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Christoph Kuzmics and Jan-Henrik Steg

Center for Mathematical Economics (IMW)
Bielefeld University
Universitätsstraße 25
D-33615 Bielefeld · Germany

e-mail: imw@uni-bielefeld.de
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On public good provision mechanisms with dominant strategies and balanced budget

Christoph Kuzmics[*]   Jan-Henrik Steg[†]

Abstract

Consider a mechanism for the binary public good provision problem that is dominant strategy incentive compatible (DSIC), ex-post individually rational (EPIR), and ex-post budget balanced (EPBB). Suppose this mechanism has the additional property that the utility from participating in the mechanism to the lowest types is zero for all agents. Such a mechanism must be of a threshold form, in which there is a fixed threshold for each agent such that the public good is not provided if there is an agent with a value below her threshold and is provided if all agents' values exceed their respective threshold. There are mechanism that are DSIC, EPIR, and EPBB that are not of the threshold form. Mechanisms that maximize welfare subject to DSIC, EPIR, and EPBB must again have the threshold form. Finally, mechanisms that are DSIC, EPIR, EPBB and that furthermore satisfy the condition that there is at least one type profile in which all agents can block the provision of the public good, also must be of the threshold form. As we allow individuals' values for the public good to be negative and positive, our results cover examples including bilateral trade, bilateral wage negotiations, a seller selling to a group of individuals (who then have joint ownership rights), and rezoning the use of land.

Keywords: Public good provision, asymmetric information, dominant strategy.

JEL subject classification: C72, D82, H41

1 Introduction

We are interested in a binary decision problem, between a status quo and one fixed alternative, that concerns multiple agents. Keeping the status-quo is costless. Implementing the alternative requires a fixed non-negative money amount. Each agent has a private net value (only known to this agent) for switching from the status quo to the alternative. This net value can be positive, zero, or negative. This setup covers the following examples.

[*]University of Graz, email: christoph.kuzmics@uni-graz.at

[†]Center for Mathematical Economics, Bielefeld University, email: jsteg@uni-bielefeld.de

Public good provision If all net values are commonly known to be non-negative, this setup is known as the public good provision problem, initiated by d'Aspremont and Gerard-Varet (1979) and Güth and Hellwig (1986). To give a concrete example: People living in a city contemplate the building of a new bridge. All people are in principle in favor of having this new bridge. The cost of building this bridge is positive.

Bilateral trade This is as in Myerson and Satterthwaite (1983). A seller owns an object. She contemplates selling it to one buyer. The status quo is not selling, i.e. the object stays with the seller. The alternative is selling, i.e. the object goes to the buyer. The seller's value for the good is positive, hence her net value for the alternative is negative. The buyer's value for the object, and thus her net value for the alternative, is positive. The cost of implementing the alternative is zero.

Bilateral wage negotiations An employer made a job offer to an employee. Only the salary has not yet been decided. The status quo is that the employee refuses this job offer. The alternative is that she takes it. The net value of the alternative is positive for the employer and negative for the employee. The cost of implementing the alternative is zero.

A seller selling to a group A seller owns an object. To give concrete examples, suppose it is the family castle or a private park. A group of individuals (e.g. the villagers of the village that houses the family castle or private park) is potentially interested in buying this object together, possibly for touristic reasons. The status quo is that the object remains with the seller, the alternative is that is goes to the villagers. The villagers would thus derive positive net value for implementing the alternative, the seller would have a negative net value for the alternative. The cost of implementing the alternative is zero (or positive and small due to legal fees).

Rezoning land A land-owner owns a piece of land, e.g. currently zoned as forest, and would like to use this land for a different purpose that affects neighbors (and possibly the state through a concern for others, including future generations). The status quo is that this land remains zoned as forest land. The alternative is that it is open to other uses, e.g. for property development. The land-owner has a positive net value for the alternative, the neighbors have a negative net value for the alternative. The cost of implementing the alternative is zero (or positive and small due to legal fees).

Supporting the social planner we are interested in identifying welfare maximizing mechanisms for these problems subject to some feasibility and plausibility constraints.

1.1 What we know

Our starting point is the textbook treatment of the public good provision problem in Börgers (2015).[1]

[1] Börgers (2015) restricts attention to the case in which all agents have a positive net value for the alternative, but this is immaterial for the main results.

Suppose we assume that agents have a commonly known belief about the value of all agents and that this belief is such that all agents' values are independently and identically distributed with some given distribution function. Myerson and Satterthwaite (1983) have shown that, for such cases, there is no Bayesian mechanism with a Bayesian Nash equilibrium that induces an efficient production of the public good (or efficient implementation of the alternative, given the language used above) and at the same time ensures a balanced budget (see also Börgers, 2015, Propositions 3.7 and 3.12).[2] One way to see this (as in Börgers, 2015) is to show that among all efficient, incentive compatible, and individually rational mechanisms, the pivot mechanism of Green and Laffont (1977), a special case of the so-called VCG mechanism inspired by Vickrey (1961), Clarke (1971), and Groves (1973), is one that minimizes the expected deficit that results, and that this expected deficit is typically strictly larger than zero.

Güth and Hellwig (1986) have provided "second best" mechanisms that maximize expected welfare subject to incentive compatibility, individual rationality, and budget balance constraints. They also identified profit maximizing mechanisms under incentive compatibility and individual rationality constraints. See also Börgers (2015, Propositions 3.7 and 3.9).

We are here interested in dominant strategy incentive compatible (DSIC) mechanisms. We are interested in these for the usual reasons. Such a mechanism seems more applicable in practice, it is simple, and it does not depend on fine details of the problem such as the beliefs people have about values of others and beliefs of others.[3] We also require that the decision to participate in the mechanism is a dominant strategy and, thus, impose the so-called ex-post individual rationality (EPIR) constraint.

These restrictions alone do not change anything. The above mentioned pivot mechanism is in fact DSIC and EPIR already. Also the second best welfare maximizing mechanism and the profit maximizing mechanism have versions that satisfy the DSIC and EPIR constraints. Thus, while it can of course not be easier to obtain the efficient implementation of the alternative (the public good) under the more stringent DSIC and EPIR constraints, it turns out to be no harder either.

[2]Myerson and Satterthwaite (1983) have in fact established this result for the bilateral trade special case of the general setup. The logic behind their result extends directly to the public good provision problem and all problems considered here.

[3]For the recent literature on robust mechanism design (see e.g. Bergemann and Morris, 2005) dominant strategy mechanisms also play a special role. For instance Chung and Ely (2007) show that in the context of auctions a profit maximizing auctioneer can restrict attention to dominant strategy mechanisms if she is interested in maximizing the minimum profit under a high degree of uncertainty about the environment. Note, however, that Börgers (2013) shows that nevertheless there is a non-dominant strategy mechanism that is in some sense weakly superior to dominant strategy mechanisms.

1.2 What we do

Among all DSIC and EPIR mechanisms, we are particularly interested in those that are (exact) ex-post budget balanced (exact EPBB). Ex-post budget balance requires that the successful implementation of the mechanism never produces a deficit. In most of the examples set out above it seems difficult to know how one would deal with a realized deficit. At the least, EPBB mechanisms avoid the need for a costly additional insurance policy (yet non-existent, we believe) to absorb the risk inherent in a mechanism that is ex-ante budget balanced (EABB), but not EPBB. As any welfare-maximizing mechanism cannot leave a surplus, either, the budget needs to be even exactly balanced for such purposes.

We, thus, aim to characterize deterministic direct mechanisms (appealing to the well-known revelation principle) that are DSIC, EPIR, and exact EPBB. In particular we are also interested in a "third best" mechanism that maximizes welfare subject to these three constraints.

Our first result (Proposition 1) provides a necessary condition for a direct mechanism to be DSIC, EPIR, exact EPBB, under the additional condition that the lowest value type of each agent expects a zero utility in all circumstances, which means that this type is just indifferent between participating and not participating in this mechanism. Every such mechanism must be of a threshold form, in which every agent has a fixed pivotal value and the alternative is implemented only if all agents have a value that is greater than or equal to their pivotal value and the alternative is implemented if all agents' values exceed their pivotal value. The contribution of each agent is the respective pivotal value if implementation happens (and zero else).[4]

Why is this useful? It turns out that many mechanisms have the feature that the lowest type of each agent expects a zero utility in all circumstances. For instance, all mechanisms that Börgers (2015) calls canonical since they are reminiscent of the pivot mechanism have this feature. Moreover we show (in Lemma 3) that the DSIC and EPIR implementation of the second best welfare maximizing mechanism also must have this feature, and thus, as it is not a threshold mechanism, it cannot be ex-post budget balanced (which we state as Corollary 1).

Threshold mechanisms are not the only ones that are DSIC, EPIR, and exact EPBB as we show by an example, which thus necessarily violates the condition that the lowest type of each agent expects a zero utility in all circumstances.

Nevertheless, using our first result, we then move on to establish our second result that

[4] A threshold mechanism in which the alternative is implemented if and only if all agents' values are greater than or equal to their pivotal value is DSIC, EPIR, exact EPBB, and satisfies that the lowest value type of each agent expects a zero utility in all circumstances. In the appendix, in Theorem 1 we provide a detailed necessary and sufficient condition for a mechanism to satisfy these four conditions. For this one has to specify carefully when the alternative is implemented for the event that, while all agents' values are greater than or equal to their pivotal value, at least one agent's value is exactly equal to her pivotal value.

the “third best” welfare maximizing mechanism, i.e. the mechanism that maximizes expected welfare subject to DSIC, EPIR, and EPBB, must again be of the threshold kind (Proposition 2).

