RUHR

ECONOMIC PAPERS

Hamed M. Moghadam

# Price and Non-Price Competition in Oligopoly – An Analysis of Relative Payoff Maximizers

#575

# Imprint

## Ruhr Economic Papers

Published by

Ruhr-Universität Bochum (RUB), Department of Economics
Universitätsstr. 150, 44801 Bochum, Germany

Technische Universität Dortmund, Department of Economic and Social Sciences
Vogelpothsweg 87, 44227 Dortmund, Germany

Universität Duisburg-Essen, Department of Economics
Universitätsstr. 12, 45117 Essen, Germany

Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI)
Hohenzollernstr. 1-3, 45128 Essen, Germany

## Editors



## Editorial Office



## Ruhr Economic Papers #575

Responsible Editor: Wolfgang Leininger



ISSN 1864-4872 (online) – ISBN 978-3-86788-664-2

Ruhr Economic Papers #575

Hamed M. Moghadam

# Price and Non-Price Competition in Oligopoly – An Analysis of Relative Payoff Maximizers

Ruhr Graduate School in Economics

technische universität dortmund

## Bibliografische Informationen der Deutschen Nationalbibliothek

Die Deutsche Bibliothek verzeichnet diese Publikation in der deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über: *http://dnb.d-nb.de* abrufbar.

http://dx.doi.org/10.4419/86788664
ISSN 1864-4872 (online)
ISBN 978-3-86788-664-2

Hamed M. Moghadam[1]

# Price and Non-Price Competition in Oligopoly – An Analysis of Relative Payoff Maximizers

## Abstract

*Do firms under relative payoffs maximizing (RPM) behavior always choose a strategy profile that results in tougher competition compared to firms under absolute payoffs maximizing (APM) behavior? In this paper we will address this issue through a simple model of symmetric oligopoly where firms select a two dimensional strategy set of price and a non-price variable known as quality simultaneously. In conclusion, our results show that equilibrium solutions of RPM and APM are distinct. We further characterize the comparison between these two equilibrium concepts. In particular, RPM does not always lead to stricter competition compared to the Nash equilbrium (APM). In fact, the comparison between two equilibrium concepts is influenced by the parameters of demand curve and cost function. The conditions, derived in this paper, determine under which circumstances RPM induces more competition or less competition w.r.t the price or non-price dimension.*

*JEL Classification: C73, D21, D43, L13, L15*

*Keywords: Relative payoffs maximizing (RPM); quality; price; oligopoly*

*September 2015*

*1 Hamed Markazi Moghadam, RGS Econ and TU Dortmund.* – *I am indebted to Wolfgang Leininger and Lars Metzger for their helpful comments and moreover I would like to thank the conference participants at Thessaloniki, Corvinus University of Budapest, SING11 and European Meeting on Game Theory in St. Petersburg.*– *All correspondence to: Hamed Markazi Moghadam, TU Dortmund University, Department of Economics and Social Science, 44221 Dortmund Germany, e-mail: hamed.moghadam@tu-dortmund.de*

## 1. Introduction

It is a well-known result by Schaffer (1988[14],1989[15]) that the concept of finite population evolutionary stable strategy (FPESS) can be characterized by *relative* payoff maximization and this solution concept is different from Nash equilibrium or absolute payoff maximization. As Schaffer explained agents in economic and social environment survive in evolutionary process, if they can perform better than their opponents and so players adhere to *relative* payoffs maximizing (RPM) rather than *absolute* payoffs maximizing (APM) behavior. The behavior implied by RPM or spiteful behavior (Hamilton (1970)[10]) leads to more competition between firms in a cournot oligopoly game and as Vega Redondo (1997)[21] revealed Walrasian equilibrium turns out to be unique stochastically stable state in the symmetric Cournot oligopoly.

In this paper, we consider a symmetric oligopoly game where each firm has a two dimensional strategy set of price and non-price. Using a non-price strategy by firms in oligopoly competition is common since firms can be exceedingly competitive in price strategy. Therefore firms may also decide to compete in another dimension of non-price strategy. Hereafter we refer to this non-price strategy as quality. Moreover firm‘s cost function structure, considered in the present paper, follows from the literature in IO in which two different assumptions are made about the nature of cost function. First, we assume that quality improvement requires fixed costs, while variable costs do not alter with quality, for instance, when firms invest in R&D and advertising activities to improve quality. Second, we study variable cost of quality improvement let's say a situation that firms entail more expensive raw materials or inputs to produce an output. Both equilibrium concepts of FPESS and Nash are analyzed and compared under these two assumptions.

Vega-Redondo(1997)[21], Tanaka(1999[17],2001[19]), Apesteguia et al. (2010)[4] and Leininger and Moghadam (2014)[13] investigate the equivalence of evolutionary equilibrium and Walrasian competitive equilibrium in symmetric and asymmetric Cournot oligopoly. Moreover Tanaka (2000)[18] studies evolutionary game theoretical models for price-setting and for quantity-setting oligopoly with differentiated goods. Tanaka(2000, Proposition 2)[18] show that if the goods of firms are substitutes, FPESS price $p^*$ is between Nash equilibrium price $p^N$ and competitive price and if the goods are complements, FPESS price is higher than both Nash equilibrium and competitive price. However, in our model with the existence of non-price strategy, when goods are substitutes RPM firms may engage in less price competition i.e. $p^* > p^N$ ( In fact, it uses a non-price strategy to soften the price competition). And when goods are complements, RPM firms may possibly decide on more price competition i.e. $p^* < p^N$.

The plan of paper is as follows: in the next section we explain the model and its assumptions. Section 3 analyzes the existence of FPESS and Nash equilibria and their distinctions in the model of quality improvement with fixed cost while section 4 examines the link between these equilibrium concepts in a model of producing quality with variable costs. Then section 5 concludes the paper.

