate the new installation of heating boilers, especially of condensing boilers, for the years 2015 to 2020 based on the trend of the sales figures of the years 2008 to 2013. The average growth rates of gas condensing boilers in Germany was 6.5% per annum for the last five years, while the sales of oil condensing boilers shrunk with 4.3% per annum (BRG, 2013). With that in mind we calculate the number of designated new installation of gas and oil condensing boilers until 2020. Since the Ecodesign Directive (2009/125/EC) permits the new installation of non-condensing boilers, with some minor exceptions. from 2016 on, the predicted installations of condensing boilers was corrected for that figures. With another correction for the installation in SFH, based on the share of the housing stock, we calculated about 3.16 million new installed oil and gas condensing boilers - potential P2H systems - in SFH from 2015 to 2020.

For the regional distribution of these installations in the case of gas, the distribution of the housing stock across federal states was used (Destatis, 2013b), while in the case of oil the distribution of oil fired boilers was available. Finally, daily heat load profiles for every federal state were generated. The corresponding hourly heat profiles were then calculated by using typical percentage shares of gas use depending on the outside temperature (BDEW, 2013)). In conclusion, with that method we obtain hourly heat load profiles for SFH in 2020 with potential P2H-systems in regional resolution.

4.3. Simulation of individual cost savings through P2H in 2020

Any assessment of the economy-wide potential of decentralized P2H requires a profitability analysis at the individual level. Only if the average gas or oil heating system owner gains a considerable economic benefit from modifying his system, a number of hybrid systems sufficient to influence aggregate electricity demand will be installed. In addition to the level of investment expenses, system profitability is primarily influenced by the evolution of electricity prices relative to prices of the conventional heating sources gas and heating oil. The cost advantage of a hybrid system in terms of annual heating expenses will be the greater, the longer the periods in which the switch to electricity as a heating source pays off. In this regard, the observed increase of supply-side volatility in German electricity trading is a contributing factor. At the wholesale level, the uncertainty concerning RES-E volumes (e.g. due to difficulties in forecasting wind speed) has already translated into growing price volatility. However, in German retail trading, end-user tariffs so far lack the flexibility that could induce final users to adjust their electricity use to unexpected changes in the supply situation. Most suppliers merely offer a general differentiation into day-tariffs and night-tariffs, which are obviously of little help in coping with unexpected imbalances in the grid. However, the current development in load management suggests that this might change soon. The European Union has urged in its third energy package from 2011 member states to boost the deployment of smart meters at final user level. Moreover, the ongoing technological development with respect to the communication infrastructure will surely reduce the start-up costs of smart metering in the future. When the infrastructure is finally widely established, competition should ensure that tariff flexibility will increase in the interest of customers.

Therefore, we examine in our simulations a future scenario where price signals from whole-sale trading are translated into electricity prices at final user level at short notice. Precisely, we analyse the case where hourly prices on the EPEX day-ahead market are completely passed on to final user prices for the corresponding hour of the next day. In the day-ahead auctions, bids for the next day can be made until 12h of each day. For this reason, the information on tomorrow's prices cannot be expected to be available when the decisions on the heating modes for the hours of the current day are made. Our basic assumption is therefore that these decisions merely rest on the knowledge of hourly electricity prices for the current day. In addition to wholesale prices, the electricity price for final consumers is also in the future likely to include taxes and other components under government control. Currently, the following components enter the price calculation: the (regulated) network charge, the Renewable Energies Act levy (EEG-levy), the electricity tax, the concession fee, the cogeneration levy and finally the value added tax. As the evolution of each of these components is associated with a high degree of political uncertainty,

we do not restrict ourselves to specific price scenarios. Instead, we carry out simulations over a range of different levels of the parameter defining the tax burden on electricity prices per kWh, where the current situation serves as a reference case.