Our third result helps to better understand the previous one. We first note that one condition implying that the lowest type of each agent expects a zero utility in all circumstances in a mechanism is that the mechanism is such that all agents have veto power (can block the implementation of the alternative) in all circumstances (i.e. for all type profiles). We then show (in Proposition 3) that a much weaker condition than this full veto power condition suffices to imply the threshold nature of DSIC, EPIR, and exact EPBB mechanisms. It suffices that there is one configuration (a single profile) in which all agents have veto power. We call this condition an instance of full veto power (IFVP).

1.3 Relationship to Serizawa (1999)

Serizawa (1999) investigates direct mechanisms in what he calls *public good economies*: There is a public good which can be produced at any level between zero and some fixed upper bound. To produce it agents have to give up some of their consumption of a private good. A given production technology then turns the sum of the “donated” levels of private good into levels of the public good. There is no money (the private good plays the role of money). Agents can have one of a large set of possible “classical” preferences over pairs of private and public good consumption levels. An agent’s preference is her private information.

Serizawa (1999) shows that any *symmetric* DSIC, EPIR, and exact ex-post budget balanced mechanism must be such that the level of the provision of the public good is determined by a *minimum demand rule* that can be described as follows. Transfers depend only on the level of the public good provided and they are equal for all agents and such that their sum is just enough to produce the desired level of the public good. The level of the public good provided is then determined in a way that can be described as an ascending auction. We start with zero production of the public and increase its production gradually until one agent does not want any more of the public good. This *minimum demand* of the public good is then implemented.

The threshold mechanisms we identify in our binary setting can be seen as such minimum demand mechanisms adapted to the binary setting. In our characterizations, however, we do not need the assumption of symmetric transfer rules to obtain our results. Indeed imposing symmetry would not be very satisfying in at least four of our five explicitly mentioned examples above. However, in Corollary 3 (to our Theorem 1) in the Appendix, we also provide an analogous result to the one of Serizawa (1999): in our setting a mechanism that is DSIC, EPIR, exact EPBB and *symmetric* must also be of the threshold kind, in fact with equal cost sharing. Moreover, we obtain a very transparent characterization of how to resolve ties in a

DSIC way. Note that our result is not a corollary to the result in Serizawa (1999), given our model restrictions. These restrictions make the DSIC condition much weaker, as an agent in our setting has much fewer types and, thus, much fewer strategies compared with agents in the model of Serizawa (1999).

The paper proceeds as follows. In Section 2 we provide the model. Section 3 provides some (mostly known) preliminary results in a useful form. Section 4 states and proves the main results. Section 5 concludes with the implications of our findings for the various examples laid out above. The Appendix completes the analysis by a detailed study of tie-breaking rules for threshold mechanisms.

2 Setup

We follow the setup of Börgers (2015, chap. 4). Throughout the main parts of the paper we use the language of public good provision. The reader should keep in mind, however, that we allow that the public "good" is actually a "bad" for some agents.[5]

A *public good problem* with private values is a tuple consisting of the following ingredients: A set I of N agents; for each agent $i \in I$ a set of possible private values (for the indivisible, non-excludable public good) $\theta_i \in \Theta_i = [\underline{\theta}_i, \bar{\theta}_i] \subset \mathbb{R}$, which is private information to the agent; the cost of providing the public good $c \geq 0$. Let $\Theta = \prod_{i \in I} \Theta_i$ and, for all $i \in I$, let $\Theta_{-i} = \prod_{j \in I \setminus \{i\}} \Theta_j$ with typical element θ_{-i}. We will frequently use the maximum value profiles $\bar{\theta}_I := (\bar{\theta}_i; i \in I) \in \Theta$ and $\bar{\theta}_{-i} := (\bar{\theta}_j; j \in I \setminus \{i\}) \in \Theta_{-i}$.

For a public good problem, a *decision rule* can be written as a function q from the set of value-profiles, Θ, to the set $\{0, 1\}$, where a 1 indicates the provision of the public good and a 0 indicates that the public good is not provided.

A direct mechanism for a public good problem consists of a decision rule and a set of transfer functions, t_i, one for each agent $i \in I$, where the transfer (possibly negative) is a money amount that is taken from the agent and given to the mechanism designer. The transfer functions are functions from the set of value profiles Θ to $\mathbb{R}$. The agents are asked to report their private values independently and the decision rule and the transfer functions are applied to the reported profile. Agent i's utility from a profile of reported values $\theta' \in \Theta$ is then $\theta_i q(\theta') - t_i(\theta')$. The utility from not participating in the mechanism is 0 for every agent.

A direct mechanism for a public good problem is *ex-post budget balanced* (EPBB) if, for all reported value-profiles, the sum of all transfers to the designer covers the cost of providing the public good if it is provided, and if the sum of transfers is also nonnegative otherwise, i.e. if $\sum_{i \in I} t_i(\theta) \geq cq(\theta)$ for all $\theta \in \Theta$. It is *exact* EPBB if equality holds in all cases.

A direct mechanism is *dominant strategy incentive compatible* (DSIC) if "truth-telling"

[5] Although we deviate from Börgers (2015) in this respect, we refer in Section 3 to some of his proofs that do not depend on positive values.

(i.e. stating one's type) is always optimal. It is *ex-post individually rational* (EPIR) if, for any value-profile, every agent expects a weakly higher payoff from participating in the mechanism than from not participating.

3 Preliminary Results

We begin by defining two objects for any direct mechanism that will be central in the analysis.

Definition 1. For a direct mechanism $(q, t_1, \dots, t_N)$, the *pivotal value* of agent $i \in I$ given the other agents' values θ_{-i} is

$$\hat{\theta}_i(\theta_{-i}) := \inf\{\theta'_i \in [\underline{\theta}_i, \bar{\theta}_i] \mid q(\theta'_i, \theta_{-i}) = 1\} \wedge \bar{\theta}_i$$

(with $\inf \emptyset = +\infty$). We further let

$$\tau_i(\theta_{-i}) := \underline{\theta}_i q(\underline{\theta}_i, \theta_{-i}) - t_i(\underline{\theta}_i, \theta_{-i})$$

denote the utility for the lowest value instance of agent i if reporting truthfully.

With these objects, we obtain a very useful characterization of direct mechanisms that are incentive compatible, a different way to write Proposition 4.5 of Börgers (2015).

Lemma 1. *A direct mechanism is dominant strategy incentive compatible (DSIC) if and only if for every $i \in I$, q is nondecreasing in θ_i and*

$$t_i(\theta) = \hat{\theta}_i(\theta_{-i}) q(\theta) - \tau_i(\theta_{-i}). \tag{3.1}$$

Proof. The given representation is derived from Proposition 2.2 (also 3.2 and 4.2) of Börgers (2015) – which states that a direct mechanism is DSIC if and only if for every $i \in I$, $q(\theta_i, \theta_{-i})$ is nondecreasing in θ_i and

$$t_i(\theta_i, \theta_{-i}) = t_i(\underline{\theta}_i, \theta_{-i}) + \theta_i q(\theta_i, \theta_{-i}) - \underline{\theta}_i q(\underline{\theta}_i, \theta_{-i}) - \int_{\underline{\theta}_i}^{\theta_i} q(x, \theta_{-i})\, dx$$

for all $\theta_{-i} \in \Theta_{-i}$ – considering q being $\{0,1\}$-valued. With q nondecreasing in θ_i, it jumps to 1 at some point $\hat{\theta}_i(\theta_{-i}) \in \Theta_i$ (possibly left-continuously, i.e. never where $\hat{\theta}_i(\theta_{-i}) = \bar{\theta}_i$). Thus we now have $\int_{\underline{\theta}_i}^{\theta_i} q(x, \theta_{-i})\, dx = (\theta_i - \hat{\theta}_i(\theta_{-i})) q(\theta_i, \theta_{-i})$ in the transfers, which yields (3.1). $\square$

Given $q(\cdot)$ and θ_{-i}, the pivotal value $\hat{\theta}_i(\theta_{-i})$ of agent i is fixed and the transfer of any DSIC direct mechanism is then completely determined by the utility for the lowest value, $\tau_i(\theta_{-i})$. Note that $\hat{\theta}_i(\theta_{-i})$ here is nonincreasing in any θ_j, $j \neq i$, a consequence of q being nondecreasing in θ. Using the definition of $\tau_i(\theta_{-i})$, the possible transfers for agent i can also be described in terms of the transfer for the lowest value $t_i(\underline{\theta}_i, \theta_{-i})$.

Precisely, if $q(\theta) = 0$ or $q(\underline{\theta}_i, \theta_{-i}) = 1$, then $q(\theta) = q(\underline{\theta}_i, \theta_{-i})$ by monotonicity of q, so reporting $\theta_i \geq \underline{\theta}_i$ does not influence the decision rule outcome. Then $t_i(\theta) = t_i(\underline{\theta}_i, \theta_{-i})$ is the transfer for the lowest value type (noting in particular that $q(\underline{\theta}_i, \theta_{-i}) = 1 \Rightarrow \hat{\theta}_i(\theta_{-i}) = \underline{\theta}_i$). If $q(\theta) = 1$ and $q(\underline{\theta}_i, \theta_{-i}) = 0$, however, then agent i pays additionally the pivotal value to change the outcome, $t_i(\theta) = \hat{\theta}_i(\theta_{-i}) + t_i(\underline{\theta}_i, \theta_{-i})$.

