On freedom, lack of information and the preference for easy choices*

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Abstract

This paper is devoted to the study of opportunity sets comparisons when the sets may contain options whose characteristics are not completely known. We propose a suitable environment in which this problem can be approached, and provide axiomatic characterizations of several rules for ranking sets in such a context.

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1 Introduction

A growing number of works have been concerned with the formal analysis of the value of freedom of choice, especially during the last two decades (see, among many others, Arlegi et al. (2005), Bossert (1997, 2000), Bossert et al. (1994), Jones and Sugden (1982), Pattanaik and Xu (1990, 1998, 2000), Puppe (1995, 1996), Sen (1991, 1993), and Sugden (1998))[1]. In most of those works the central object of analysis is the opportunity set enjoyed by the agent, which is taken as the reference to evaluate the freedom of choice he/she enjoys. An opportunity set is interpreted as the set of mutually exclusive options from which the decision maker has the power to choose one. In general, those works follow an axiomatic methodology: starting from conditions that fit well with the possible motivations for the preference for freedom of choice, they characterize rules to compare and rank sets in terms of freedom.

An implicit assumption in the mentioned models is that the decision maker has complete information about all alternatives in the opportunity sets. However, there are many real life situations where agents choose among some alternatives that they know well, while having also the possibility to choose options that are not completely well-known.

Actually, one finds easily a wide range of economic decisions made in such an environment. For example, choices in supermarkets are usually made among different brands of the same product, some of which have already been experienced by the consumer while others are unknown. A firm could have the possibility to choose among different production plans, for some of

[1] For a complete survey, including a discussion about the reasons for which freedom of choice could be valuable, see Barberà et al. (2004).

which the expected flow of profits they generate can be fairly determined, while for others a market research would be necessary. An employer may have different candidates for a job, having detailed reports of some while lacking enough references for others. And the same could apply for many examples in everyday choices made by consumers, such as buying a new car or choosing a dish from a menu in a restaurant.

In choice situations like those there are reasons to think that the decision maker might be averse to have such kind of not completely known or badly determined options, and we observe a *preference for easy choice situations*, in which such disturbing options are not present. This means that very often there is a trade-off between the desire of freedom of choice and a certain aversion to the presence of badly defined options. Bare introspection suggests that, though wanting to enjoy freedom of choice, the presence of too many brands at the shelf of the supermarket, or too many meals in the menu might be annoying. Some classical authors in the filed of Organization Theory argue that it is precisely the human necessity of being coherent in his decisions and following clear goals which motivates his preference for simpler problems (see for example Friedman (1954) or Krulee (1955)). Thus, the presence of such options is an obstacle for feeling as making accurate decisions, unless certain costs for gathering additional information are paid.

Taking into account such a circumstance leads to a deviation from the kind of rules characterized in the freedom of choice literature: all such rules share a "monotonicity" property, according to which any enlargement of an opportunity set always leads to an improvement in terms of freedom, either strict of weak. In our case, however, the availability of more options could make a choice situation worse under some circumstances. Thus, what is meant by effective freedom of choice is bounded by the aversion to the presence of such perturbing alternatives that make choices more complicate.

Obviously, the effects of these opposite forces can translate into different trade-off possibilities. In the following sections we provide axiomatic characterizations of several possible rankings each displaying reasonable ways to trade with both effects. In Section 2 we introduce some notation and definitions. In Section 3 we characterize axiomatically a rule for ranking sets that takes into account only the number of the options that are known to the decision maker. Changing one of the axioms results in a family of rankings that is uniquely based on the number of known options and the number of unknown options, weighting positively the former and negatively the later; this family is introduced in Section 4. We present the axiomatic characterizations of three particular rules of this family in Section 5, and conclude in Section 6 with some final remarks. All proofs are collected in the Appendix.

2 Basic setup

Let X be a non-empty set of alternatives (finite or infinite) with $|X| \geq 2$, and $\mathcal{X}$ be the set of all finite subsets of X. We will denote the elements of $\mathcal{X}$ by $A, B \ldots$. The interpretation of each element of $\mathcal{X}$ is that of an *opportunity set*: the set of (mutually exclusive) options enjoyed by the decision maker.

We will distinguish two categories of alternatives: those, whose relevant aspects are sufficiently well determined, and those, for which there is lack of information about their relevant characteristics that is large enough. For fluency reasons we will use a "known-unknown" terminology in order to label the options. We will define as "known" options those, whose relevant characteristics in order to be evaluated and compared with other options are known by the agent. We will define as "unknown" the remaining options.