Since we consider hybrid systems in which an electric heater is combined with either a gas or an oil-fired condensing boiler system, the cost-minimizing heating modes and the resulting cost savings are likewise influenced by the development of heating oil and gas prices. Long-run forecasting of these prices is subject to more or less the same level of uncertainty as price forecasting for the underlying primary energy carriers oil and natural gas. A consideration of different price scenarios seems therefore appropriate. For both energy carriers, we compare a Status Quo scenario with a Trend scenario. In the Status Quo scenario, we set the heating oil and the gas price equal to the average of their monthly prices reported by the Federal Statistical Office of Germany within the last five years. The resulting prices are 6.79 Ct/kWh for heating oil and 6.24 Ct/kWh for gas. For the Trend scenario, we derived own forecasts from time series analysis. For heating oil, we fitted an ARIMA (1,1,1)-model to the seasonally-adjusted series of monthly household prices in Germany for the time span 1991-2014. The resulting model was used to predict the average monthly price of light heating oil in 2020, which was obtained as 9.05 Ct/kWh. For gas prices, no sufficiently long series were available. Therefore, we applied our heating oil simulations also to the case of gas by assuming that both prices experience the same trend in terms of percentage changes. Given that both prices have until 2011 for a long time been legally linked and exhibit a similar risk exposure, this seems to us a justifiable assumption. The resulting trend price for gas in 2020 is 8.31 Ct/kWh.

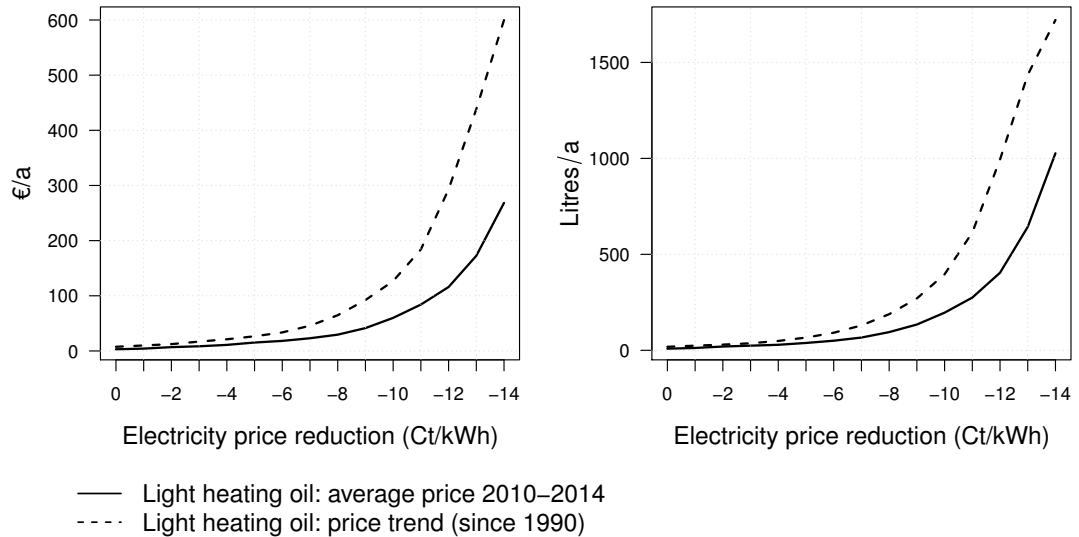
To simulate the optimal heating behaviour under these price parameters, a simple dynamic optimization approach was implemented. The objective function is the minimization of heating expenses across the year, with the amounts of electricity and gas/heating oil used for heating purposes in a certain hour as decision variables. The dynamics of this decision result from the presence of a thermal storage, which allows a decoupling of heat generation and consumption. This enables households to exploit intertemporal differences in electricity prices, but only within limits. In addition to the restricted foresight of electricity prices, further technical constraints have been considered. One is the limited capacity of the electric heater, which restricts the quantity of heat gained from electricity within an hour. Heating rods suitable for these applications are typically available with capacities in a range from 3 to 9 kWh. We analyse in the base-case simulations a capacity of 6 kWh, which is varied in a subsequent sensitivity analysis. Another constraint is the limited capacity of the thermal storage. We consider a storage capacity of 500 litres in the base case. A further technical restriction is a constraint to the quantity of heat stored at each instant, which is defined by the upper and the lower temperature limit of the

thermal storage. For these limits, experience-based values of 85° C and 35° C were applied. Furthermore, the hourly level of standing losses of heat energy stored within the thermal storage is accounted for by applying the approximation proposed by the norm DIN EN 304. A formal description of all model variables, parameters and equations is provided in Appendix A3.