With the representation of transfers in terms of $\tau_i(\theta_{-i})$, we can express Proposition 4.3 of Börgers (2015) as follows, which states that it suffices to consider the lowest value types for individual rationality.

Lemma 2. *A direct mechanism is DSIC and ex-post individually rational (EPIR) if and only if* (3.1) *holds with $\tau_i(\theta_{-i}) \geq 0$ for every $i \in I$ and $\theta_{-i} \in \Theta_{-i}$.*

In many important cases, indeed $\tau_i \equiv 0$ for all agents, i.e. the lowest value types are just indifferent to participate, like in the famous pivot mechanism. More generally, any of the mechanisms Börgers (2015) calls *canonical* in his Definition 4.4 are DSIC and EPIR exactly because they satisfy (3.1) with $\tau_i \equiv 0$.[6] Further specific examples are all DSIC and EPIR implementations of second best mechanisms for welfare maximization, which are characterized in terms of *expected* transfers in Proposition 3.8 of Börgers (2015).

Lemma 3. *Consider a belief under which the agents' valuations are independently distributed, according to distribution functions F_i on each Θ_i, $i \in I$, which are regular.*[7] *Assume also $\sum_I \underline{\theta}_i < c < \sum_I \bar{\theta}_i$, to exclude the trivial cases when the public good should always or never be supplied. Then any second best, welfare maximizing mechanism can be implemented DSIC and EPIR if and only if we concretize transfers using* (3.1) *with $\tau_i \equiv 0$ for all $i \in I$ and $\theta \in \Theta$.*

Proof. The characterization of the second best mechanism given in Proposition 3.8 of Börgers (2015) has three items (i)–(iii), the first two concerning q. The transfers are only characterized in expectation by item (iii). We will show that given the decision rule q of any second best mechanism, one can choose transfers $t_1, \cdots, t_N$ to make the mechanism DSIC and EPIR subject to item (iii) if and only if using (3.1) with $\tau_i \equiv 0$ for all $i \in I$.

The decision rule $q(\theta)$ satisfying item (i) is clearly nondecreasing as required by Lemma 1 for any DSIC mechanism. Transfers must satisfy (3.1). Item (iii) states that in expectation,

[6] Note that $t_i(\theta) = \hat{\theta}_i(\theta_{-i})q(\theta)$ for all $\theta \in \Theta$ also *implies* $\tau_i(\theta) = 0$ for all $\theta \in \Theta$, obviously if $q(\underline{\theta}_i, \theta_{-i}) = 0$ and due to $\hat{\theta}_i(\theta_{-i}) = \underline{\theta}_i$ if $q(\underline{\theta}_i, \theta_{-i}) = 1$. In particular for the pivot mechanism (see Börgers, 2015, Definition 3.8), where $q^*(\theta) = 1 \Leftrightarrow \sum_I \theta_i \geq c$ and the transfers can be rewritten as $t_i(\theta) = q^*(\theta) \max\{\underline{\theta}, c - \sum_{j \neq i} \theta_j\} = q^*(\theta)\hat{\theta}_i(\theta_{-i})$, we thus get (3.1) with $\tau_i(\theta_{-i}) = 0$ for all $\theta \in \Theta$.

[7] A distribution function F_i on Θ_i is called *regular* if it has a positive density $f_i(\theta_i) > 0$ and if the function $\theta_i - (1 - F_i(\theta_i))/f_i(\theta_i)$ is strictly increasing in θ_i.

the transfers for each value θ_i of agent $i \in I$ must also satisfy

$$\int_{\Theta_{-i}} t_i(\theta_i, \theta_{-i}) f_{-i}(\theta_{-i})\, d\theta_{-i} = \int_{\Theta_{-i}} \left[\theta_i q(\theta_i, \theta_{-i}) - \int_{\underline{\theta}_i}^{\theta_i} q(x, \theta_{-i})\, dx\right] f_{-i}(\theta_{-i})\, d\theta_{-i},$$

where $f_{-i}(\theta_{-i}) = \prod_{j \neq i} f_j(\theta_j)$ is the positive density of the belief over the other agents' values. Given $\int_{\underline{\theta}_i}^{\theta_i} q(x, \theta_{-i})\, dx = (\theta_i - \hat{\theta}_i(\theta_{-i}))q(\theta_i, \theta_{-i})$ from the proof of Lemma 1, under DSIC this becomes

$$\int_{\Theta_{-i}} \big[\underbrace{\hat{\theta}_i(\theta_{-i}) q(\theta_i, \theta_{-i}) - t_i(\theta_i, \theta_{-i})}_{=\tau_i(\theta_{-i})}\big] f_{-i}(\theta_{-i})\, d\theta_{-i} = 0.$$

Under EPIR, $\tau_i(\theta_{-i}) \geq 0$ by Lemma 2, whence item (iii) is satisfied if and only if $\tau_i(\theta_{-i}) = 0$ for any $\theta_{-i} \in \Theta_{-i}$ (with probability one), $i \in I$. □

4 Mechanisms with a Balanced Budget

One main concern in designing mechanisms to provide a public good is a balanced budget, in particular for a welfare-maximizing designer. Together with the frequently encountered condition that the lowest value types are just indifferent to participate, $\tau_i \equiv 0$, demanding an *ex-post* balanced budget has a strong implication for the structure of qualifying mechanisms.

Proposition 1. *Consider a direct mechanism that is DSIC and EPIR with $\tau_i \equiv 0$ for all $i \in I$. Then the mechanism has an* exact *ex-post balanced budget (EPBB) if and only if there is a critical value $\tilde{\theta}_i \in [\underline{\theta}_i, \bar{\theta}_i]$ for every $i \in I$ such that $q(\theta) = 0$ if $\theta_i < \tilde{\theta}_i$ for some $i \in I$ and $q(\theta) = 1$ if $\theta_i > \tilde{\theta}_i$ for all $i \in I$, and $t_i(\theta) = \tilde{\theta}_i q(\theta)$ for all $i \in I$ and $\theta \in \Theta$, and where $\sum_{i \in I} \tilde{\theta}_i = c$ if $q(\theta) = 1$ for some $\theta \in \Theta$.*

In these mechanisms, the public good will be provided only if every agent i has a value that is greater than or equal to a fixed individual critical value $\tilde{\theta}_i$, and it will be provided if all announced values exceed the critical values. We will call them *threshold mechanisms*.

In particular, Proposition 1 implies that any mechanism that is DSIC, EPIR, exact EPBB, and is such that it satisfies $\tau_i \equiv 0$ for all $i \in I$ (i.e. agents with the lowest possible values are indifferent between participating in the mechanism and not participating) must be of such a threshold form. The threshold form is thus a necessary condition for a mechanism to satisfy all four stated conditions.

The threshold form alone, as stated here, is not sufficient for a mechanism to satisfy the four stated conditions. Note that in our implicit definition of a threshold mechanism with a given profile of thresholds $\tilde{\theta}_i$ we have not specified whether or not the public good is provided when all values are greater than or equal to their thresholds and at least one value is exactly equal to its respective threshold. If we assume that the distribution of values is atomless then this event has zero measure. Nevertheless not all possible specifications of the decision rule in this zero-measure set of value profiles will lead to the mechanism satisfying the stated four conditions. In the Appendix, in Theorem 1, we provide a detailed necessary and sufficient condition for a mechanism to satisfy the four stated conditions. One can, however, provide an immediate simple sufficient (and not necessary) condition for a threshold mechanism to satisfy the four state conditions. Suppose that, for a given profile of thresholds $\tilde{\theta}_i$, the public good is provided if and only if $\theta_i \geq \tilde{\theta}_i$ for all $i \in I$. Then every such mechanism is DSIC, EPIR, exact EPBB, and is such that it satisfies $\tau_i \equiv 0$ for all $i \in I$ (i.e. agents with the lowest possible values are indifferent between participating in the mechanism and not participating).

Proof of Proposition 1. Sufficiency is immediate. To show necessity, first note that as q is nondecreasing under DSIC by Lemma 1, $q(\theta) = 1 \Rightarrow q(\theta_i, \bar{\theta}_{-i}) = 1$ for all $\theta \in \Theta$ and thus $\hat{\theta}_i(\theta_{-i}) \geq \hat{\theta}_i(\bar{\theta}_{-i})$ for all $i \in I$. Also $q(\theta) = 1 \Rightarrow q(\bar{\theta}_I) = 1$, and thus, if $\tau_i(\theta_{-i}) = \tau_i(\bar{\theta}_{-i}) = 0$ in (3.1), we have $t_i(\theta) = \hat{\theta}_i(\theta_{-i}) \geq \hat{\theta}_i(\bar{\theta}_{-i}) = t_i(\bar{\theta}_I)$ for all $i \in I$ and $\theta \in \Theta$ with $q(\theta) = 1$. However, if exact EPBB holds and $q(\theta) = 1$, then $c = \sum_I t_i(\theta) = \sum_I t_i(\bar{\theta}_I)$ and thus indeed $t_i(\theta) = \hat{\theta}_i(\theta_{-i}) = \hat{\theta}_i(\bar{\theta}_{-i}) = t_i(\bar{\theta}_I)$ for all $i \in I$. Now we choose $\tilde{\theta}_i = \hat{\theta}_i(\bar{\theta}_{-i}) \in [\underline{\theta}_i, \bar{\theta}_i]$ for all $i \in I$ (i.e. $\tilde{\theta}_i = \bar{\theta}_i$ for all $i \in I$ if $q(\theta) = 0$ for all $\theta \in \Theta$), which ensures $\sum_{i \in I} \tilde{\theta}_i = c$ by exact budget balance if there is any $\theta \in \Theta$ with $q(\theta) = 1$.