## 2. The Model

We assume an industry of $i = 1, ..., n$ firms, each offering quantity amount $x_i$ of a product that may vary in its quality $q_i$ and its price $p_i$. It is also assumed that non-price variable or quality is a measurable attribute with values in the interval $[0, \infty)$. The quality level has a lower bound that is known as zero quality or minimum technologically feasible quality level.

Following Dixit (1979)[8], demand functions for goods can be written as follow

$$x_i = D_i(\mathbf{p}, \mathbf{q}), i = 1, ..., n \tag{1}$$

where $\mathbf{p} = [p_1, p_2, ..., p_n]$ and $\mathbf{q} = [q_1, q_2, ..., q_n]$ In which an increase in any $p_j$ and $q_j$ raises or lowers each $x_i$ dependent on whether product pair $(i, j)$ are complements or substitutes. Moreover we assume that the demand function $D_i$ for each firm $i$ is affected more by changes in its own price and quality than those of its competitors (Tirole 1990)[20].

**Assumption 1.** $D_i(\boldsymbol{p}, \boldsymbol{q})$ *are continuous, twice differentiable and concave functions, and satisfy the following relations:*

$$\frac{\partial D_i}{\partial p_i} < 0, \ and \ |\frac{\partial D_i}{\partial p_i}| > |\frac{\partial D_j}{\partial p_i}| \quad \forall i = 1, ..., n, j \neq i.$$
$$\frac{\partial D_i}{\partial q_i} > 0, \ and \ \frac{\partial D_i}{\partial q_i} > |\frac{\partial D_j}{\partial q_i}| \quad \forall i = 1, ..., n, j \neq i.$$

$$\begin{cases} \frac{\partial D_j}{\partial p_i} < 0, \frac{\partial D_j}{\partial q_i} > 0 & if\ (i,j) are\ complements\ goods \\ \frac{\partial D_j}{\partial p_i} > 0, \frac{\partial D_j}{\partial q_i} < 0 & if\ (i,j) are\ substitutes\ goods \end{cases}$$

Concerning the cost function, on the one hand in the economic literature, Shaked and Sutton (1987)[16] highlight that quality improvement can consist of increase in fixed cost and/or variable cost. More recently, also Berry and Waldfogel (2010)[6] study product quality and market size. They consider two different types of industries including industries producing quality mostly with variable costs (like restaurant industry) and industries that produce quality mostly with fixed costs (like daily newspapers). Further, Brecard (2010)[7] investigates also the effects of the introduction of a unit production cost in vertical model with fixed cost of quality improvement. On the other hand, in management literature, for instance Banker et al.(1998)[5] assume that total cost function of firm i is affected by quality choice in both fixed cost and variable cost. Further they assert that it is frequently phenomenon that variable production costs decline when quality is improved; e.g., when a higher quality (higher precision) product produced by robot have less needs of direct labor hours. In general, here we assume that the quality level selected by firm influences its total cost through two distinctive ways; that is to say, fixed cost or variable cost like the following assumption 2.

**Assumption 2.** *The cost function for firm i as a general rule is*

$$C_i(x_i, q_i) = F_i(q_i) + V_i(x_i, q_i) \tag{2}$$

*And it is assumed to be one of the following two forms*

1. $\frac{\partial V_i}{\partial q_i} = 0$ *i.e.* $C_i(x_i, q_i) = F_i(q_i) + V_i(x_i)$

2. $\frac{\partial F_i}{\partial q_i} = 0$ *i.e.* $C_i(x_i, q_i) = V_i(x_i, q_i)$

*$F(.)$ and $V(.)$ are increasing and convex functions with respect to each of their arguments and all fixed costs that are not related to the quality, without loss of generality, are normalized to zero.*

## 3. Quality improvement with fixed costs

In this section we assume that the quality affects technology through only fixed cost i.e.$C_i(x_i, q_i) = F_i(q_i) + V_i(x_i)$. We analyze the outcomes of Nash equilibrium and evolutionary equilibrium of this game and then we provide a comparison between the two equilibrium concepts and discuss the results.

Profit function of firm $i$ is defined as follow

$$\pi_i(\mathbf{p}, \mathbf{q}) = p_i D_i(\mathbf{p}, \mathbf{q}) - F_i(q_i) - V_i(D_i(\mathbf{p}, \mathbf{q})) \quad i = 1, 2, ..., n \tag{3}$$

The strategic variables are price and quality. Since the interaction between price and quality strategies of the firms only occur through the common demand function, price vector $\mathbf{p} = [p_1, p_2, ..., p_n]$ and quality vector $\mathbf{q} = [q_1, q_2, ..., q_n]$ can be written from the point of view of firm $i$, respectively as $[p_i, \mathbf{p}_{-i}]$ and $[q_i, (\mathbf{q}_{-i})]$

Let the number of firms $n$ be fixed. Consider a simultaneous move game where each firm chooses a pair of quality- price. We assume that all firms produce a strictly positive quantity in equilibrium. So we have the following definition of Nash equilibrium.

**Definition 1.** *A Nash equilibrium in an oligopoly competition is given by a price vector $\boldsymbol{p}^N$ and quality vector $\boldsymbol{q}^N$ such that each firm maximizes its profit.*

$$(p_i^N, q_i^N) = \arg\max_{p_i, q_i} \pi_i(p_i, q_i, \boldsymbol{p}_{-i}^N, \boldsymbol{q}_{-i}^N) \quad \forall i = 1, ..., n \tag{4}$$

The first order conditions for the profit maximization with respect to $p_i$ and $q_i$ are as follows:

$$\frac{\partial \pi_i}{\partial p_i} = D_i + \frac{\partial D_i}{\partial p_i} p_i - \frac{\partial V_i}{\partial D_i} \frac{\partial D_i}{\partial p_i} = 0 \tag{5}$$

$$\frac{\partial \pi_i}{\partial q_i} = \frac{\partial D_i}{\partial q_i} p_i - \frac{\partial F_i}{\partial q_i} - \frac{\partial V_i}{\partial D_i}\frac{\partial D_i}{\partial q_i} = 0 \tag{6}$$

Equation (5) is the familiar equality between marginal revenue and marginal cost. Besides equation (6) states that the marginal revenue related with one unit increase in quality level is equal to the marginal cost of producing this quality.