5. Results

For heating oil and gas, separate simulations of the maximum annual heating cost savings from implementing the hybrid system were performed. In line with expectation, estimated results for 2020 prove to be highly sensitive both to the price scenarios specified for heating oil and gas and to the future tax burden on electricity prices. Figure 4 graphs the level of cost savings from an oil-electricity hybrid system as dependent on the extent of future reductions in the regulated components of the electricity price. It becomes immediately clear that without a serious tax relief for electricity used in hybrid systems savings will be negligible. This holds even if heating oil prices continued to follow their past trend path in the long-term. If electricity prices were reduced by an amount of 5 Ct/kWh, which is in the range of the current EEG levy, savings of merely 15.28 €/a (Status Quo scenario) or 26.61 €/a (Trend scenario) could be achieved under the given simulation scenario.

Figure 4: Annual savings of heating expenses under an oil/electricity hybrid system



At the same time, savings increase to a more than proportional degree with the size of the tax

relief. They climb up to 59.98 €/a (Status Quo scenario) and 126.71 €/a (Trend scenario) under a relief of 10 Ct/kWh. One of the reasons for this pattern is the increasing opportunity to exploit the cost-reducing properties of the thermal storage: as the time spans during which electricity represents the cheaper heat source become longer, heat energy generated by the electric heater can be effectively accumulated in the storage, thereby reducing the need for switching to heating oil in subsequent high-price situations. For this reason, savings in terms of litres of heating oil exhibit a similar pattern. Given a tax relief of 10 Ct/kWh, 217.31 litres of heating oil could be saved by an average household per year.

Figure 5: Annual savings of heating expenses under a gas/electricity hybrid system

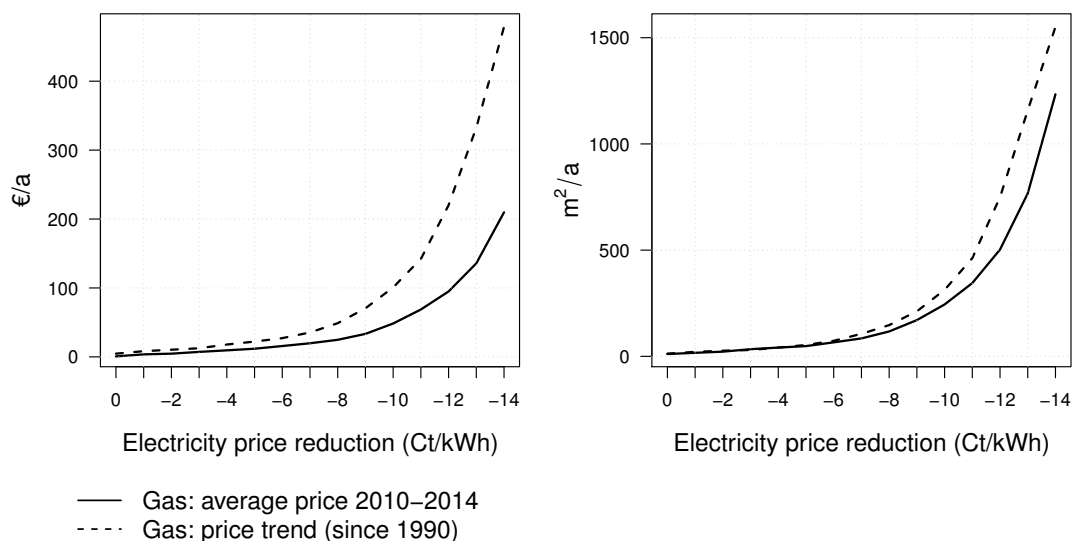


Figure 5 documents a qualitatively similar picture for gas. However, the absolute level of estimated savings is slightly lower than for owners of oil-fired systems under each scenario. For instance, under the same 10 Ct/kWh reduction scenario, annual heating expenses merely decline by 48.41 €/a (Gas prices: Status Quo) or 100.79 €/a (Gas prices: Trend) compared to a pure gas heating. This is primarily due to the lower usage costs of gas per kWh of heat generated.