Next, to verify that $t_i(\theta) = \tilde{\theta}_i q(\theta)$ for all $i \in I$ also if $q(\theta) = 0$, note that in the latter case $t_i(\theta) = \tau_i(\theta_{-i})$ by (3.1), where $\tau_i(\theta_{-i}) = 0$ by hypothesis for all $i \in I$.

It remains to show that the identified values $\tilde{\theta}_i$ allow the claimed characterization of q. On the one hand, it always holds that $q(\theta) = 0$ if $\theta_i < \tilde{\theta}_i = \hat{\theta}_i(\bar{\theta}_{-i})$ for some $i \in I$, because $\hat{\theta}_i(\bar{\theta}_{-i}) \leq \hat{\theta}_i(\theta_{-i})$ for all $\theta \in \Theta$ as argued at the beginning. On the other hand, we now have $q(\theta) = 1$ if $\theta_i > \tilde{\theta}_i$ for all $i \in I$. Indeed, suppose the latter condition holds (so necessarily $\bar{\theta}_i > \tilde{\theta}_i$ for all $i \in I$). With $\theta_1 > \tilde{\theta}_1$, $q(\theta_1, \bar{\theta}_2, \ldots, \bar{\theta}_N) = 1$ holds by definition. Then $\hat{\theta}_2(\theta_1, \bar{\theta}_3, \ldots, \bar{\theta}_N) = \tilde{\theta}_2$ as observed above, such that also $q(\theta_1, \theta_2, \bar{\theta}_3, \ldots, \bar{\theta}_N) = 1$. Iterating this procedure until $i = N$, we obtain $q(\theta) = 1$. $\square$

In Lemma 3 we saw that the second best, welfare maximizing mechanism subject to an *ex-ante* balanced budget can be implemented DSIC and EPIR, but only if the lowest value types are indifferent to participate, $\tau_i \equiv 0$. As requiring an ex-post balanced budget would actually imply an *exact* balance, any EPBB implementation would have to be a threshold mechanism by Proposition 1, which the second best mechanism is generally not.

Corollary 1. *Consider a belief as in Lemma 3. Then the second best, welfare maximizing mechanism can satisfy EPBB only exactly. Thus, if the probability of type profiles for which the good is provided is in* $(0, 1)$ *(i.e. the outcome is not trivial) and if the distribution functions* F_i *have continuous densities, then there is no implementation that is DSIC and EPIR and EPBB.*

Proof. The second best mechanism maximizes expected welfare subject to the budget being balanced ex-ante. In any optimum, the budget constraint must be binding (see p. 54 in Börgers, 2015), i.e.

$$\int_{\Theta} \Big[\sum\nolimits_I t_i(\theta) - cq(\theta) \Big] \, dF(\theta) = 0.$$

Thus, imposing the EPBB condition $\sum_I t_i(\theta) \geq cq(\theta)$, this also has to be binding, resp. *exact* for every $\theta \in \Theta$ (with probability one). By Lemma 3 and Proposition 1, any implementation that is DSIC, EPIR and EPBB would then have to be a threshold mechanism. Further, as F is assumed to have a density on Θ and hypothesized to assign positive measure to $\{\theta \in \Theta \mid q(\theta) = 1\} \subset \prod_I [\tilde{\theta}_i, \bar{\theta}_i]$, we would have $\tilde{\theta}_i < \bar{\theta}_i$ for all $i \in I$. On the other hand, as F is hypothesized not to assign full measure to $\{\theta \in \Theta \mid q(\theta) = 1\} \supset \prod_I (\tilde{\theta}_i, \bar{\theta}_i]$, we would have $\underline{\theta}_i < \tilde{\theta}_i$ for some $i \in I$. Note that in a threshold mechanism, for any $\theta \in \prod_I (\tilde{\theta}_i, \bar{\theta}_i]$ it holds that $\hat{\theta}_i(\theta_{-i}) = \tilde{\theta}_i$ for all $i \in I$.

However, the $\hat{\theta}_i(\theta_{-i})$ implied by Proposition 3.8 in Börgers (2015) *cannot* be constant in that sense. It requires that $q(\theta) = 1$ only if $\sum_I \lambda\psi_i(\theta_i) + \theta_i \geq (1 + \lambda)c$, and if the inequality is strict, then $q(\theta) = 1$. Here $\lambda > 0$ and the functions $\psi_i(\theta_i) = \theta_i - (1 - F_i(\theta_i))/f_i(\theta_i)$ are assumed strictly increasing (cf. our fn. 7 and Assumption 3.2 in Börgers (2015)) and continuous by hypothesis. As we would have $q(\theta) = 1$ for all $\theta \in \prod_I (\tilde{\theta}_i, \bar{\theta}_i]$ in a threshold mechanism, then $\sum_I \lambda\psi_i(\tilde{\theta}_i) + \tilde{\theta}_i \geq (1 + \lambda)c$ by continuity. Now consider one of the $i \in I$ with $\tilde{\theta}_i \in (\underline{\theta}_i, \bar{\theta}_i)$. Then we would have $q(\theta_i, \bar{\theta}_{-i}) = 1$ for some $\theta_i < \tilde{\theta}_i$ by $\bar{\theta}_j > \tilde{\theta}_j$ for all $j \neq i$ and continuity of ψ_i, contradicting $\hat{\theta}_i(\bar{\theta}_{-i}) = \tilde{\theta}_i$ as noted above. □

To obtain any mechanisms that are DSIC, EPIR and exact EPBB but not of the threshold kind, it is necessary to have $N > 2$ agents.[8]

That there are such mechanisms is illustrated by the following example, which also necessarily features $q(\underline{\theta}_i, \theta_{-i}) = 1$ for some $i \in I$ and some $\theta \in \Theta$ (see Lemma 4 below). The latter is easily achieved by making the decision rule independent of one agent's value. The good will be provided if the other (two) agents together have a sufficiently high valuation. In that case each of the latter will contribute the value that would just suffice for provision, given the other agent's value. The provision threshold for the "active" agents' aggregate valuation is chosen such that the difference of those agents' contributions to the cost of providing the good is less than the lowest value for the "passive" agent, who is thus willing to participate. Depending on the parameterization, the transfer for the passive agent can be positive, asking for a *contribution* or negative, paying out a *surplus*.

Example 1. Let $N = 3$, $\Theta = [\underline{\theta}, \bar{\theta}]^3$, and assume $c \in (3\underline{\theta}, 2\underline{\theta} + \bar{\theta}]$. The lower bound is

[8] The crucial fact for $N = 2$ is that $t_i(\theta)$ in our Lemma 1, which is constant in θ_i on $\{\theta \in \Theta \mid q(\theta) = 1\}$, now is also constant in $\theta_{-i} = \theta_j$ by $t_i(\theta_1, \theta_2) = c - t_j(\theta_1, \theta_2)$ (without knowing monotonicity of all t_i as in the proof of Proposition 1). Hence, if $q(\theta) = 1$, then $q(\bar{\theta}_I) = 1$ by monotonicity and thus $t_i(\theta) = t_i(\bar{\theta}_I)$. Therefore Theorem 1 in the Appendix applies. Cf. also Proposition 4.8 of Börgers (2015).

only chosen such that it is not efficient to supply the good for all value profiles, whereas the upper is needed for the following mechanism to be individually rational. Now define a direct mechanism by the decision rule $q(\theta) = \mathbf{1}_{\theta_1+\theta_2\geq b}$ with $b := \bar{\theta} + \frac{1}{2}(\underline{\theta} + \bar{\theta})$, and

$$t_1(\theta) = \begin{cases} b - \theta_2 & \text{if } \theta_1 + \theta_2 \geq b, \\ 0 & \text{else,} \end{cases}$$

$$t_2(\theta) = \begin{cases} b - \theta_1 & \text{if } \theta_1 + \theta_2 \geq b, \\ 0 & \text{else, and} \end{cases}$$

$$t_3(\theta) = \begin{cases} c - t_1(\theta) - t_2(\theta) = c + \theta_1 + \theta_2 - 2b & \text{if } \theta_1 + \theta_2 \geq b, \\ 0 & \text{else.} \end{cases}$$

Since $b \in (2\underline{\theta}, 2\bar{\theta})$, the sets of type profiles for which the public good is provided resp. not provided both have nonempty interior. q is obviously nondecreasing in θ_i, $i \in I$. (3.1) holds for $i = 1, 2$ with $\tau_i \equiv 0$, since $q(\underline{\theta}, \theta_{-i}) = 0 = t_i(\underline{\theta}, \theta_{-i})$ for both by the choice of b. For $i = 3$ we have $q(\theta) = q(\underline{\theta}, \theta_{-3})$, hence $\hat{\theta}_3(\theta_{-3})q(\theta) = \underline{\theta}q(\underline{\theta}, \theta_{-3})$, and $t_3(\theta) = t_3(\underline{\theta}, \theta_{-3})$. Thus (3.1) holds with $\tau_3(\theta_{-3}) = q(\underline{\theta}, \theta_{-3})[\underline{\theta} - c - \theta_1 - \theta_2 + 2b]$. Now $\tau_3 \geq 0 \; \forall \theta \in \Theta \Leftrightarrow c \leq 2\underline{\theta} + \bar{\theta}$, which is ensured by our assumption on c. In summary, the mechanism is DSIC and EPIR. It is also exact EPBB by construction. Nevertheless $\tau_3(\theta_{-3}) > 0$ for all $\theta_1 + \theta_2 \in [b, 2\bar{\theta})$, whence Proposition 1 does not apply.