In symmetric infinite population games, it is widely verified that the concept of evolutionary stable strategy is a refinement of Nash equilibrium. However, in finite population framework, the behavior implied by evolutionary stability may have distinctive features from Nash strategic behavior. The reason for this is as follow: when one player mutates from ante-adopted strategy to a new strategy in a population with small number of players, both incumbent and mutant players do not encounter a same population profile. In fact, mutant player confronts with a homogenous profile of $n-1$ incumbent players and incumbent players face a profile of one single mutant and $n-2$ other incumbent players.

Recall that firm's strategy choices are two dimensions including price and quality levels. Then consider a state of the system where all firms' strategy sets are the same and suppose that one firm experiments a new different strategy. We say that a state is evolutionary stable, if no mutant firm which chooses a different strategy can realize higher profits than the firms which employ the incumbent strategy. In other words, no mutant strategy can invade a population of incumbent strategists successfully.

Formally, consider a state where all firms choose the same strategies $(p^*, q^*)$. This state $(p^*, q^*)$ is *finite population evolutionarily stable strategy* (FPESS) when one mutant firm (an experimenter) chooses a different strategy $(p^m, q^m) \neq (p^*, q^*)$ its profit must be smaller than the profits of incumbent firms (the rest of firms). Formally speaking, we have the following definition by assuming firm $i$ as mutant firm:

**Definition 2.** *$(p^*, q^*)$ is FPESS if $\pi_i(\boldsymbol{p}, \boldsymbol{q}) < \pi_j(\boldsymbol{p}, \boldsymbol{q}) \forall j \neq i$ and all $(p^m, q^m) \neq (p^*, q^*), i = 1, ..., n$. Where $\boldsymbol{p} = [p^*, ..., p^*, p_i = p^m, p^*, ..., p^*]$ and $\boldsymbol{q} = [q^*, ..., q^*, q_i =$*

$q^m, q^*, ..., q^*]$. [2]

According to Schaffer (1989)[15], in a so-called *playing the field* game, we can also find a FPESS as the solution of following *relative* payoff optimization problem

$$(p^*, q^*) = \arg\max_{p^m, q^m} \varphi_i = \pi_i(\mathbf{p}, \mathbf{q}) - \pi_j(\mathbf{p}, \mathbf{q}) \tag{7}$$

Note that this means a FPESS is a Nash equilibrium for relative payoffs maximizing (RPM) firms.

**Proposition 1.** *Consider an industry where firms produce quality with fixed costs. If the following condition* $-2\frac{\partial D_i}{\partial p_i}\frac{\partial^2 F_i}{\partial q_i^2} > (\frac{\partial D_i}{\partial q_i})^2$ *holds, then FPESS equilibrium and symmetric Nash equilibrium both exist and are different. In Nash equilibrium, a firm's marginal rate of substitution between quality and price is* $\frac{\partial F_i}{\partial q_i}/D_i$. *Whereas, in FPESS equilibrium, the relative marginal rate of substitution between quality and price is* $\frac{\partial F_i}{\partial q_i}/D_i$.

*Proof.* First of all, rephrasing both FOCs of equations 5 and 6, we obtain

$$-\frac{\frac{\partial D_i}{\partial q_i}}{\frac{\partial D_i}{\partial p_i}} = \frac{\partial F_i}{\partial q_i}/D_i \tag{8}$$

Furthermore, by substitution of equation (3) in the optimization problem of (7), we obtain

$$\varphi_i = D_i(\mathbf{p}, \mathbf{q})p^m - V_i(D_i(\mathbf{p}, \mathbf{q}) - F_i(q^m) - D_j(\mathbf{p}, \mathbf{q})p^* + V_j(D_j(\mathbf{p}, \mathbf{q}) - F_j(q^*)$$

Given that $p_i = p^m$ and $q_i = q^m$, and $\forall j \neq i p_j = p^*$ and $q_j = q^*$.

Then the first order conditions for maximization of $\varphi$ with respect to $p^m$ and $q^m$ are as follows:

$$\frac{\partial \varphi_i}{\partial p_i} = D_i + \frac{\partial D_i}{\partial p_i}p^m - \frac{\partial V_i}{\partial D_i}\frac{\partial D_i}{\partial p_i} - \frac{\partial D_j}{\partial p_i}p^* + \frac{\partial V_j}{\partial D_j}\frac{\partial D_j}{\partial p_i} = 0$$
$$\frac{\partial \varphi_i}{\partial q_i} = \frac{\partial D_i}{\partial q_i}p^m - \frac{\partial F_i}{\partial q_i} - \frac{\partial V_i}{\partial D_i}\frac{\partial D_i}{\partial q_i} - \frac{\partial D_j}{\partial q_i}p^* + \frac{\partial V_j}{\partial D_j}\frac{\partial D_j}{\partial q_i} = 0$$

[2]Clearly this definition includes any one-dimensional deviation (like $(p^m, k^*)$ and $(p^*, k^m)$) by mutant.

In symmetric situations, by imposing $p^m = p^*, q^m = q^*$ and $\frac{\partial V_i}{\partial D_i} = \frac{\partial V_j}{\partial D_j}$, FOC's can be rewritten as

$$D_i + (p^* - \frac{\partial V_i}{\partial D_i})(\frac{\partial D_i}{\partial p_i} - \frac{\partial D_j}{\partial p_i}) = 0 \tag{9}$$

$$(p^* - \frac{\partial V_i}{\partial D_i})(\frac{\partial D_i}{\partial q_i} - \frac{\partial D_j}{\partial q_i}) = \frac{\partial F_i}{\partial q_i} \tag{10}$$

Then combining these two equations we obtain

$$-\frac{\frac{\partial D_i}{\partial q_i} - \frac{\partial D_j}{\partial q_i}}{\frac{\partial D_i}{\partial p_i} - \frac{\partial D_j}{\partial p_i}} = \frac{\partial F_i}{\partial q_i}/D_i \tag{11}$$

By comparing with the solution from Nash equilibrium, i.e. $\frac{\partial F_i}{\partial q_i}/D_i = -\frac{\frac{\partial D_i}{\partial q_i}}{\frac{\partial D_i}{\partial p_i}}$, since the left-hand sides of both are distinct, so the solutions for Nash and FPESS equilibrium will be different.