Note that, according to this terminology, an option could be labelled as "unknown" even if the decision maker has big pieces of information about it, but not sufficiently relevant: an employer may have a very detailed information about certain aspects of a candidate, such as information about his private life, but we will label the candidate as unknown if the employer has not the necessary information about what is relevant for her choice, namely, the information about the candidate's labor skills.

In our context the choice problem can be determined by two aspects: the opportunity set the agent enjoys (which is an element of $\mathcal{X}$), and the set of alternatives in X that are known to the decision maker, which is also an element of $\mathcal{X}$. In order to avoid some trivialities, we assume further that there is at least one option in X that is known to the agent. Formally, we are interested in the elements of $\mathcal{X} \times \mathcal{X}_{\emptyset}$, where $\mathcal{X}_{\emptyset} := \mathcal{X} \setminus \{\emptyset\}$. We call each $(A, C) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$ an *extended opportunity set* and attach to it the following interpretation: the decision maker has the opportunity to choose from A having only enough information about the options collected in C. Comparisons of extended opportunity sets will be represented by a binary relation $\succsim$ defined on $\mathcal{X} \times \mathcal{X}_{\emptyset}$. For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \succsim (B, D)$ should be read as "having the possibility to choose from A when knowing the options in C is weakly preferred to having the possibility to choose from B when knowing the options in D". The asymmetric and symmetric parts of $\succsim$ will be denoted by $\succ$ and $\sim$, respectively. We want for $\succsim$ to capture both the preference for freedom of choice and the possible aversion to the presence of unknown options. We denote by $\mathcal{P}$ the set of all reflexive, transitive and complete binary relations on $\mathcal{X} \times \mathcal{X}_{\emptyset}$.

3 The known-options-based rule

We start our analysis by introducing the following four axioms a preference relation $\succsim \in \mathcal{P}$ may satisfy:

Empty choice (EC): For all $C, D \in \mathcal{X}_{\emptyset}$,

$(\emptyset, C) \sim (\emptyset, D)$;

Simple monotonicity (SM): For all $x \in X$ and all $y \in C \in \mathcal{X}_{\emptyset}$,

$(\{x, y\}, C) \succ (\{x\}, C)$;

Simple neutrality (SN): For all $x \in X$ and all $y \notin C \in \mathcal{X}_{\emptyset}$,

$(\{x, y\}, C) \sim (\{x\}, C)$;

Independence (IND): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, and all $x \in X \setminus A, y \in X \setminus B$ with $x \in C \Leftrightarrow y \in D$,

$(A, C) \succsim (B, D) \Leftrightarrow (A \cup \{x\}, C) \succsim (B \cup \{y\}, D)$.

According to EC the decision maker is indifferent between any two situations in which he has no options to choose from, regardless the amount of information he might have about the (unavailable) alternatives.

SM considers the adition of a new known option in a situation where the decision maker has no freedom of choice as a strict improvement. This condition is a translation to our context of a well-known axiom in the literature on freedom of choice, initially introduced by Pattanaik and Xu (1990), in which *any* additional alternative improves the situation.

According to SN the adition of a new option to a situation in which the decision maker already enjoys one alternative does not affect his or her freedom if the added option is unknown. It could be interpreted as if, in the situation described, the aditional availability of the unknown option would not annoy the decision maker, but either increase effectively his freedom of choice.

IND displays the influence that adding (or dropping out) options of the same type (either known or unknown) has on the ranking over two extended opportunity sets, namely, that the ranking is preserved. This axiom adapts to our context similar axioms that can often be found in the axiomatic characterizations of rankings of opportunity sets in terms of freedom of choice. In particular, compared with such axioms, IND restricts the requirement of coherence of the preference to those cases in which what is added (dropped out) to (from) both sets is "similar" in the sense that it is either a known option in both sets or an unknown one.

As we will see in our first theorem, combining the four axioms introduced so far provides a characterization of the *known-options-based rule* $\succsim^1 \in \mathcal{P}$ defined as follows:

For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,

$$(A, C) \succsim^1 (B, D) \text{ iff } |A \cap C| \geq |B \cap D| .$$

Theorem 1 $\succsim \in \mathcal{P}$ *satisfies EC, IND, SM, and SN if and only if* $\succsim = \succsim^1$.

In order to show the *independence* of the axioms used for the characterization of $\succsim^1$, consider the following four examples. The reader can easily check that each example satisfies all axioms except one:

($\neg$EC): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff (1) $|C| < |D|$, or (2) $|C| = |D|$ and $|A \cap C| \geq |B \cap D|$.

($\neg$IND): Let $|X| \geq 3$. For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff (1) if $|A| \geq 3$ and $|B| \geq 3$, then $(A, C) \sim (B, D)$, (2) if $|A| \geq 3$ and $|B| < 3$, then $(A, C) \succ (B, D)$, (3) if $|A| < 3$ and $|B| < 3$, then $\succsim = \succsim^1$.