Indeed, here $t_1(\theta) = \hat{\theta}_1(\theta_{-1})$ is strictly decreasing in θ_2 if $q(\theta) = 1$, i.e. whenever $\theta_2 \in [b - \theta_1, \bar{\theta})$. This interval is nonempty for all $\theta_1 > \frac{1}{2}(\underline{\theta} + \bar{\theta})$; analogously when switching roles. Contrarily, $t_3(\theta)$ is strictly increasing in $\theta_1 + \theta_2$ when $q(\theta) = 1$, i.e. whenever $\theta_1 + \theta_2 \in [b, 2\bar{\theta})$.

Depending on the relative size of $\underline{\theta}$ and $\bar{\theta}$ (and the location of c) we can accommodate both a case where $t_3 > 0$ whenever the good is provided, as well as the opposite, $t_3 < 0$.[9]

Welfare maximization subject to EPBB leads again to threshold mechanisms, even without requiring an *exactly* balanced budget or any $\tau_i = 0$; these properties now arise endogenously.

Proposition 2. *For any belief, i.e. distribution over the value profile set Θ, and any direct mechanism that is DSIC, EPIR and EPBB, there is some mechanism of the threshold form as in Proposition 1 that yields at least the same expected welfare, in particular one that provides the public good if and only if $\theta_i \geq \tilde{\theta}_i$ for all $i \in I$. Further, if the belief has full support and the decision rule of the initial mechanism is not of the form in Proposition 1, than there is some mechanism of that form yielding strictly greater expected welfare.*

[9] Specifically, when $q(\theta) = 1$, i.e. $\theta_1 + \theta_2 \geq b$, then $t_3(\theta) \geq c - b > 3\underline{\theta} - b$. The latter is nonnegative if $\underline{\theta} \geq \frac{3}{5}\bar{\theta}$. On the other hand, when $q(\theta) = 1$, then $t_3(\theta) \leq c + 2\bar{\theta} + 2b = c - (\underline{\theta} + \bar{\theta})$, which we can make negative by choosing $3\underline{\theta} < c < \underline{\theta} + \bar{\theta}$ with $\bar{\theta} > 2\underline{\theta}$.

Proof. By Lemmas 1 and 2, the problem to maximize expected welfare given some distribution F on Θ and subject to the mechanism being DSIC, EPIR and EPBB is

$$\left.\begin{aligned} &\max_{q,t_1,\ldots,t_N} \int_\Theta \Big(\textstyle\sum_I [\theta_i q(\theta) - t_i(\theta)]\Big)\, dF(\theta) \\ &\text{s.t.} \quad q \text{ nondecreasing}, \\ &\qquad\quad t_i(\theta) \le \hat{\theta}_i(\theta_{-i}) q(\theta) \text{ for all } i \in I, \\ &\qquad\quad \textstyle\sum_I t_i(\theta) \ge c q(\theta). \end{aligned}\right\} \tag{4.1}$$

The last (budget) constraint can be assumed binding without loss, since decreasing any $t_i(\theta)$ can only increase the objective value and relax the second constraint. Further, the constraints (4.1) imply $q(\theta)(\sum_I \hat{\theta}_i(\theta_{-i}) - c) \ge 0$, which only concerns q. Letting the latter replace the second constraint in (4.1), we obtain the following relaxed problem without transfers:

$$\left.\begin{aligned} &\max_q \int_\Theta \Big(\textstyle\sum_I \theta_i - c\Big) q(\theta)\, dF(\theta) \\ &\text{s.t.} \quad q \text{ nondecreasing}, \\ &\qquad\quad \Big(\textstyle\sum_I \hat{\theta}_i(\theta_{-i}) - c\Big) q(\theta) \ge 0. \end{aligned}\right\} \tag{4.2}$$

Consider now some $q, t_1, \ldots, t_N$ feasible in the first problem, with the budget constraint binding. Then q yields the same objective value in the relaxed problem. With q nondecreasing, we have $q(\theta) = 1 \Rightarrow \theta_i \ge \hat{\theta}_i(\theta_{-i}) \ge \hat{\theta}_i(\bar{\theta}_{-i})$ for all $i \in I$ (see the proof of Proposition 1). Moreover, $q(\theta) = 1$ also implies $q(\bar{\theta}_I) = 1$ and then $\sum_I \hat{\theta}_i(\bar{\theta}_{-i}) \ge c$ by (4.2), such that one can find some $\tilde{\theta}_i \le \hat{\theta}_i(\bar{\theta}_{-i})$ for every $i \in I$ with $\sum_I \tilde{\theta}_i = c$. Then, defining a new decision rule by $q^*(\theta) = 1$ if and only if $\theta_i \ge \tilde{\theta}_i$ for all $i \in I$, we have $q(\theta) = 1$ only if $q^*(\theta) = 1$ and $q^*(\theta) = 1$ only if $\sum_I \theta_i \ge c$. Therefore, in the relaxed problem q^* attains at least the value that q attains. Moreover, if F has full support and there is any $\theta \in \Theta$ with $\theta_i > \tilde{\theta}_i$ for all $i \in I$ and $q(\theta) = 0$ (in particular, if q is not of the threshold form as in Proposition 1), then q^* is even a strict improvement, because then also $q(\theta') = 0$ and $\sum_I \theta'_i > c$ for any profile from $\{\theta' \in \Theta \mid \tilde{\theta}_i < \theta'_i \le \theta_i, i \in I\}$, which has positive F-measure.

q^* is obviously nondecreasing and induces $\hat{\theta}^*_i(\theta_{-i}) = \tilde{\theta}_i$ for all $i \in I$ whenever $q^*(\theta) = 1$. It can be made feasible in the first problem with constraints (4.1) by choosing transfers $t^*_i(\theta) = \tilde{\theta}_i q^*(\theta)$ for all $i \in I$ and $\theta \in \Theta$, which satisfy (3.1) with associated $\tau^*_i \equiv 0$ (cf. fn. 6). Thus, $(q^*, t^*_1, \ldots, t^*_N)$ is DSIC by Lemma 1, EPIR by Lemma 2 and exact EPBB by construction. It yields the same objective value in both problems by exact EPBB, so the (weak) improvement of passing from q to q^* can be realized in the first problem. □

In an attempt to provide additional intuition why non-threshold mechanisms cannot be welfare maximizing, we now provide an intuitive characterization of all alternative mechanisms. We first note that a (far from necessary) sufficient condition to guarantee that $\tau_i \equiv 0$ (i.e. that all lowest types expect a zero utility from the mechanism and are, thus, indifferent between participating and not participating) is that in the mechanism every agent i can block the provision of the public good at any profile of values by announcing the lowest value $\underline{\theta}_i$.

Lemma 4. *Consider a direct mechanism that is EPIR and EPBB. If $q(\underline{\theta}_i, \theta_{-i}) = 0$ for some $i \in I$ and $\theta \in \Theta$, then $\tau_i(\theta_{-i}) = 0$.*

Proof. If $q(\underline{\theta}_i, \theta_{-i}) = 0$ for some $i \in I$ and $\theta \in \Theta$, then EPIR requires $t_j(\underline{\theta}_i, \theta_{-i}) \leq 0$ for all $j \in I$, whereas EPBB requires $\sum_{j\in I} t_j(\underline{\theta}_i, \theta_{-i}) \geq 0$, which together implies $t_i(\underline{\theta}_i, \theta_{-i}) = \tau_i(\theta_{-i}) = 0$. □

It turns out that there is a much weaker sufficient condition to guarantee that $\tau_i \equiv 0$. We say that a mechanism satisfies the condition of an *instance of full veto power* (IFVP) if there is at least one profile of valuations where every agent has a "veto power" to block the provision of the public good. In other words a mechanism satisfies IFVP if there is a $\theta \in \Theta$ such that $q(\theta) = 1$ and $q(\underline{\theta}_i, \theta_{-i}) = 0$ for all $i \in I$. This seems a mild condition and one that one would like to impose. Consider a profile of valuations that sum up to a value just slightly greater than the cost of the public good provision. Then one would probably like the mechanism to be highly sensitive to each agent's valuation for at least one such profile.

Conversely, if a mechanism is DSIC, EPIR and exact EPBB, but not of the threshold kind, then, whenever the public good is provided, there is at least one agent who cannot influence the decision.