To ensure the existence of a unique equilibrium as well, it is required to check that second order conditions have negative definite Hessian matrix.

Further, we need to be ensured about the solvability condition. Particularly, consider the following Hessian matrix

$$H_i = \begin{pmatrix} 2\frac{\partial D_i}{\partial p_i} + \frac{\partial^2 D_i}{\partial p_i^2}(p_i - \frac{\partial V_i}{\partial D_i}) - \frac{\partial^2 V_i}{\partial D_i^2}(\frac{\partial D_i}{\partial p_i})^2 & \frac{\partial D_i}{\partial q_i} + \frac{\partial^2 D_i}{\partial p_i \partial q_i}(p_i - \frac{\partial V_i}{\partial D_i}) - \frac{\partial^2 V_i}{\partial D_i^2}(\frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i}) \\ \frac{\partial D_i}{\partial q_i} + \frac{\partial^2 D_i}{\partial p_i \partial q_i}(p_i - \frac{\partial V_i}{\partial D_i}) - \frac{\partial^2 V_i}{\partial D_i^2}(\frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i}) & -\frac{\partial^2 F_i}{\partial q_i^2} + \frac{\partial^2 D_i}{\partial q_i^2}(p_i - \frac{\partial V_i}{\partial D_i}) - \frac{\partial^2 V_i}{\partial D_i^2}(\frac{\partial D_i}{\partial q_i})^2 \end{pmatrix}$$

It can be rewritten as follow

$$H_i = \begin{pmatrix} 2\frac{\partial D_i}{\partial p_i} & \frac{\partial D_i}{\partial q_i} \\ \frac{\partial D_i}{\partial q_i} & -\frac{\partial^2 F_i}{\partial q_i^2} \end{pmatrix} + (p_i - \frac{\partial V_i}{\partial D_i}) \begin{pmatrix} \frac{\partial^2 D_i}{\partial p_i^2} & \frac{\partial^2 D_i}{\partial p_i \partial q_i} \\ \frac{\partial^2 D_i}{\partial p_i \partial q_i} & \frac{\partial^2 D_i}{\partial q_i^2} \end{pmatrix} - \frac{\partial^2 V_i}{\partial D_i^2} \begin{pmatrix} (\frac{\partial D_i}{\partial p_i})^2 & \frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i} \\ \frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i} & (\frac{\partial D_i}{\partial q_i})^2 \end{pmatrix}$$

Since $D_i(.)$ is concave and $(p_i - \frac{\partial V_i}{\partial D_i}) > 0$ then it is required only that the first matrix be negative definite. That means we obtain the following condition

$$-2\frac{\partial D_i}{\partial p_i}\frac{\partial^2 F_i}{\partial q_i^2} > (\frac{\partial D_i}{\partial q_i})^2 \tag{12}$$

The above condition guarantee that $|H_i| < 0$ and solvability condition is satisfied. To ensure the existence of a unique evolutionary equilibrium as well, it is required to check that second order conditions have negative definite Hessian matrix.

$$H_i = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Where

$$a_{11} = 2\frac{\partial D_i}{\partial p_i} + \frac{\partial^2 D_i}{\partial p_i^2}(p_i - \frac{\partial V_i}{\partial D_i}) - \frac{\partial^2 D_j}{\partial p_i^2}(p^* - \frac{\partial V_j}{\partial D_j}) - \frac{\partial^2 V_i}{\partial D_i^2}(\frac{\partial D_i}{\partial p_i})^2 + \frac{\partial^2 V_j}{\partial D_j^2}(\frac{\partial D_j}{\partial p_i})^2$$

$$a_{12} = a_{21} = \frac{\partial D_i}{\partial q_i} + \frac{\partial^2 D_i}{\partial p_i \partial q_i}(p_i - \frac{\partial V_i}{\partial D_i}) - \frac{\partial^2 D_j}{\partial p_i \partial q_i}(p^* - \frac{\partial V_j}{\partial D_j}) - \frac{\partial^2 V_i}{\partial D_i^2}(\frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i}) + \frac{\partial^2 V_j}{\partial D_j^2}(\frac{\partial D_j}{\partial p_i}\frac{\partial D_j}{\partial q_i})$$

$$a_{22} = -\frac{\partial^2 F_i}{\partial q_i^2} + \frac{\partial^2 D_i}{\partial q_i^2}(p_i - \frac{\partial V_i}{\partial D_i}) - \frac{\partial^2 D_j}{\partial q_i^2}(p^* - \frac{\partial V_j}{\partial D_j}) - \frac{\partial^2 V_i}{\partial D_i^2}(\frac{\partial D_i}{\partial q_i})^2 + \frac{\partial^2 V_j}{\partial D_j^2}(\frac{\partial D_j}{\partial q_i})^2$$