($\neg$SM): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff $|A \cap C| \leq$

$|B \cap D|$.

($\neg$SN): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \succsim (B, D)$ iff (1) $|A \cap C| > |B \cap D|$, or (2) $|A \cap C| = |B \cap D|$ and $|A \setminus C| \leq |B \setminus D|$.

4 A family of rules

The axiom of simple neutrality represents one possible way to take into account the effect of adding an unknown option to the decision maker's opportunity set consisting of a single element. The next axiom displays another possibility.

Simple aversion (SA): For all $x \in X$ and all $y \notin C \in \mathcal{X}_{\emptyset}$,

$(\{x, y\}, C) \prec (\{x\}, C)$.

SA represents a very elementary way to express the idea of preference for an easier choice. This axiom says that having an additional unknown option to choose from "bothers" the decision maker, rather than leaving him indifferent.

It turns out that replacing SN by SA in the characterization displayed by Theorem 1 does not result in a well defined rule for ranking extended opportunity sets; it rather generates a family of rules that are based on two unique numbers: the number of known options and the number of unknown options in the corresponding sets, weighting possitively the former and negatively the later.

Theorem 2 *Let* $\succsim \in \mathcal{P}$ *satisfy EC, IND, SM, and SA. Then, for all* $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$,

(1) $|A \cap C| > |B \cap D|$ *and* $|A \setminus C| < |B \setminus D|$ *imply* $(A, C) \succ (B, D)$,

(2) $|A \cap C| \geq |B \cap D|$ *and* $|A \setminus C| \leq |B \setminus D|$ *imply* $(A, C) \succsim (B, D)$.

5 Possible trade-offs

Our aim in this section is to characterize three particular rules for ranking extended opportunity sets that are contained in the family of rankings described above, each of them solving in a particular way the trade-off between the preference for known alternatives and the aversion to the unknown ones. The first two rules combine those aspects lexicographically, while the third rule is of an additive nature.

5.1 The known-options-priority rule

Let us consider a situation in which $(A, C) \succ (B, D)$ for some $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$ and in which the opportunity set B is included in $A \cap C$, i.e. B contains only options that would be known in the situation (A, C). The idea of robustness of the strict preference displayed by the next axiom requires in this case that adding a new option to A does not change the original ranking.

Robustness 1 (ROB1): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$ with $B \subset (A \cap C)$ and for all $x \in X$,

$(A, C) \succ (B, D) \Rightarrow (A \cup \{x\}, C) \succ (B, D)$.

In other words, ROB1 says that, if all what the decision maker knows in situation (B, D) is already known in situation (A, C), but also all that is unknown in (B, D) is known in (A, C), then a preference for (A, C) (which is rather plausible) should be robust enough as to support the incorporation of a new option in A even if such an additional option is unknown.

We have shown in Section 4 that EC, IND, SM, and SA generate a family of rules to rank extended opportunity sets. Adding ROB1 to these axioms results in the characterization of the *known-options-priority rule* $\succsim^2 \in \mathcal{P}$ defined as follows:

For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,

$$(A, C) \succsim^2 (B, D) \text{ iff } \begin{cases} |A \cap C| > |B \cap D|, \\ \text{or} \\ |A \cap C| = |B \cap D| \text{ and } |A \setminus C| \leq |B \setminus D|. \end{cases}$$

Theorem 3 *$\succsim \in \mathcal{P}$ satisfy EC, IND, SM, SA, and ROB1 if and only if $\succsim = \succsim^2$.*

The *independence* of the axioms used for the characterization of $\succsim^2$ can be checked by means of the following five examples:

($\neg$EC): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff (1) $|C| < |D|$, or (2) $|C| = |D|$ and $(A, C) \succsim^2 (B, D)$.

($\neg$IND): Let $|X| \geq 3$. For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, (1) if $|A| \geq 3$ and $|B| \geq 3$, then $(A, C) \sim (B, D)$, (2) if $|A| \geq 3$ and $|B| < 3$, then $(A, C) \succ (B, D)$, (3) if $|A| < 3$ and $|B| < 3$, then $\succsim = \succsim^2$.

($\neg$SM): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff $|A \setminus C| \leq |B \setminus D|$.

($\neg$SA): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff $|A| \geq |B|$.

($\neg$ROB1): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff (1) $|A \setminus C| < |B \setminus D|$, or (2) $|A \setminus C| = |B \setminus D|$ and $|A \cap C| \geq |B \cap D|$.

5.2 The unknown-options-priority rule

Let us consider now another notion of robustness, as shown by the following axiom.