Proposition 3. *If a direct mechanism is DSIC, EPIR, exact EPBB and satisfies IFVP, then its decision rule q is of the threshold form (as in Proposition 1).*

Proof. Consider a direct mechanism that is DSIC, EPIR and exact EPBB and suppose there exists a profile $\theta' \in \Theta$ such that $q(\theta') = 1$ and $q(\underline{\theta}_i, \theta'_{-i}) = 0$ for all $i \in I$. We will show that q must be of the form in Proposition 1. By Lemma 4, we must have $\tau_i(\theta'_{-i}) = 0$ for all $i \in I$ and thus $t_i(\theta') = \hat{\theta}_i(\theta'_{-i})$ by Lemma 1. With $q(\theta') = 1$, also $q(\bar{\theta}_I) = 1$ by monotonicity and $\hat{\theta}_i(\theta'_{-i}) \geq \hat{\theta}_i(\bar{\theta}_{-i})$ for all $i \in I$ as argued in the proof of Proposition 1. Further, $\tau_i(\theta'_{-i}) = 0 \leq \tau_i(\bar{\theta}_{-i})$ by Lemma 2. Applying Lemma 1 also to $\bar{\theta}_I$, thus $t_i(\theta') \geq t_i(\bar{\theta}_I)$ for all $i \in I$. However, as $\sum_{i\in I} t_i(\theta') = c = \sum_{i\in I} t_i(\bar{\theta}_I)$ by exact EPBB, we must have $t_i(\theta') = t_i(\bar{\theta}_I)$ and correspondingly for the components $\hat{\theta}_i(\theta'_{-i}) = \hat{\theta}_i(\bar{\theta}_{-i}) =: \tilde{\theta}_i$ and $\tau_i(\theta'_{-i}) = \tau_i(\bar{\theta}_{-i}) = 0$ for all $i \in I$. Thus $\sum_{i\in I} t_i(\theta') = \sum_{i\in I} \tilde{\theta}_i = c$.

If $\theta_i < \tilde{\theta}_i$ for some $i \in I$, then $q(\theta) = 0$ due to $\hat{\theta}_i(\bar{\theta}_{-i}) \leq \hat{\theta}_i(\theta_{-i})$. It remains to show that $q(\theta) = 1$ if $\theta_i > \tilde{\theta}_i$ for all $i \in I$. By the monotonicity of q it suffices to show $q(\min(\theta, \theta')) = 1$ (where the minimum is taken coordinatewise). Note that the hypothesis $q(\theta') = 1$ implies indeed $\theta'_i \geq \hat{\theta}_i(\theta'_{-i}) = \tilde{\theta}_i$ for all $i \in I$. Also note that $q(\underline{\theta}_i, \theta_{-i}) = 0$ for all $i \in I$ and any profile $\theta \leq \theta'$ again by the monotonicity of q, such that by the arguments above also $\hat{\theta}_i(\theta_{-i}) = \tilde{\theta}_i$ for all $i \in I$ if $q(\theta) = 1$. Now consider the profile θ'. If $\tilde{\theta}_1 < \theta'_1$, fix an arbitrary $\theta_1 \in (\tilde{\theta}_1, \theta'_1]$, whence $q(\theta_1, \theta'_{-1}) = 1$ by $\theta_1 > \tilde{\theta}_1 = \hat{\theta}_1(\theta'_{-1})$. If $\tilde{\theta}_1 = \theta'_1$, set $\theta_1 = \theta'_1$, so still $q(\theta_1, \theta'_{-1}) = 1$. In both cases $(\theta_1, \theta'_{-1}) \leq \theta'$ and thus $\hat{\theta}_2(\theta_1, \theta'_3, \ldots, \theta'_N) = \tilde{\theta}_2$ as remarked before, such that we can again choose arbitrary $\theta_2 \in (\tilde{\theta}_2, \theta'_2]$ or $\theta_2 = \tilde{\theta}_2 = \theta'_2$ with $q(\theta_1, \theta_2, \theta'_3, \ldots, \theta'_N) = 1$. Iterating until $i = N$ yields that indeed $q(\min(\theta, \theta')) = 1$ for any profile θ with $\theta_i > \tilde{\theta}_i$ for all $i \in I$. $\square$

5 Conclusion

In this concluding section we come back to the five examples we described in the Introduction and explain what form the welfare-maximizing mechanism subject to DSIC, EPIR, and EPBB must have in each of these examples.

Public good provision In this example all values θ_i are commonly known to be positive and there is a cost $c > 0$ of providing the public good. The welfare-maximizing mechanism subject to DSIC, EPIR, and EPBB is then such that there are thresholds $\tilde{\theta}_i > 0$ such that the good is provided if and only if all agents have a value that exceeds their respective threshold, i.e. if and only if $\theta_i \geq \tilde{\theta}_i$ for all $i \in I$. Each agent then pays the threshold value and these thresholds must sum up to the cost c. That is, the public good is provided if and only if each agent agrees to pay a fixed (individual) share of the cost.

Bilateral trade Here there are only two agents, a seller and a buyer. The seller has a negative value for the public "good" (of moving the object from the seller to the buyer), while the buyer has a positive value. The costs of moving the object from seller to buyer is zero. Then the welfare-maximizing mechanism subject to DSIC, EPIR, and EPBB is such that there is a fixed price $p > 0$ and the sale takes place at this price if and only if the absolute value of the seller's value is below the price and the buyers's value is above the price.

Bilateral wage negotiations This is similar to the bilateral trade example. The mechanism maximizing welfare subject to DSIC, EPIR, and EPBB is such that there is a fixed wage w and the employee gets and accepts the job if and only if the employee's value for hiring her is below the wage and the employee values the outside option, the status quo, less than the offered wage.

A seller selling to a group In this example the seller has a negative value for the public "good" of handing over the ownership rights of her castle or park to the villagers, who in turn have a positive value. The welfare-maximizing mechanism subject to DSIC, EPIR, and EPBB is then such that there is a fixed price $p > 0$ and there is a threshold $\tilde{\theta}_i > 0$ for each villager, such that the ownership rights are transferred if and only if the seller values her castle or park less than the price and the individual values of all villagers exceed their respective threshold, with the sum of all thresholds equal to p. The seller receives p and the villagers pay the value of their respective threshold to the seller. This is under the assumption of no legal costs.

Rezoning land Here the welfare-maximizing mechanism subject to DSIC, EPIR, and EPBB is such that there is a fee $f > 0$ and payments $\tilde{\theta}_i$ such that the rezoning happens if and only if the land-owner's value for rezoning exceeds f and each neighbors' negative value for the rezoning is compensated by their respective payment. The land-owner pays the fee which is then paid out according to the threshold values to the neighbors. The sum of the thresholds must of course be equal to the fee. This is under the assumption of no legal costs.

A Appendix

In Section 4 we have not fully specified which outcome is chosen by a threshold mechanism if ties $\theta_i = \tilde{\theta}_i$ occur. Now we provide a complete characterization also for those cases. Therefore it suffices to consider a weaker property than requiring the lowest value types to be indifferent, i.e. $\tau_i \equiv 0$ as we did before. Instead, we now use a *cost sharing* principle: There is a fixed contribution for every agent i to pay whenever the public good is supplied (or more generally, whenever the alternative is implemented and where a contribution can also be negative). Ties are then resolved by choosing any set of *tie-breaking coalitions* $C \subset I$ and providing the public good if and only if all members of any such coalition C announce valuations strictly above their thresholds (and all other agents announce at least their threshold values). In particular, if $\emptyset$ is chosen as tie-breaking coalition, the good will be provided whenever every agent i announces at least the value $\tilde{\theta}_i$. We prove Theorem 1 at the end of this Appendix.

Theorem 1. *A direct mechanism is DSIC, EPIR, exact EPBB and satisfies $t_i(\theta) = t_i(\theta')$ for all $i \in I$ and $\theta, \theta' \in \Theta$ with $q(\theta) = q(\theta') = 1$ if and only if there is a critical value $\tilde{\theta}_i \in \mathbb{R}$ for every $i \in I$ and a nonempty set of tie-breaking coalitions $T \in \mathcal{P}(I)$ such that $q(\theta) = 1$ if and only if $\theta_i \geq \tilde{\theta}_i$ for all $i \in I$ and $\theta_i > \tilde{\theta}_i$ for all i in some $C \in T$, and such that $t_i(\theta) = \tilde{\theta}_i q(\theta)$ for all $i \in I$ and $\theta \in \Theta$, and where $\sum_{i \in I} \tilde{\theta}_i = c$ if there is any $\theta \in \Theta$ with $q(\theta) = 1$.*

Cost sharing holds by Proposition 1 if $\tau_i \equiv 0$ for all $i \in I$ and we then get the following strengthening of Theorem 1, that the thresholds have to satisfy $\tilde{\theta}_i \in [\underline{\theta}_i, \bar{\theta}_i]$.

Corollary 2. *A direct mechanism is DSIC, EPIR, exact EPBB and satisfies $\tau_i \equiv 0$ for all $i \in I$ if and only if it is of the form as in Theorem 1 with $\tilde{\theta}_i \in [\underline{\theta}_i, \bar{\theta}_i]$ for every $i \in I$.*

Proof. For sufficiency we only have to show that $\tau_i(\theta) = 0$ for any $i \in I$ and $\theta \in \Theta$; the other properties are ensured by Theorem 1. If $q(\underline{\theta}_i, \theta_{-i}) = 0$, then by hypothesis $t_i(\underline{\theta}_i, \theta_{-i}) = 0$, implying $\tau_i(\theta_{-i}) = 0$. If $q(\underline{\theta}_i, \theta_{-i}) = 1$, then by hypothesis $t_i(\underline{\theta}_i, \theta_{-i}) = \tilde{\theta}_i \geq \underline{\theta}_i$, implying $\tau_i(\theta_{-i}) \leq 0$. The latter must be binding by Lemma 2, given that EPIR holds by Theorem 1.