This matrix is the summation of following matrixes

$$H_i = \begin{pmatrix} 2\frac{\partial D_i}{\partial p_i} & \frac{\partial D_i}{\partial q_i} \\ \frac{\partial D_i}{\partial q_i} & -\frac{\partial^2 F_i}{\partial q_i^2} \end{pmatrix} + (p_i - \frac{\partial V_i}{\partial D_i}) \begin{pmatrix} \frac{\partial^2 D_i}{\partial p_i^2} & \frac{\partial^2 D_i}{\partial p_i \partial q_i} \\ \frac{\partial^2 D_i}{\partial p_i \partial q_i} & \frac{\partial^2 D_i}{\partial q_i^2} \end{pmatrix} - (p^* - \frac{\partial V_j}{\partial D_j}) \begin{pmatrix} \frac{\partial^2 D_j}{\partial p_i^2} & \frac{\partial^2 D_j}{\partial p_i \partial q_i} \\ \frac{\partial^2 D_j}{\partial p_i \partial q_i} & \frac{\partial^2 D_j}{\partial q_i^2} \end{pmatrix}$$

$$-\frac{\partial^2 V_i}{\partial D_i^2} \begin{pmatrix} (\frac{\partial D_i}{\partial p_i})^2 & \frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i} \\ \frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i} & (\frac{\partial D_i}{\partial q_i})^2 \end{pmatrix} + \frac{\partial^2 V_j}{\partial D_j^2} \begin{pmatrix} (\frac{\partial D_j}{\partial p_i})^2 & \frac{\partial D_j}{\partial p_i}\frac{\partial D_j}{\partial q_i} \\ \frac{\partial D_j}{\partial p_i}\frac{\partial D_j}{\partial q_i} & (\frac{\partial D_j}{\partial q_i})^2 \end{pmatrix}$$

Since demand system by assumption 1 is concave, it is only sufficient that the first matrix be negative definite and we have also $\frac{\partial D_i}{\partial p_i} < 0$. Therefore solvability condition is identical to condition (12) derived in Nash equilibrium, that is $-2\frac{\partial D_i}{\partial p_i}\frac{\partial^2 F_i}{\partial q_i^2} > (\frac{\partial D_i}{\partial q_i})^2$.

□

Since the concept of evolutionary stability is based on relative performance, then its difference from Nash equilibrium depends on pair comparison types either substitute or complement. A strategy set that can survive in the game with relative payoff maximizing, first of all is not a Nash strategy set and second this difference with Nash equilibrium strategy set is subject to which pair of goods are competing.

In order to understand better, how these two equilibrium concepts are different, we need to make two assumptions about the structure of demand and cost functions. Firstly, we assume that fixed cost $F_i(q_i) = (f + \psi q_i^2)$ is increasing and convex in quality level $q_i$ since improving product's quality level require an initial investment by firms. Without loss of generality, we normalize $f$ to zero. So with a standard linear variable cost, the cost function for firm $i$ is assumed like the following form

$$C_i(x_i, q_i) = \psi q_i^2 + \nu x_i \tag{13}$$

Second, let's assume also demand function has a linear form as follows:

$$x_i = a - p_i + q_i + \beta \sum_{j=1, j\neq i}^{n} p_j - \gamma \sum_{j=1, j\neq i}^{n} q_j \tag{14}$$

Where $|\beta| < 1, |\gamma| < 1$ and $\beta \neq \gamma$.

The first and second restrictions on $\beta$ and $\gamma$ are implied by assumption 1 in which we assume that the demand function for firm $i$ is affected more by changes in its own price and quality than those of its competitors. And the third restriction ensures that solutions for Nash and FPESS equilibrium are different.

In the following propositions, we characterize the comparison between two equilibrium concepts.

**Proposition 2.** *Suppose that cost function and linear demand function are like equations (13) and (14). Then FPESS equilibrium leads to more quality competition compared to the Nash equilibrium quality i.e.*

$$q^* > q^N \quad \textit{if and only if} \quad \gamma > \bar{\gamma} = \tfrac{\beta}{2+\beta-n\beta}$$

*Proof.* See appendix 1. □

**Proposition 3.** *Suppose that cost function and linear demand function are like equations (13) and (14). Then FPESS equilibrium leads to more price competition compared to the Nash equilibrium i.e.*

$$p^* < p^N \quad \textit{if and only if} \quad \beta > \bar{\beta} = \tfrac{(1+\gamma-n\gamma)\gamma}{2\psi}$$

*Proof.* See appendix 1. □

Propositions 2 and 3 demonstrate that FPESS (RPM) does not always lead to tougher competition compared to the Nash equilbrium. In other words, RPM firm engages in more price (quality) competition if the price (quality) effect of other competitors on the demand of good $i$ i.e.$\beta(\gamma)$ is greater than the threshold $\bar{\beta}(\bar{\gamma})$. The conditions, derived in these two propositions, determine under which circumstances FPESS equilibrium induce more competition or less competition w.r.t the price or non-price dimension.

Therefore, the comparison between two equilibrium concepts is influenced by the parameters of demand curve and cost function. To see in more detail dissimilarities concerning both strategic behavior and evolutionary behavior, we further study the following numerical example. This numerical example helps us to recognize better how variations in $\beta$, $\gamma$ and market size $n$ can influence the comparison between FPESS and Nash equilibrium. Note that if the price effect of good $j$ on the demand of good $i$ ($\beta$) or the quality effect of good $j$ on the demand of good $i$ ($\gamma$) are both positive, the goods of the firms are substitutes and if $\beta$ and $\gamma$ are negative, each two goods are complements.

**Example 1.** *This numerical example is performed with fixed scenario* $\psi = 1/2$ *but varying* $\beta, \gamma$ *and* $n$*. Feasible ranges in case of substitute goods are* $0.01 < \beta < 1, 0.01 < \gamma < 1$ *and in case of complements goods are* $-1 < \beta < -0.01, -1 < \gamma < -0.01$ *and* $1 < n < 20$*. We plot the regions that satisfy the conditions of propositions 2 and 3 simultaneously in order to see what sort of parameters comply with the following circumstances:*

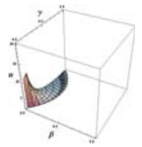
a) $p^* > p^N$ *and* $q^* > q^N$

b) $p^* < p^N$ *and* $q^* < q^N$

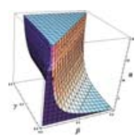
c) $p^* > p^N$ *and* $q^* < q^N$

d) $p^* < p^N$ *and* $q^* > q^N$

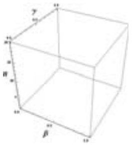
3D plots in the figures 1 and 2 illustrate all above circumstances (a – d) for substitute goods and complements goods respectively. We obtain the following results.