Robustness 2 (ROB2): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$ with $A \subset (B \setminus D)$ and for all $x \in X$,

$(A,C) \succ (B,D) \Rightarrow (A,C) \succ (B \cup \{x\}, D)$.

According to this axiom, in a situation in which the opportunity set A consist only of options that would be unknown in the situation (B,D) and (A,C) is strictly better than (B,D), then the ranking is preserved when a new option is added to B. In other words, if all that is unknown in situation (A,C) is also unknown in (B,D), but also all that is known in (A,C) is unknown in (B,D), then a preference of (A,C) over (B,D) should be robust enough as to support the incorporation to B of a new option, even if it is known.

As shown in our next theorem, adding ROB2 to EC, IND, SM, and SA results in the characterization of the *unknown-options-priority rule* $\succsim^3 \in \mathcal{P}$ defined as follows:

For all $(A,C), (B,D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,

$$(A,C) \succsim^3 (B,D) \text{ iff } \begin{cases} |A \setminus C| < |B \setminus D|, \\ \text{or} \\ |A \setminus C| = |B \setminus D| \text{ and } |A \cap C| \geq |B \cap D|. \end{cases}$$

Theorem 4 *$\succsim \in \mathcal{P}$ satisfy EC, IND, SM, AV, and ROB2 if and only if $\succsim = \succsim^3$.*

In order to show the *independence* of the axioms used for the characterization of $\succsim^3$, consider the following examples:

($\neg$EC): For all $(A,C), (B,D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A,C) \succsim (B,D)$ iff (1) $|C| < |D|$, or (2) $|C| = |D|$ and $(A,C) \succsim^3 (B,D)$.

($\neg$IND): Let $|X| \geq 3$. For all $(A,C), (B,D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, (1) if $|A| \geq 3$ and $|B| \geq 3$, then $(A,C) \sim (B,D)$, (2) if $|A| \geq 3$ and $|B| < 3$, then $(A,C) \succ (B,D)$, (3) if $|A| < 3$ and $|B| < 3$, then $\succsim = \succsim^3$.

(¬SM): For all $(A,C),(B,D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A,C) \succsim (B,D)$ iff $|A \setminus C| \leq |B \setminus D|$.

(¬SA): For all $(A,C),(B,D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A,C) \succsim (B,D)$ iff $|A| \geq |B|$.

(¬ROB2): Take $\succsim = \succsim^2$.

5.3 The dichotomy rule

Let us consider now the following axiom (see also Dimitrov et al. (2004)).

Dichotomy (DI): For all $(A,C),(B,C) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, all $x \in A \cap C$ and all $y \in X \setminus (B \cup C)$,

$(A,C) \sim (B,C) \Rightarrow (A \setminus \{x\}, C) \sim (B \cup \{y\}, C)$.

DI says that if two situations (A,C) and (B,C) are indifferent, then, the indifference is preserved if we take out an option from A that is known to the decision maker and, at the same time, we add to B an unknown option. This axiom establishes a kind of prefect substitution between known and unknown options in certain situations: loosing freedom by removing a known option is, somehow, equivalent to losing ease in choice by adding an unknown option.

We are ready now to present the characterization of the *dichotomy rule* $\succsim^4 \in \mathcal{P}$ defined as follows:

For all $(A,C),(B,D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$,

$$(A,C) \succsim^4 (B,D) \text{ iff } |A \cap C| - |A \setminus C| \geq |B \cap D| - |B \setminus D|.$$

Theorem 5 *$\succsim \in \mathcal{P}$ satisfy EC, IND, SM, and DI if and only if $\succsim = \succsim^4$.*

In order to check the *independence* of the axioms used for the characterization of $\succsim^4$, the reader can consider the following examples:

(¬EC): For all $(A,C),(B,D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A,C) \succsim (B,D)$ iff (1) $|C| < |D|$, or (2) $|C| = |D|$ and$(A, C) \succsim^4 (B, D)$.

($\neg$IND): Let $X = \{x, y\}$, and let the the following ranking $\succcurlyeq$ on $\mathcal{X} \times \mathcal{X}_\emptyset$: $(\{x,y\},\{x\}) \succ (\{x,y\},\{y\}) \succ (\{x,y\},\{x,y\}) \succ (\{x\},\{x\}) \succ (\{x\},\{y\}) \succ (\{x\},\{x,y\}) \succ (\{y\},\{x\}) \succ (\{y\},\{y\}) \succ (\{y\},\{x,y\}) \succ (\emptyset,\{x\}) \sim (\emptyset,\{y\}) \sim (\emptyset,\{x,y\})$.

($\neg$SM): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \sim (B, D)$.

($\neg$DI): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset : (A, C) \succsim (B, D)$ iff $|A| \geq |B|$.