Concerning necessity, if the hypothesis holds, then Proposition 1 implies that with its delivered threholds $\tilde{\theta}_i = t_i(\theta) = t_i(\theta')$ for all $i \in I$ and $\theta, \theta' \in \Theta$ with $q(\theta) = q(\theta') = 1$, such that the hypothesis of Theorem 1 holds, too. As shown in Step 3 of its proof, the threholds delivered by Theorem 1 satisfy indeed $\tilde{\theta}_i = \hat{\theta}_i(\bar{\theta}_{-i}) \in [\underline{\theta}_i, \bar{\theta}_i]$ if $\tau_i(\theta) = 0$ for all $i \in I$ whenever $q(\theta) = 1$. □

As a final result we show that considering an alternative well known condition again implies the full characterization of threshold mechanisms as in Theorem 1. A direct mechanism is *symmetric* if $t_i(\theta) = t_j(\theta)$ holds for any $\theta \in \Theta$ and $i, j \in I$ with $\theta_i = \theta_j$. As in Serizawa (1999), symmetry, DSIC and exact EPBB together imply *equal* cost sharing, i.e. $t_i(\theta) = (c/N)q(\theta)$ for all $i \in I$, if we assume symmetric type spaces. This assumption is not innocuous regarding the diversity of applications we cover in this paper. For instance, in bilateral trade, the buyer generally has a positive valuation for the outcome that the good changes the owner and the seller a negative one. By adding "dummy payments", the value intervals can be shifted towards each other, but not changed in length. Introducing more types makes DSIC more stringent, as there are more deviations to consider.

Whereas the latter fact limits the scope of applications of the following Corollary 3 (in comparison to our previous results), it is at the same time the reason why the result we obtain is interesting. Due to equal cost sharing, a mechanism that is symmetric, DSIC, EPIR and exact EPBB must be of the threshold form as we show, which means that the outcome of q is determined by a *minimum demand rule.*[10] Hence, we arrive at an analogous characterization to that in Theorem 3 of Serizawa (1999) for public good economies. That his considered class of preferences is more complex than ours does not mean, that our result is a special case of his, however, exactly due to the fact that larger type spaces lead to stronger implications of DSIC. Moreover, our characterization of tie-breaking rules that are DSIC in terms of tie-breaking coalitions is very transparent.

Corollary 3. *Assume $\Theta = [\underline{\theta}, \bar{\theta}]^N$. A direct mechanism is DSIC, EPIR, exact EPBB and symmetric if and only if it is of the threshold form as in Theorem 1, where $\tilde{\theta}_i = c/N$ for all $i \in I$ if there is any $\theta \in \Theta$ with $q(\theta) = 1$.*

Proof. Concerning sufficiency, symmetry is obvious. The other properties follow from Theorem 1. For necessity we first show that any direct mechanism with the listed properties satisfies $t_i(\theta) = c/N$ for all $i \in I$ whenever $q(\theta) = 1$ by mimicking Step 1 in the proof of

[10]As defined by Serizawa (1999), if all transfers $t_i(\theta)$ of a mechanism depend on θ only by the value of $q(\theta)$, the latter is determined by a minimum demand rule if it is the minimum of the demands of all agents $i \in I$ for the public good, which are formed as follows. Each agent $i \in I$ considers the choice set $\{(q(\theta), t_i(\theta)) \mid \theta \in \Theta\}$, which here consists of one or two pairs, depending on whether there are any profiles with $q(\theta) = 0$ or $q(\theta) = 1$, respectively. If there is a true choice, agent i clearly demands (one unit of) the public good if θ_i exceeds the difference of t_i for the alternatives with $q = 1$ and $q = 0$ and clearly demands no (unit of the) public good if θ_i falls short of that difference. If θ_i equals the difference, i is indifferent and the (single-valued) demand may depend on the full profile θ (which is called a *tie-breaking rule* and required to be DSIC itself).

Thus, in our case a mechanism with all transfers t_i depending only on the value of q is a minimum demand rule if and only if it is of the threshold form, with each $\tilde{\theta}_i$ the difference of t_i for $q = 1$ and $q = 0$. By Theorem 1 we have established that any mechanism that is DSIC, EPIR, exact EPBB and with t_i depending only on the value of q is of the threshold form with $t_i(\theta) = \tilde{\theta}_i q(\theta)$.

Proposition 1 of Serizawa (1999). Therefore, suppose by way of contradiction that $q(\theta) = 1$, but $t_i(\theta) \neq c/N$ for some $i \in I$, say $i = 1$ w.l.o.g. Then also $q(\bar{\theta}, \theta_{-1}) = 1$ by monotonicity and $t_1(\bar{\theta}, \theta_{-1}) = t_1(\theta) \neq c/N$ by (3.1). By exact EPBB, also $t_i(\bar{\theta}, \theta_{-1}) \neq c/N$ for some other $i > 1$, say $i = 2$ w.l.o.g. As before, then further $t_2(\bar{\theta}, \bar{\theta}, \theta_j; j > 3) = t_2(\bar{\theta}, \theta_{-1}) \neq c/N$, but now also $t_1(\bar{\theta}, \bar{\theta}, \theta_j; j > 3) = t_2(\bar{\theta}, \bar{\theta}, \theta_j; j > 3)$ by symmetry. Iterating the argument, one arrives at $t_1(\bar{\theta}^N) = \cdots = t_N(\bar{\theta}^N) \neq c/N$, contradicting exact EPBB. (Analogously one can prove that $t_i(\theta) = 0$ for all $i \in I$ if $q(\theta) = 0$, then replacing each θ_i by $\underline{\theta}$ instead of $\bar{\theta}$.)

Now our Theorem 1 yields the threshold form, with $\tilde{\theta}_i = t_i(\theta)$ for all $i \in I$ whenever $q(\theta) = 1$, which now means $\tilde{\theta}_i = c/N$. □

Proof of Theorem 1. We begin by showing sufficiency. Any mechanism of the described form clearly satisfies $t_i(\theta) = t_i(\theta') = \tilde{\theta}_i$ for all $i \in I$ and $\theta, \theta' \in \Theta$ with $q(\theta) = q(\theta') = 1$ and also the exact EPBB condition $\sum_I t_i(\theta) = cq(\theta)$ holds by hypothesis. Any q of the described form is nondecreasing. Now consider any $\theta \in \Theta$ and $i \in I$. If $q(\theta) = 0$, then by monotonicity $q(\underline{\theta}_i, \theta_{-i}) = 0 = t_i(\underline{\theta}_i, \theta_{-i}) = t_i(\theta)$ and (3.1) holds with $\tau_i(\theta_{-i}) = 0$. Now suppose $q(\theta) = 1$. If $q(\underline{\theta}_i, \theta_{-i}) = 0$, then again $\tau_i(\theta_{-i}) = 0$ and $\hat{\theta}_i(\theta_{-i}) = \tilde{\theta}_i = t_i(\theta)$, yielding (3.1). If $q(\underline{\theta}_i, \theta_{-i}) = 1$, then necessarily $\tilde{\theta}_i \leq \underline{\theta}_i$, implying $\tau_i(\theta_{-i}) \geq 0$. Further, now $\hat{\theta}_i(\theta_{-i}) = \underline{\theta}_i$, such that (3.1) holds by $t_i(\theta) = t_i(\underline{\theta}_i, \theta_{-i}) = \tilde{\theta}_i$. Together, DSIC follows from Lemma 1 and EPIR from Lemma 2.

In the remainder, necessity is proven in three steps.

Step 1: Consider any $i \in I$ and $\theta \in \Theta$ with $q(\theta) = 1$, so $q(\bar{\theta}_I) = 1$ by monotonicity given DSIC (see Lemma 1) and thus $t_i(\theta) = t_i(\bar{\theta}_I)$ by hypothesis. We want to show that given also EPIR and EPBB we must further have

$$\hat{\theta}_i(\theta_{-i}) = \hat{\theta}_i(\bar{\theta}_{-i}). \tag{A.1}$$

We know $\hat{\theta}_i(\theta_{-i}) \geq \hat{\theta}_i(\bar{\theta}_{-i})$ from the proof of Proposition 1. Thus, on the one hand, for $q(\underline{\theta}_i, \theta_{-i}) = 1$ we get $\underline{\theta}_i = \hat{\theta}_i(\theta_{-i}) \geq \hat{\theta}_i(\bar{\theta}_{-i}) \geq \underline{\theta}_i$ by definition of $\hat{\theta}_i(\cdot)$, implying (A.1). On the other hand, with $t_i(\theta) = t_i(\bar{\theta}_I)$ and (3.1) we get $\tau_i(\theta_{-i}) - \tau_i(\bar{\theta}_{-i}) = \hat{\theta}_i(\theta_{-i}) - \hat{\theta}_i(\bar{\theta}_{-i}) \geq 0$. Hence, (A.1) obtains also for the remaining case $q(\underline{\theta}_i, \theta_{-i}) = 0$, because $\tau_i(\theta_{-i}) = 0$ by Lemma 4, whereas $\tau_i(\bar{\theta}_{-i}) \geq 0$ by Lemma 2 (which now must hold with equality, too).