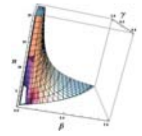
(a) $p^* > p^N$ and $q^* > q^N$



(b) $p^* < p^N$ and $q^* < q^N$

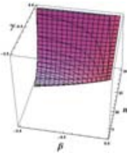


(c) $p^* > p^N$ and $q^* < q^N$

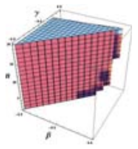


(d) $p^* < p^N$ and $q^* > q^N$

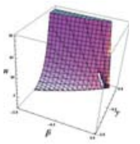
Figure 1: In Case of Substitute Goods



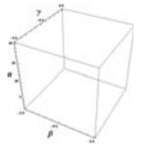
(a) $p^* > p^N$ and $q^* > q^N$



(b) $p^* < p^N$ and $q^* < q^N$



(c) $p^* > p^N$ and $q^* < q^N$



(d) $p^* < p^N$ and $q^* > q^N$

Figure 2: In Case of Complement Goods

**Result 1.** *If goods of firms are substitutes, FPESS equilibrium cannot lead to less price competition and less quality competition compared to Nash equilibrium.*

**Result 2.** *If goods of firms are complements, FPESS equilibrium cannot lead to more price competition and more quality competition compared to Nash equilibrium.*

Results 1 and 2 are demonstrated by figures 1-(c) and 2-(d) respectively. Since FPESS is a Nash equilibrium for relative payoff maximizing (RPM) firms, therefore one interpretation of these results is that, If goods are substitutes, less price competition (higher prices) and less quality competition are not feasible for a firm under RPM behavior. However if goods are complements, more price competition and more quality competition are not feasible for a firm under RPM behavior.

Tanaka (2000)[18] studies evolutionary game theoretical models for price-setting and for quantity-setting oligopoly with differentiated goods. Tanaka(2000, Proposition 2) shows that in price-setting oligopoly with differentiated goods, the difference between FPESS price and Nash price is due to whether the goods of firms are substitutes or complements. Thus, if the goods of firm are substitutes, FPESS price is less than Nash equilibrium price and if the goods are complements, FPESS price is higher than Nash equilibrium i.e. $p^* > p^N$ if goods are complements($\beta, \gamma < 0$); and $p^* < p^N$ if goods are substitutes ($\beta, \gamma > 0$). However, in our model with the existence of non-price strategy, when goods are substitutes a RPM firm may choose a less price competition i.e. $p^* > p^N$ (see Figure 1-(a); in fact it uses a non-price strategy to soften price competition). And when goods are complements, a RPM firm may possibly decide on more price competition i.e. $p^* < p^N$ (see Figure 2-(b)).

## 4. Quality improvement with variable costs

In this section, we study the effect of producing quality with variable costs on equilibriums comparison. This can be a situation that firms entail costly raw materials or inputs to produce an output. The variables in this model are characterized by $\hat{q}, \hat{p}$ and $\hat{x}$ so as to distinguish between them and their counterparts in the previous section.

The cost function for each firm here is given by $C_i(\hat{x_i}, \hat{q_i}) = V_i(\hat{x_i}, \hat{q_i})$. Accordingly, profit function for each firm $i$ defined as follows:

$$\pi_i(\hat{\mathbf{p}}, \hat{\mathbf{q}}) = \hat{p_i} D_i(\hat{\mathbf{p}}, \hat{\mathbf{q}}) - V_i(D_i(\hat{\mathbf{p}}, \hat{\mathbf{q}}), \hat{q_i})$$

Where price vector and quality vector defined respectively $\hat{\mathbf{p}} = [\hat{p_1}, \hat{p_2}, ..., \hat{p_n}]$ and $\hat{\mathbf{q}} = [\hat{q_1}, \hat{q_2}, ..., \hat{q_n}]$.

Then we derive the first order conditions that characterize the symmetric Nash equilibrium.

$$\frac{\partial \pi_i}{\partial \hat{p_i}} = D_i + \frac{\partial D_i}{\partial \hat{p_i}} \hat{p_i} - \frac{\partial V_i}{\partial D_i} \frac{\partial D_i}{\partial \hat{p_i}} = 0 \tag{15}$$

$$\frac{\partial \pi_i}{\partial \hat{q_i}} = \frac{\partial D_i}{\partial \hat{q_i}} \hat{p_i} - \frac{\partial V_i}{\partial \hat{q_i}} - \frac{\partial V_i}{\partial D_i} \frac{\partial D_i}{\partial \hat{q_i}} = 0 \tag{16}$$

Now we look for FPESS equilibrium using definition 2 and solving optimization problem (7), so we obtain the first order conditions as before:

$$\frac{\partial \varphi_i}{\partial \hat{p_i}} = D_i + \frac{\partial D_i}{\partial \hat{p_i}} \hat{p}^m - \frac{\partial V_i}{\partial D_i} \frac{\partial D_i}{\partial \hat{p_i}} - \frac{\partial D_j}{\partial \hat{p_i}} \hat{p}^* + \frac{\partial V_j}{\partial D_j} \frac{\partial D_j}{\partial \hat{p_i}} = 0$$
$$\frac{\partial \varphi_i}{\partial \hat{q_i}} = \frac{\partial D_i}{\partial \hat{q_i}} \hat{p}^m - \frac{\partial V_i}{\partial \hat{q_i}} - \frac{\partial V_i}{\partial D_i} \frac{\partial D_i}{\partial \hat{q_i}} - \frac{\partial D_j}{\partial \hat{q_i}} \hat{p}^* + \frac{\partial V_j}{\partial D_j} \frac{\partial D_j}{\partial \hat{q_i}} = 0$$