6 Concluding remarks

Our model assumes the preference for freedom of choice but also the possibility of an aversion to alternatives whose characterisitics are unsufficiently known, that is, what we have called a *preference for easy choices*. Thus, there are several ways how the decision maker may evaluate opportunity sets containing what we have called "known" and "unknown" options. The previous rules display plausible solutions to this problem springing up from a common axiomatic basis, reflected by the axioms of Empty Choice (EC), Independence (IND), and Simple Monotonicity (SM). Adding Simple Neutrality (SN) to this basis defines the known-options-based rule, by which it is the number of known options what determines the ranking. However, when replacing SN by Simple Aversion (SA) a family of rules that takes into account the ease to choose by weighting negatively the number of unknown options arises. EC, IND, SM, and SA constitute the axiomatic core for the following characterizations. Taking on either of the Robustness axioms (ROB1 and ROB2) to that core we obtain, respectively, the characterization of two lexicographic rules: the known-options-priority-rule and the unknown-options-priority rule. If, instead of any of the robustness axioms we use dichotomy (DI), then an additive rule that maximizes the difference between the number of the known options and the number of the unknown ones is obtained.

7 Appendix

This section collects the proofs of all theorems that appear in the text.

Theorem 1 *$\succsim \in \mathcal{P}$ satisfies EC, IND, SM, and SN if and only if $\succsim = \succsim^1$.*

We will prove first the following two lemmas.

Lemma 1 *Let $\succsim \in \mathcal{P}$ satisfy IND and SM. Then $(A \cup E, C) \succ (A, C)$ for all $(A, C) \in \mathcal{X} \times \mathcal{X}_\emptyset$ and all $E \subseteq (C \setminus A) \setminus \{\emptyset\}$.*

Proof of Lemma 1. Take $\succsim \in \mathcal{P}$ as above and let $E = \{e_1, \ldots, e_n\}$. We will distinguish the following two cases:

(1) $A \neq \emptyset$. Let $A = \{a_1, \ldots, a_m\}$. By SM, $(\{a_1, e_1\}, C) \succ (\{a_1\}, C)$. By IND applied $(m-1)$-times, $(A \cup \{e_1\}, C) \succ (A, C)$. Again by SM, $(\{a_1, e_2\}, C) \succ (\{a_1\}, C)$, and by IND applied m-times, $(A \cup \{e_1, e_2\}, C) \succ (A \cup \{e_1\}, C)$. Repeating the same step $(n-2)$-times and by transitivity, $(A \cup E, C) \succ (A, C)$.

(2) $A = \emptyset$. Take $x \neq e_1 \in E$ (note that such an element exists because $|X| \geq 2$). By SM, $(\{x, e_1\}, C) \succ (\{x\}, C)$. By IND $(\{e_1\}, C) \succ (\emptyset, C)$, i.e. $(A \cup \{e_1\}, C) \succ (A, C)$. Again by IND applied $(n-1)$-times, and by transitivity we reach $(A \cup E, C) \succ (A, C)$. ■

Lemma 2 *Let $\succsim \in \mathcal{P}$ satisfy IND and SN. Then $(A \cup E, C) \sim (A, C)$ for all $(A, C) \in \mathcal{X} \times \mathcal{X}_\emptyset$ and all $E \subseteq X \setminus (A \cup C)$.*

Proof of Lemma 2. The proof is similar to the proof of Lemma 1 applying

SN instead of SM. ■

Proof of Theorem 1. Clearly, $\succsim^1$ satisfies these axioms. Suppose now that $\succsim \in \mathcal{P}$ satisfies EC, IND, SM, and SN. We have to prove that

(1) $|A \cap C| > |B \cap D|$ implies $(A, C) \succ (B, D)$, and

(2) $|A \cap C| = |B \cap D|$ implies $(A, C) \sim (B, D)$.

(1) Let $|A \cap C| > |B \cap D|$. By EC, $(\emptyset, C) \sim (\emptyset, D)$. If $|B \cap D| = 0$ (i.e. $B \cap D = \emptyset$), $(A \cap C, C) \succ (\emptyset, C)$ follows from Lemma 1 with $A \cap C$ in the role of E. By transitivity, $(A \cap C, C) \succ (B \cap D, D)$. If $|B \cap D| = s > 0$, applying IND s-times results in $((A \cap C)_s, C) \sim (B \cap D, D)$, where $(A \cap C)_s$ is any subset of $A \cap C$ with s elements. By Lemma 1 with $(A \cap C) \setminus (A \cap C)_s$ in the role of E, we have $(A \cap C, C) \succ ((A \cap C)_s, C)$ that, by transitivity, results in $(A \cap C, C) \succ (B \cap D, D)$. By Lemma 2 with $A \setminus (A \cap C)$ in the role of E, we obtain $(A, C) \sim (A \cap C, C)$. By the same argument, and with $B \setminus (B \cap D)$ in the role of E, we have $(B, D) \sim (B \cap D, D)$. By transitivity, $(A, C) \succ (B, D)$.