5.1. Sensitivity analysis

The maximal savings presented above were obtained under a certain set of technical specifications. As these specifications can differ to a considerable extent among real-world households, it is important to demonstrate how seriously changes to the technical model parameters affect the results. Our sensitivity analysis focuses on two central features: the technical capacity of the heating rod and the storage capacity of the thermal storage. In general, the capacity of the

heating rod influences the degree to which households are able to exploit situations of very low electricity prices. With a higher performance, more electricity can be transformed into heat at these instants and stored for later high-price periods. Thermal storage capacities are likewise important in this. They define the maximum amount of heat energy that can be stored at each instant and thus the maximum time span that could be bridged during high-price periods without having to resort to conventional energy carriers.

Our sensitivity check consists of first raising the heating rod capacity and second doubling the capacity of the thermal storage. The alternative simulations have been conducted for the electricity price scenario of a medium-level tax relief of 5 Ct/kWh. Table 1 reports the results in terms of annual savings (€/a) for both types of hybrid systems. Altogether the differences are of a fairly small magnitude. Trivially, increases in storage capacities only matter if capacity limits are reached at many times during the year under the basic 500 l storage scenario. This is not the case in the simulations made. The impact of an increase in transformation capacities is likewise very limited. A more powerful version of a heating rod merely raises annual savings by 2 to 3%. Here it is the limitedness of hourly heat demand (together with storage losses) that reduces incentives to operate the heating rod at its maximum capacity. Therefore, additional savings from installing even more powerful hybrid components will become smaller and smaller, not mentioning the increasing technical difficulties of their integration into the heating system of a family house. It seems thus needless to undertake further simulations in this parameter area.

Table 1: Sensitivity of annual savings of heating expenses

System type	Oil/electricity hybrid				Gas/electricity hybrid			
	Status Quo scenario		Trend scenario		Status Quo scenario		Trend scenario	
Capacity thermal storage	500l	1000l	500l	1000l	500l	1000l	500l	1000l
Capacity heating rod	Annual savings of heating expenditures (€)							
6 kW	15.28	17.21	26.61	29.23	11.83	11.84	22.20	24.58
9 kW	15.42	17.36	27.41	30.06	12.11	12.13	22.66	25.08

6. Discussion

We have examined the economic incentives to use decentralized P2H in gas and oil condensing boiler systems. Our analysis shows that without a considerable tax relief the financial incentives to invest in such technologies are not sufficiently large. Given the complex environment in which we operate, our results are nevertheless subject to a range of assumptions, whose criticality we need to address. First of all, this concerns our electricity price simulations. As

mentioned before, they are based on two main assumptions, of which the second one, the procedure of upscaling the generation patterns of 2012, should be discussed in more detail. By simply upscaling the actual generation of wind and solar power in 2012, we implicitly assume that the additional RES-E capacities will exhibit the same generation patterns as the existing ones. This seems unrealistic, because additional wind turbines and solar panels will not be installed at the same locations. Instead, future instalment will have to focus on places with less decent conditions concerning wind speeds or solar irradiation. As a consequence, we probably overestimate future volatility in electricity wholesale prices and thereby also the level of cost savings from P2H. However, integrating a location analysis into our approach would require adding a range of critical assumptions concerning the future RES-E locations.

Furthermore, we do not consider adaptations of the dispatchable power plant fleet due to structural change in the electricity system like the expansion of RES-E capacities, grid improvements, progress regarding RES-E forecast quality and demand response measures. Hence, we rather conducted a conservative estimation of the prices and therefore provide an upper bound for the price reducing effect of RES-E. Another important limitation of our analysis is that we do not model the feedback of P2H-related electricity consumption on the wholesale electricity price. The more P2H systems will be installed, the greater is the demand for electricity in times of low electricity prices and the lower are the incentives to use the hybrid technology.