The only source of difference in FOC's arises from cost structure and in symmetric situations, by imposing $\hat{p}^m = \hat{p}^*$ and $\hat{q}^m = \hat{q}^*$, therefore we obtain our first order conditions like the following form:

$$D_i + (\hat{p}^* - \frac{\partial V_i}{\partial D_i})(\frac{\partial D_i}{\partial \hat{p_i}} - \frac{\partial D_j}{\partial \hat{p_i}}) = 0 \tag{17}$$

$$(\hat{p}^* - \frac{\partial V_i}{\partial D_i})(\frac{\partial D_i}{\partial \hat{q_i}} - \frac{\partial D_j}{\partial \hat{q_i}}) - \frac{\partial V_i}{\partial \hat{q_i}} = 0 \tag{18}$$

**Proposition 4.** *Consider an industry where quality is produced with only variable costs. If the following condition* $-2\frac{\partial D_i}{\partial \hat{p_i}} \frac{\partial^2 V_i}{\partial \hat{q_i}^2} > (\frac{\partial D_i}{\partial \hat{q_i}})^2$ *holds,then FPESS equilibrium and symmetric Nash equilibrium both exist and are different.*

*Proof.* The symmetric Nash equilibrium (rephrasing equation (15) and (16)) implies the following equality

$$\frac{\frac{\partial D_i}{\partial \hat{q}_i}}{\frac{\partial D_i}{\partial \hat{p}_i}} = \frac{\partial V_i}{\partial \hat{q}_i}/D_i$$

While combining two FOC's of FPESS equilibrium (equation (17) and (18)), we get

$$\frac{\frac{\partial D_i}{\partial \hat{q}_i} - \frac{\partial D_j}{\partial \hat{q}_i}}{\frac{\partial D_i}{\partial \hat{p}_i} - \frac{\partial D_j}{\partial \hat{p}_i}} = \frac{\partial V_i}{\partial \hat{q}_i}/D_i$$

Hence by comparison of both equalities, it is straightforward to see that two equilibriums are different.

With the same reasoning and calculation like the proposition 1, we can obtain the following solvability condition for both Nash and FPESS equilibriums.

$$-2\frac{\partial D_i}{\partial \hat{p}_i}\frac{\partial^2 V_i}{\partial \hat{q}_i{}^2} > (\frac{\partial D_i}{\partial \hat{q}_i})^2$$

This condition guarantees that Hessian matrix is negative definite.

□

**Remark.** Note that, if we assume that the marginal cost is invariant with respect to output level $x$ and it varies with the quality in the linear form i.e. $C_i(\hat{x_i}, \hat{q}_i) = V_i(\hat{x_i}, \hat{q}_i) = (\nu + \epsilon\hat{q}_i)\hat{x_i}$, then both first order conditions of equations (15) and (16) ( and also equations (17) and (18)) are identical. So we get a one equation and two unknowns, that is considered an underdetermined system where it has either infinitely many solutions or is inconsistent. In this case , the marginal rate of substitution between quality and price in absolute and relative terms are constant and equal.

$$\frac{\frac{\partial D_i}{\partial \hat{q}_i} - \frac{\partial D_j}{\partial \hat{q}_i}}{\frac{\partial D_i}{\partial \hat{p}_i} - \frac{\partial D_j}{\partial \hat{p}_i}} = \frac{\frac{\partial D_i}{\partial \hat{q}_i}}{\frac{\partial D_i}{\partial \hat{p}_i}} = \frac{\partial V_i}{\partial \hat{q}_i}/D_i = \epsilon$$

Our evolutionary analysis can be simply applied in an oligopoly-technology model of price competition with technology choice rather than quality choice, e.g. see Vives (2008)[22] and Acemoglu and Jensen (2013)[1]. In this type of games, firms decide about technology choice besides setting output or price. In fact, firm $i$ incurs a similar cost of $C_i(x_i, a_i) = F_i(a_i) + V_i(a_i, x_i)$ by choosing technology $a_i$ together with the quantity $x_i$ but the demand is not affected by the technology choice $a_i$.

Notice that in the present paper we do not study the dynamic concept of evolutionary stability. Alos-Ferrer and Ania (2005, Proposition 4)[2], based on a result of Ellison(2000)[9], show that in a symmetric N-player game with finite strategy set, a strictly globally stable ESS is the unique stochastic stable state of the imitation dynamics with experimentation. Further, Leininger (2006)[12] shows that any ESS of a quasi-submodular generalized aggregative game is strictly globally stable. Since the oligopoly game under analysis is an aggregative game and by assuming that our payoff function satisfies quasi-submodularity property then a static solution of FPESS would be sufficient in this context.

## 5. Conclusion

In the present paper, we have applied a concept of finite population evolutionarily stable strategy (FPESS) by Schaffer (1989)[15], in which agents in economic and social environment adhere to *relative* payoff maximizing rather than *absolute* payoff maximizing behavior, in an oligopoly framework. A simple model of firm competition with simultaneous price and quality choices has been analyzed with the aim of comparing FPESS and Nash equilibriums in oligopoly market. In general, the notion of FPESS and Nash equilibrium are not related. Ania (2008)[3] and Hehenkamp et al. (2010)[11] study this relationship in different classes of games and particularly in the framework of Bertrand oligopoly with homogenous product. Here in this paper, we consider a symmetric oligopoly game where each firm has two dimensional strategy set of price and quality and then we ask the following question whether firms under relative payoffs maximizing (RPM) behavior (implied by FPESS) choose a strategy profile closer to competitive strategy or under absolute payoffs maximizing (APM) behavior.