(2) Let $|A \cap C| = |B \cap D|$. As before, by applying EC and IND, we get either $(\emptyset, C) \sim (\emptyset, D)$ (if $|A \cap C| = |B \cap D| = 0$) or $(A \cap C, C) \sim (B \cap D, D)$ (if $|A \cap C| = |B \cap D| > 0$). By Lemma 2 with $A \setminus C$ in the role of E we have $(A, C) \sim (A \cap C, C)$. By the same argument, and with $B \setminus D$ in the role of E, we have $(B, D) \sim (B \cap D, D)$. Thus, by transitivity, $(A, C) \sim (B, D)$. ■

Theorem 2 *Let $\succsim \in \mathcal{P}$ satisfy EC, IND, SM, and SA. Then, for all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,*

(1) $|A \cap C| > |B \cap D|$ and $|A \setminus C| < |B \setminus D|$ imply $(A, C) \succ (B, D)$,

(2) $|A \cap C| \geq |B \cap D|$ and $|A \setminus C| \leq |B \setminus D|$ imply $(A, C) \succsim (B, D)$.

Note first that the following lemma holds true.

Lemma 3 *Let $\succsim \in \mathcal{P}$ satisfy IND and SA. Then $(A \cup E, C) \prec (A, C)$ for all*

$(A, C) \in \mathcal{X} \times \mathcal{X}_\emptyset$ *and all* $E \subseteq (X \setminus (A \cup C)) \setminus \{\emptyset\}$.

Proof of Lemma 3. The proof is similar to the proof of Lemma 1 by applying SA instead of SM. ■

Proof of Theorem 2. (1) Let $|A \cap C| > |B \cap D|$ and $|A \setminus C| < |B \setminus D|$. Like in the first part of the proof of Theorem 1 it can be proved, by using EC, IND and SM, that $(A \cap C, C) \succ (B \cap D, D)$. By Lemma 3 we have $(B \cap D, D) \succ (B, D)$. Let $|A \setminus C| = u$. Starting with $(A \cap C, C) \succ (B \cap D, D)$ and applying u-times IND we get $(A, C) \succ ((B \cap D) \cup (B \setminus D)_u, D)$, where $(B \setminus D)_u$ is any subset of $B \setminus D$ with u elements. By Lemma 3 with $(B \setminus D) \setminus (B \setminus D)_u$ in the role of E, $((B \cap D) \cup (B \setminus D)_u, D) \succ (B, D)$. By transitivity, $(A, C) \succ (B, D)$.

(2) The case in which $|A \cap C| > |B \cap D|$ and $|A \setminus C| < |B \setminus D|$ was proved in the previous paragraph. Thus, we will distinguish the three remaining possible cases:

(2.1) $|A \cap C| > |B \cap D|$ and $|A \setminus C| = |B \setminus D|$,

(2.2) $|A \cap C| = |B \cap D|$ and $|A \setminus C| < |B \setminus D|$, and

(2.3) $|A \cap C| = |B \cap D|$ and $|A \setminus C| = |B \setminus D|$.

(2.1) Like in the first part of the proof of Theorem 1 it can be proved, by using EC, IND and SM, that $(A \cap C, C) \succ (B \cap D, D)$. If $|A \setminus C| = |B \setminus D| = 0$, then $(A, C) \succ (B, D)$ follows directly. If $|A \setminus C| = |B \setminus D| = u > 0$, by IND repeated u-times, $(A, C) \succ (B, D)$.

(2.2) Let $|A \setminus C| = u$. From EC and applying u-times IND, $(A \setminus C, C) \sim ((B \setminus D)_u, D)$, where $(B \setminus D)_u$ is any subset of $B \setminus D$ with u elements. By Lemma 3 with $(B \setminus D) \setminus (B \setminus D)_u$ in the role of E, $((B \setminus D)_u, D) \succ (B \setminus D, D)$. By transitivity, $(A \setminus C, C) \succ (B \setminus D, D)$. Applying IND $|A \cap C| = |B \cap D|$-times, $(A, C) \succ (B, D)$.