In addition, the simulated heat profiles for the year 2020 demand further considerations. As mentioned in section 4.2, the final energy consumption was calculated using values for an assumption-based average SFH with a certain thermal insulation status, a certain boiler configuration and a corresponding specific energy demand. In this regard, we did not account for the construction of new buildings and for all possible refurbishment measures affecting the energy efficiency of the building, which would reduce heat demand in the future. In fact, the energy demand in space heating and DHW has steadily decreased, even though the living area has increased (Destatis, 2013a,b). Hence, it is likely that the need for energy in the respective buildings in 2020 is overestimated.

Another limitation of our analysis is the prerequisite of an existing ICT at household level (e.g. smart meters) in 2020. Despite an explicit political will on both the national and the European level to foster the roll-out of such information and communication technologies, there are still concerns e.g. regarding data protection laws. Based on in-depth expert interviews Muench et al. (2014) provide an analysis of barriers which hamper the roll-out of smart technologies. In line with Römer et al. (2012) they identify among others a stable regulatory and legal framework as one key enabler for a better diffusion of these technologies. Furthermore, it is at least questionable if the electricity retailers have incentives to provide real time price signals with

an hourly resolution. Retailers may prefer to offer dynamic tariffs in the form of on-peak and off-peak tariffs. By this, they could smooth their load profiles without exposing their customers to extreme price signals which could lead to non-consumption. In all, we can conclude from the majority of these caveats that we still overestimate the expected savings through P2H at household level to some degree.

7. Conclusion and policy implications

The purpose of this paper was to assess the future economic potential of P2H in decentralized heating systems in Germany. Savings of heating expenses by switching from conventional gas and oil condensing boiler systems to hybrid systems in which boilers are equipped with an electric heater were simulated for the year 2020. Our analysis shows that in spite of a great technical demand potential, the likely financial benefits of P2H-hybrid systems at household level will continue to be small, if the electricity used for heating purposes is not heavily subsidized. This is the case even though electricity wholesale prices can be predicted to become both lower on average and more volatile. The main reason is the size of the publicly influenced components of the electricity price at final user level in Germany. Against this backdrop, policy makers have to decide whether creating a more attractive environment for decentralized P2H is compatible with other policy goals in the context of the German energy transition. Policy instruments, as the electricity tax-relief for P2H purposes, can and should be judged by their societal opportunity costs as well as their adequacy regarding related policy goals. Currently, several energy and environmental goals are pursued with differing and sometimes conflicting instruments in Germany. For example, in addition to the decarbonization targets in the electricity and heating sector there are ambitious targets for the development of e-mobility and energy-efficiency in the housing sector in Germany (Die Bundesregierung (2009), EEWärmeG §1). In this regard, it could be argued that subsidizing the refurbishment of fossil fuel operated heating systems is to some extent counterproductive, at least against the backdrop of ambitious decarbonization targets if there are less carbon-intensive technologies (e.g. solar thermal energy or improved thermal insulation). Nevertheless, at the very end it remains a political choice.

According to the discussed limitations of our analysis there are remaining questions for future research. It would be interesting to develop a fundamental market model which incorporates the interplay of different flexibility options, the feedback on the electricity prices as well as the societal opportunity costs of different flexibility options. Another important step would be a spatially disaggregated analysis of the P2H potential, especially with regard to regions with high RES-E in-feed at distribution grid level.

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Appendices

Appendix A1: Simulation of electricity spot market

$$P_t = g_t(D_t - W_t - S_t) + Z_t \quad (1)$$

$$g_t = a_0 + a_1 R_t + a_2 R_t^2 + a_3 R_t^3 \quad (2)$$

$$Z_t = \phi_1 Z_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1} \quad (3)$$

$$\epsilon_t = \sigma_t \eta_t \quad (4)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (5)$$

$$\eta_t \sim t(\nu) \quad (6)$$

Table A1: Polynomial regression: Multiple R-squared (0.7413), adjusted R-squared (0.7413)