Mainly, non-price competition; i.e. quality competition is typically used to soften price competition. For example one competes spitefully w.r.t the non-price variable in order to do the best reply or even collude to some extent with respect to the price variable. As we know RPM usually leads to tougher competition, than APM. Hence according to this logic firms should engage more in quality competition than in price competition. The conditions derived for the comparison of RPM behavior and APM (best reply) behavior determine under which circumstances RPM firms engage more in quality competition or in price competition or in both. If goods are substitutes, RPM cannot lead to less price competition (higher prices) and less quality competition compared to APM equilibrium. That is, less price competition and less quality competition are not feasible for a firm under RPM behavior. However if goods are complements, more price competition and more quality competition are not feasible for a firm under RPM behavior.

**Appendix A.**

**The proof of propositions 2 and 3**

Assuming the cost function (13) and the linear demand function (14), equations (5) and (6) can be rewritten like the following

$$a-(1-(n-1)\beta)p^N+(1-(n-1)\gamma)q^N=p^N-\nu$$

$$p^N=\nu+2\psi q^N$$

Rearranging above equations,they yield $q^N$ and $p^N$ as follow:

$$q^N=\frac{a-(1-(n-1)\beta)\nu}{2\psi(2-(n-1)\beta)-(1-(n-1)\gamma)}$$

$$p^N=\nu+\frac{a-(1-(n-1)\beta)\nu}{(2-(n-1)\beta)-\frac{((1-(n-1)\gamma)}{2\psi}}$$

Likewise, equations (9)and (10) can be also inscribed along these lines:

$$(-1-\beta)(p^*-\nu)+a-(1-(n-1)\beta)p^*+(1-(n-1)\gamma)q^*=0$$

$$p^*=\nu+\frac{2\psi}{(1+\gamma)}q^*$$

After solving for $q^*$ and $p^*$ , we obtain

$$q^*=\frac{a-(1-(n-1)\beta)\nu}{\frac{2\psi}{1+\gamma}(2-(n-2)\beta)-(1-(n-1)\gamma)}$$

$$p^*=\nu+\frac{a-(1-(n-1)\beta)\nu}{(2-(n-2)\beta)-\frac{((1-(n-1)\gamma)(1+\gamma)}{2\psi}}$$

Therefore,we have $q^*>q^N$ if and only if $2\psi(2-(n-1)\beta)>\frac{2\psi}{1+\gamma}(2-(n-2)\beta)$. Simplifying this inequality, we get the condition of proposition 2

$$\gamma>\bar{\gamma}=\frac{\beta}{2+\beta-n\beta}$$

And we have $p^* < p^N$ if and only if $(2-(n-2)\beta) - \frac{((1-(n-1)\gamma)(1+\gamma)}{2\psi} > (2-(n-1)\beta) - \frac{((1-(n-1)\gamma)}{2\psi}$ and this inequality leads to the following condition

$$\beta > \bar{\beta} = \frac{(1+\gamma-n\gamma)\gamma}{2\psi}$$

**References**


[1] Acemoglu, D., Jensen, M.K., 2013. Aggregate Comparative Statics. Games and Economic Behavior 81, 27-49.

[2] Alós-Ferrer, C., Ania, A.B., 2005. The evolutionary stability of perfectly competitive behavior. EconTheory 26, 497-516.

[3] Ania, A., 2008. Evolutionary stability and Nash equilibrium in finite populations with an application to price competition. Journal of Economic Behavior and Organization 65, 472-488.

[4] Apesteguia, J., Huck, S., Oechssler, J., Weidenholzer, S., 2010. Imitation and the evolution of Walrasian behavior: Theoretically fragile but behaviorally robust. Journal of Economic Theory 145, 1603-1617.

[5] Banker, R.D., Khosla, I., Sinha, K.K.,1998. Quality and competition. Management Science 44 (9), 1179-1192.

[6] Berry, S., and Waldfogel, J., 2010. Quality and market size. Journal of Industrial Economics 58(1):1-31.

[7] Brecard, D.,2010. On production costs in vertical differentiation models, Economics Letters, 109, 183-186.

[8] Dixit, A., 1979. Quality and quantity competition. Review of Economic Studies. 46(4):587-599.

[9] Ellison, G., 2000. Basins of attraction, long-run stochastic stability, and the speed of step-by-step evolution. Review of Economic Studies 67, 17-45.

[10] Hamilton, W.D,1970. Selfish and spiteful behaviour in an evolutionary model. Nature 228, 1218-1220.

[11] Hehenkamp, B., Possajennikov, A., and Guse, T., 2010. On the equivalence of Nash and evolutionary equilibrium in finite populations. Journal of Economic Behavior and Organization, 73, 254-258.

[12] Leininger,W., 2006. Fending off one means fending off all: evolutionary stability in quasi-submodular games. Econ Theory 29, 713-719.

[13] Leininger, W., Moghadam, H.M., 2014. Evolutionary stability in asymmetric oligopoly: a non-Walrasian result. Ruhr economic papers #497.

[14] Schaffer, M.,1988. Evolutionarily stable strategies for a Finite Population and a variable contest size. J Theor Biol 132, 469-478.

[15] Schaffer, M.,1989. Are profit-maximisers the best survivors? J Econ Behav Org 12, 29-45.

[16] Shaked, A. and Sutton, J., 1987, Product Differentiation and Industrial Structure, Journal of Industrial Economics, 36, pp.131-146.

[17] Tanaka, Y., 1999. Long run equilibria in an asymmetric oligopoly. Economic Theory 14, 705-715.

[18] Tanaka, Y.,2000. Stochastically stable states in an oligopoly with differentiated goods: equivalence of price and quantity strategies. Journal of Mathematical Economics 34, 235-253.

[19] Tanaka, Y., 2001. Evolution to equilibrium in an asymmetric oligopoly with differentiated goods. International Journal of Industrial Organization 19, 1423 -1440.

[20] Tirole, J., 1990. The theory of industrial organization. The MIT press, Cambridge, MA.

[21] Vega-Redondo, F., 1997. The evolution of Walrasian behavior. Econometrica 65, 375-384.

[22] Vives, X., 2008. Innovation and competitive pressure. J. Ind. Econ. 56, 419-469.