(2.3) From EC and applying IND $|A \cap C| = |B \cap D|$-times, $(A \cap C, C) \sim$

$(B \cap D, D)$. Again by IND $|A \setminus C| = |B \setminus D|$-times, $(A, C) \sim (B, D)$. ■

Theorem 3 *$\succsim \in \mathcal{P}$ satisfy EC, IND, SM, SA, and ROB1 if and only if $\succsim = \succsim^2$.*

Proof of Theorem 3. It can be easily checked that $\succsim^2$ satisfies the five axioms. Suppose now that $\succsim \in \mathcal{P}$ satisfies EC, IND, SM, SA, and ROB1. We have to prove that

(1) $|A \cap C| > |B \cap D|$ implies $(A, C) \succ (B, D)$,

(2) $|A \cap C| = |B \cap D|$ and $|A \setminus C| < |B \setminus D|$ imply $(A, C) \succ (B, D)$, and

(3) $|A \cap C| = |B \cap D|$ and $|A \setminus C| = |B \setminus D|$ imply $(A, C) \sim (B, D)$.

(1) Let $|A \cap C| > |B \cap D|$. Like in the first part of the proof of Theorem 1, EC, IND, and SM imply $(A \cap C, C) \succ (B \cap D, D)$.

Let us consider now the following partitions of $A \cap C$ and $B \cap D$:

$$\begin{aligned} A \cap C &= (A \cap C)^1 \cup (A \cap C)^2 \cup (A \cap C)^3 \\ B \cap D &= (B \cap D)^1 \cup (B \cap D)^2 , \end{aligned}$$

where

$$\begin{aligned} (A \cap C)^1 &= \{x \in A \cap C \mid x \in B \cap D\} , \\ (A \cap C)^2 &= \{x \in A \cap C \mid x \in D \setminus (B \cap D)\} , \\ (A \cap C)^3 &= \{x \in A \cap C \mid x \in X \setminus D\} , \\ (B \cap D)^1 &= \{x \in B \cap D \mid x \in A \cap C\} , \\ (B \cap D)^2 &= \{x \in B \cap D \mid x \in X \setminus (A \cap C)\} . \end{aligned}$$

By construction $(A \cap C)^1 = (B \cap D)^1$. Hence, $B \cap D = (A \cap C)^1 \cup (B \cap D)^2$. Let $\left|(B \cap D)^2\right| = s_2$. We will consider two cases:

(1.1) $\left|(A \cap C)^2\right| > s_2$, and

(1.2) $\left|(A \cap C)^2\right| \leq s_2$.

(1.1) By Theorem 2,

$$\left((A\cap C)^1\cup(B\cap D)^2\,,D\right)\sim\left((A\cap C)^1\cup(A\cap C)^2_{s_2}\,,D\right),$$

where $(A\cap C)^2_{s_2}$ is any subset of $(A\cap C)^2$ with s_2 elements. Hence, $(B\cap D,D)\sim\left((A\cap C)^1\cup(A\cap C)^2_{s_2}\,,D\right)$, and by transitivity,

$$(A\cap C,C)\succ\left((A\cap C)^1\cup(A\cap C)^2_{s_2}\,,D\right).$$

By ROB1 repeatedly applied, $(A,C)\succ\left((A\cap C)^1\cup(A\cap C)^2_{s_2}\,,D\right)$. By transitivity, $(A,C)\succ(B\cap D,D)$. If $B\setminus D=\emptyset$, we have that $(A,C)\succ(B,D)$. If $B\setminus D\neq 0$, then, by Lemma 3 $(B\cap D,D)\succ(B,D)$, and by transitivity $(A,C)\succ(B,D)$.

(1.2) By Theorem 2,

$$\left((A\cap C)^1\cup(B\cap D)^2\,,D\right)\sim\left((A\cap C)^1\cup(A\cap C)^2\cup(B\cap D)^2_*\,,D\right),$$

where $(B\cap D)^2_*$ is any subset of $(B\cap D)^2$ s.t. $\left|(B\cap D)^2_*\right|=\left|(B\cap D)^2\right|-\left|(A\cap C)^2\right|$. Again by Theorem 2,

$$\begin{aligned}&\left((A\cap C)^1\cup(A\cap C)^2\cup(B\cap D)^2_*\,,D\right)\\ \sim\;&\left((A\cap C)^1\cup(A\cap C)^2\cup(A\cap C)^3_*\,,\left(D\cup(A\cap C)^3_*\right)\setminus(B\cap D)^2_*\right),\end{aligned}$$

where $(A\cap C)^3_*$ is any subset of $(A\cap C)^3$ s.t. $\left|(A\cap C)^3_*\right|=\left|(B\cap D)^2\right|-\left|(A\cap C)^2\right|$ (given that $|A\cap C|>|B\cap D|$ by hypothesis, such a subset always exists by construction). By transitivity,

$$\begin{aligned}&\left((A\cap C)^1\cup(B\cap D)^2\,,D\right)\\ \sim\;&\left((A\cap C)^1\cup(A\cap C)^2\cup(A\cap C)^3_*\,,\left(D\cup(A\cap C)^3_*\right)\setminus(B\cap D)^2_*\right),\end{aligned}$$

that is

$$\begin{aligned}&(B\cap D,D)\\ \sim\;&\left((A\cap C)^1\cup(A\cap C)^2\cup(A\cap C)^3_*\,,\left(D\cup(A\cap C)^3_*\right)\setminus(B\cap D)^2_*\right).\end{aligned}$$