Step 2: Next, to show that q can be characterized as claimed if (A.1) holds for all $i \in I$ whenever $q(\theta) = 1$, we first ignore the condition $\sum_I \tilde{\theta}_i = c$ and use $\tilde{\theta}_i = \hat{\theta}_i(\bar{\theta}_{-i})$ for all $i \in I$. If $\theta_i < \tilde{\theta}_i$ for some $i \in I$, then $q(\theta) = 0$, because with $q(\theta) = 1$ we would get $\theta_i \geq \hat{\theta}_i(\theta_{-i}) = \tilde{\theta}_i$. Now suppose $q(\theta) = 1$, so $\theta_i \geq \tilde{\theta}_i$ for all $i \in I$ by the previous fact. Let $C(\theta)$ denote those $i \in I$ with $\theta_i > \tilde{\theta}_i$. For any $i \in C(\theta)$ then $q(\theta'_i, \theta_{-i}) = 1$ for all $\theta'_i \in (\tilde{\theta}_i, \bar{\theta}_i]$ by (A.1). Taking turns over $i \in C(\theta)$ shows that $q(\theta') = 1$ for all $\theta' \in \Theta$ with $\theta'_i = \tilde{\theta}_i$ for $i \in I \setminus C(\theta)$ and $\theta'_i > \tilde{\theta}_i$ for $i \in C(\theta)$. By monotonicity of q we further obtain that $q(\theta') = 1$ whenever $\theta'_i \geq \tilde{\theta}_i$ for all $i \in I$ and $\theta'_i > \tilde{\theta}_i$ for $i \in C(\theta)$. We call the latter a tie-breaking coalition. Now let $T := \{C(\theta) \mid q(\theta) = 1\} \cup \{I\}$ be determined this way. Then $q(\theta') = 1$ if and only if $\theta'_i \geq \tilde{\theta}_i$

for all $i \in I$ and $\theta'_i > \tilde{\theta}_i$ for all i in some $C \in T$. Indeed, necessity holds with $C = C(\theta')$ by definition of the latter. Sufficiency has been shown for all $C = C(\theta)$ with $q(\theta) = 1$. Now suppose $\theta'_i > \tilde{\theta}_i$ for all $i \in C = I$. Then there is some $\theta \in \Theta$ with $q(\theta) = 1$, because otherwise $\tilde{\theta}_i = \hat{\theta}_i(\bar{\theta}_{-i}) = \bar{\theta}_i$ for all $i \in I$. Given such θ, now the sufficient condition with $C(\theta)$ is satisfied to yield $q(\theta') = 1$. Note that in particular $\emptyset \in T$ if and only if $q(\tilde{\theta}_i; i \in I) = 1$.
Step 3: Concerning the transfers, note that for all $i \in I$ we have $q(\theta) = 0 \Rightarrow q(\underline{\theta}_i, \theta_{-i}) = 0$ by monotonicity again (see Lemma 1) and thus $\tau_i(\theta_{-i}) = 0$ by Lemma 4, so (3.1) yields $t_i(\theta) = 0$ as claimed. It remains to verify that whenever $q(\theta) = 1$, then $t_i(\theta) = \tilde{\theta}_i$ for all $i \in I$ and $\sum_I \tilde{\theta}_i = c$. First suppose that $\tau_i(\theta_{-i}) = 0$ holds for all $i \in I$ if $q(\theta) = 1$. In this case, we have $t_i(\theta) = \hat{\theta}_i(\theta_{-i}) = \hat{\theta}_i(\bar{\theta}_{-i})$ for $q(\theta) = 1$ by (3.1) and (A.1), and thus indeed $t_i(\theta) = \tilde{\theta}_i$ for all $i \in I$ with the choice in Step 2. Then exact EPBB implies also $\sum_I \tilde{\theta}_i = c$ and the proof is complete in this case.

We now explain how the $\tilde{\theta}_i$ can be modified if there is any $\theta \in \Theta$ with $q(\theta) = 1$, but where the inequality $\tau_i(\theta_{-i}) \geq 0$ from Lemma 2 is strict for some i. Therefore recall from Step 1 that whenever $q(\theta) = 1$, then $q(\bar{\theta}_I) = 1$ by monotonicity and thus $t_i(\theta) = t_i(\bar{\theta}_I)$ by hypothesis, and $\hat{\theta}_i(\theta_{-i}) = \hat{\theta}_i(\bar{\theta}_{-i})$ by (A.1). Then also $\tau_i(\theta_{-i}) = \tau_i(\bar{\theta}_{-i})$ is constant for all these θ by (3.1). Given any $q(\theta) = 1$, exact EPBB thus implies $c - \sum_I \hat{\theta}_i(\bar{\theta}_{-i}) = -\sum_I \tau_i(\bar{\theta}_{-i}) \leq 0$ (see Lemma 2 again). The latter can only be strict if there are $i \in I$ with $\tau_i(\bar{\theta}_{-i}) > 0$. Denoting the latter group by I^{NV}, for all $i \in I^{\mathrm{NV}}$ then $\tau_i(\theta_{-i}) = \tau_i(\bar{\theta}_{-i}) > 0$ for *all* $\theta \in \Theta$ with $q(\theta) = 1$ as just argued (and $\tau_i(\theta_{-i}) = \tau_i(\bar{\theta}_{-i}) = 0$ for all $i \in I \setminus I^{\mathrm{NV}}$ if $q(\theta) = 1$). Thus, as $q(\underline{\theta}, \theta_{-i}) = 0 \Rightarrow \tau_i(\theta_{-i}) = 0$ by Lemma 4, it holds that $q(\theta) = 1 \Rightarrow \tau_i(\theta_{-i}) > 0 \Rightarrow q(\underline{\theta}_i, \theta_{-i}) = 1$ for all $i \in I^{\mathrm{NV}}$. On the one hand this means that $\hat{\theta}_i(\theta_{-i}) = \hat{\theta}_i(\bar{\theta}_{-i}) = \underline{\theta}_i$ for all $i \in I^{\mathrm{NV}}$. On the other hand, recalling monotonicity, we see that q is independent of $(\theta_i; i \in I^{\mathrm{NV}})$, and so are also all transfers, because we have already shown $t_i(\theta) = t_i(\bar{\theta}_I)q(\theta)$ for all $i \in I$ and $\theta \in \Theta$ ($q(\theta) = 1 \Rightarrow t_i(\theta) = t_i(\bar{\theta}_I)$ and $q(\theta) = 0 \Rightarrow t_i(\theta) = 0$). Therefore we can consider the direct mechanism $(q, t_i; i \in I \setminus I^{\mathrm{NV}})$ with cost $c - \sum_{I^{\mathrm{NV}}} t_i(\bar{\theta}_I)$, which is DSIC, EPIR, exact EPBB and satisfies $\tau_i(\theta_{-i}) = 0$ for all $i \in I \setminus I^{\mathrm{NV}}$ whenever $q(\theta) = 1$. It thus has the threshold form with $\tilde{\theta}_i = \hat{\theta}_i(\bar{\theta}_{-i})$ for all $i \in I \setminus I^{\mathrm{NV}}$ in Step 2 and some tie-breaking coalitions $T^{-\mathrm{NV}} \subset \mathcal{P}(I \setminus I^{\mathrm{NV}})$, and satisfies $t_i(\theta) = \tilde{\theta}_i q(\theta)$ for all $i \in I \setminus I^{\mathrm{NV}}$ and $\theta \in \Theta$ as shown at the beginning of Step 3, with $\tilde{\theta}_i = t_i(\theta) = t_i(\bar{\theta}_I)$ for all $i \in I \setminus I^{\mathrm{NV}}$ given any $q(\theta) = 1$. This allows to choose any $\tilde{\theta}_i \leq \underline{\theta}_i$ for $i \in I^{\mathrm{NV}}$ and $T = T^{-\mathrm{NV}} \cup \{I\}$ for the full mechanism to obtain the desired representation of q. Indeed, we now have $\theta'_i \geq \tilde{\theta}_i$ for all $i \in I$ if and only if $\theta'_i \geq \tilde{\theta}_i$ for all $i \in I \setminus I^{\mathrm{NV}}$ and $\theta'_i > \tilde{\theta}_i$ for all i in some $C \in T$ if and only if $\theta'_i > \tilde{\theta}_i$ for all i in some $C' \in T^{-\mathrm{NV}}$, noting that $\theta'_i > \tilde{\theta}_i$ for all $i \in I \in T$ implies the same for all $i \in I^{\mathrm{NV}} \in T^{-\mathrm{NV}}$.

In particular, we may choose $\tilde{\theta}_i = t_i(\bar{\theta}_I)$ for all $i \in I^{\mathrm{NV}}$ if there is any $\theta \in \Theta$ with $q(\theta) = 1$, because for these agents $q(\underline{\theta}_i, \theta_{-i}) = 1$ and thus $t_i(\bar{\theta}_I) = t_i(\underline{\theta}_i, \theta_{-i}) \leq \underline{\theta}_i$ by EPIR. Then also $t_i(\theta) = t_i(\bar{\theta}_I) = \tilde{\theta}_i$ whenever $q(\theta) = 1$, like for all $i \in I \setminus I^{\mathrm{NV}}$, such that $\sum_I \tilde{\theta}_i = c$ again by exact EPBB. $\square$

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