Coefficient	Estimate	Std. error	t value	Pr(> t)
a_0	-1.12E+02	2.51E+00	-44.49	<2e-16
a_1	7.98E-03	1.76E-04	45.25	<2e-16
a_2	-1.43E-07	4.01E-09	-35.62	<2e-16
a_3	9.71E-13	2.95E-14	32.91	<2e-16

Table A2: ARMA(1,1)-GARCH(1,1): AIC (5.474995), BIC (5.477656), SIC (5.474995), HQIC (5.475871)

Coefficient	Estimate	Std. error	t value	Pr(> t)
ϕ_1	0.7785	0.0061	128.6595	0.0000
θ_1	0.0803	0.0098	8.1808	0.0000
α_0	6.1559	0.4484	13.7294	0.0000
α_1	0.3038	0.0204	14.8714	0.0000
β_1	0.4047	0.0309	13.1014	0.0000
ν	3.9125	0.1248	31.3445	0.0000

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \epsilon_t + \theta_1 \epsilon_{t-1} \quad (3b)$$

$$Z_t = \phi_1 Z_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} \quad (3c)$$

Table A3: ARMA(2,1)-GARCH(1,1): AIC (5.475008), BIC (5.478113), SIC (5.475008), HQIC (5.47603)

Coefficient	Estimate	Std. error	<i>t</i> value	Pr(> <i>t</i>)
ϕ_1	0.7043	0.0819	8.5951	0.0000
ϕ_2	0.0602	0.0664	0.9067	0.3646
θ_1	0.1539	0.0811	1.8972	0.0578
α_0	6.1445	0.4481	13.7117	0.0000
α_1	0.3038	0.0204	14.8641	0.0000
β_1	0.4055	0.0309	13.1250	0.0000
ν	3.9109	0.1248	31.3466	0.0000

Table A4: ARMA(1,2)-GARCH(1,1): AIC (5.474954), BIC (5.478059), SIC (5.474954), HQIC (5.475977)

Coefficient	Estimate	Std. error	<i>t</i> value	Pr(> <i>t</i>)
ϕ_1	0.7853	0.0077	101.6202	0.0000
θ_1	0.0718	0.0115	6.2201	0.0000
θ_2	-0.0132	0.0099	-1.3335	0.1824
α_0	6.1193	0.4483	13.6513	0.0000
α_1	0.3034	0.0204	14.8439	0.0000
β_1	0.4072	0.0310	13.1549	0.0000
ν	3.9112	0.1248	31.3458	0.0000

Appendix A2: Simulation of heat profiles

Heat load profiles:

$$Q_D = h(T_D) \left[\frac{Q_N}{\sum_i^N h(T_{D_i})} \right] \quad \text{with} \quad h(T_D) = \left[\frac{A}{1 + \left(\frac{B}{T-40} \right)^C} + D' \right]$$

Q_D = heat load at day D

A, B, C, D' = specific regional parameters (sigmoid-coefficients)

Q_N = heat load per year (in kWh)

$h(T_{D_i})$ = sigmoid function for day D

$$T = \frac{T_t + 0.5T_{t-1} + 0.25T_{t-2} + 0.125T_{t-3}}{1 + 0.5 + 0.25 + 0.125}$$

Time series for the daily average outside temperature for reference weather stations in each federal state in 2012 (WebWerdis: <http://www.dwd.de/webwerdis>). Baden Württemberg (Stuttgart-Echterdingen), Bavaria (Regensburg), Berlin (Berlin-Tegel), Brandenburg (Neuruppin), Bremen (Bremen), Hamburg (Hamburg-Fuhlsbüttel), Hesse (Fritzlar), Lower Saxony (Hannover), Mecklenburg-Vorpommern (Schwerin), North Rhine Westphalia (Düsseldorf), Rhine-Palatinate (Hahn), Saarland (Saarbrücken-Ensheim), Saxony (Dresden-Kloetz), Saxony-Anhalt (Magdeburg), Schleswig-Holstein (Schleswig), Thuringia (Erfurt-Weimar)

Table A5: Respective values for the federal states and (re-)installations of oil[†]- and gas[‡]-fired condensing boilers 2015 - 2020.

Federal State	Q_N	Oil^{\dagger}	Gas^{\ddagger}	A	B	C	D'
Baden Württemberg	18 894	53 246	363 802	3.0385	-37.1829	5.6644	0.0957
Bavaria	18 894	67 098	451 911	3.0217	-37.1823	5.6477	0.0950
Berlin	18 894	4 244	48 317	3.0553	-37.1836	5.6810	0.09959
Brandenburg	18 894	6,803	96,635	3.0217	-37.1823	5.6477	0.1157
Bremen	18 894	1 229	19,895	3.0890	-37.1849	5.7137	0.08141
Hamburg	18 894	1 918	36 949	3.0722	-37.1842	5.6975	0.0897
Hesse	18 894	28 892	210 323	3.0553	-37.1836	5.6810	0.0950
Lower Saxony	18 894	4 462	59 686	3.0217	-37.1823	5.6477	0.1145
Mecklenburg-Vorp.	18 894	26 983	335 380	3.0217	-37.1823	5.6477	0.1096
North Rhine-Westphalia	18 894	51 222	582 652	3.0217	-37.1823	5.6477	0.0877
Rhine-Palatinate	18 894	24 891	179 059	3.0385	-37.1829	5.6644	0.0933
Saarland	18 894	6 387	45 475	3.0722	-37.1842	5.6975	0.0930
Saxony	18 894	10 888	122 215	3.0385	-37.1829	5.6644	0.1122
Saxony-Anhalt	18 894	8 060	88 108	3.0217	-37.1823	5.6477	0.1181
Schleswig-Holstein	18 894	12 557	122 215	3.0385	-37.1829	5.6644	0.1063
Thuringia	18 894	7 844	79 582	3.0217	-37.1823	5.6477	0.1169

Appendix A3: Optimization model to determine cost savings

Symbols:

Index

$t = 1, \dots, T$: Hours of the year 2020 ($T = 8760$)

$b \in \{o, g\}$: Fuel type (heating oil, gas)

Natural constant

c : Specific heat capacity of water (4182 *joule*/[*kg* · C°])

Parameters

ν : Efficiency of heating rod (set equal to 1 in the simulations)

m : Capacity of thermal storage (in *liters*)

T^{min} : Maximum temperature within the thermal storage (in C°)

T^{max} : Minimum temperature within the thermal storage (in C°)

\bar{S} : Capacity of heating rod (in *kW*)

p^b : Price of fossil fuel b (in *Euro/kWh*)

Exogenous variables

p_t^S : Electricity price in t (in *Euro/kWh*)

V_t : Heat demand in t (in *kWh*)

Endogenous variables

S_t : Amount of electricity (in *kWh*) used in heat generation in t

F_t^b : Amount of fossil fuel (in *kWh*) used in heat generation in t

T_t : Temperature within the thermal storage at the end of t (in C°)

Q_t : Heat energy (in *kWh*) generated in t

E_t : Standing losses of heat energy (in *kWh*) stored
within the thermal storage during t

Equations:

Objective function:

$$\min. \sum_{t=1}^T p_t^S S_t + p^b F_t^b \quad (1)$$

Side conditions:

$$S_t \geq 0, S_t \leq \bar{S}, F_t^b \geq 0, T_t \geq T^{min}, T_t \leq T^{max} \quad (2)$$

$$Q_t = \eta S_t + F_t^b \quad (3)$$

$$T_t = T_{t-1} + \Delta T_{t-1,t} \quad (4)$$

$$\Delta T_{t-1,t} = \frac{3600000 \frac{J}{kWh}}{c \ m} (Q_t - V_t - E_t) \quad (5)$$

$$E_t = \frac{0.08532 \frac{kWh}{C^\circ} T_{t-1} - 2.11937}{24} \quad (6)$$

Explanations:

- (1): Annual costs of heat generation (to be minimized)
- (2): Non-negativity and capacity constraints
- (3): Equation determining the amount of heat energy generated
- (4): Equation of motion for storage temperature
- (5): Equation determining the temperature changes
- (6): Approximated relation between storage temperature and the amount of stored heat lost during an hour due to heat dissipation (as defined by the norm DIN EN 304)