By transitivity,

$$\begin{aligned} & (A\cap C,C) \\ \succ\ & \left((A\cap C)^1\cup(A\cap C)^2\cup(A\cap C)^3_*,\left(D\cup(A\cap C)^3_*\right)\setminus(B\cap D)^2_*\right). \end{aligned}$$

By ROB1 repeatedly,

$$\begin{aligned} & (A,C) \\ \succ\ & \left((A\cap C)^1\cup(A\cap C)^2\cup(A\cap C)^3_*,\left(D\cup(A\cap C)^3_*\right)\setminus(B\cap D)^2_*\right). \end{aligned}$$

By transitivity, $(A,C)\succ(B\cap D,D)$. If $B\setminus D\neq\emptyset$, then, by Lemma 3 $(B\cap D,D)\succ(B,D)$. If $B\setminus D=\emptyset$, then $B\cap D=B$. Again by transitivity, $(A,C)\succ(B,D)$.

(2) Let $|A\cap C|=|B\cap D|$ and $|A\setminus C|<|B\setminus D|$. By Theorem 2 $(A\cap C,C)\sim(B\cap D,D)$. Let $|A\setminus C|=u$. By IND repeated u-times,

$$(A,C)\sim\left((B\cap D)\cup(B\setminus D)_u,D\right),$$

where $(B\setminus D)_u$ is any subset of $B\setminus D$ with u elements. By Lemma 3 with $B\setminus((B\cap D)\cup(B\setminus D)_u)$ in the role of E, $((B\cap D)\cup(B\setminus D)_u,D)\succ(B,D)$. Thus, by transitivity, $(A,C)\succ(B,D)$.

(3) Let $|A\cap C|=|B\cap D|$ and $|A\setminus C|=|B\setminus D|$. By Theorem 2, $(A,C)\sim(B,D)$. ■

Theorem 4 *$\succsim\in\mathcal{P}$ satisfy INCS, IND, SM, AV, and ROB2 if and only if $\succsim=\succsim^3$.*

Proof of Theorem 4. It is not difficult to check that $\succsim^3$ satisfies the five axioms. Suppose now that $\succsim\in\mathcal{P}$ satisfies EC, IND, SM, SA, and ROB2. We have to prove that

(1) $|A\setminus C|<|B\setminus D|$ implies $(A,C)\succ(B,D)$,

(2) $|A \setminus C| = |B \setminus D|$ and $|A \cap C| > |B \cap D|$ imply $(A, C) \succ (B, D)$, and

(3) $|A \setminus C| = |B \setminus D|$ and $|A \cap C| = |B \cap D|$ imply $(A, C) \sim (B, D)$.

(1) Let $A \setminus C = \left\{a_1^-, \ldots, a_u^-\right\}$ and $B \setminus D = \left\{b_1^-, \ldots, b_v^-\right\}$, $v > u$. By EC and IND applied u-times $(A \setminus C, C) \sim ((B \setminus D)_u, D)$, where $(B \setminus D)_u$ is any subset of $B \setminus D$ with u elements. By Lemma 3 with $(B \setminus D) \setminus (B \setminus D)_u$ in the role of E, $((B \setminus D)_u, D) \succ (B \setminus D, D)$. By transitivity, $(A \setminus C, C) \succ (B \setminus D, D)$.

Now, let us consider the following partitions of $A \setminus C$ and $B \setminus D$:

$$\begin{aligned} A \setminus C &= (A \setminus C)^1 \cup (A \setminus C)^2, \\ B \setminus D &= (B \setminus D)^1 \cup (B \setminus D)^2 \cup (B \setminus D)^3, \end{aligned}$$

where

$$\begin{aligned} (A \setminus C)^1 &= \{x \in A \setminus C \mid x \in B \setminus D\}, \\ (A \setminus C)^2 &= \{x \in A \setminus C \mid x \in X \setminus (B \setminus D)\}, \\ (B \setminus D)^1 &= \{x \in B \setminus D \mid x \in A \setminus C\} = (A \setminus C)^1, \\ (B \setminus D)^2 &= \{x \in B \setminus D \mid x \in X \setminus (A \cup C)\}, \\ (B \setminus D)^3 &= \{x \in B \setminus D \mid x \in C\}. \end{aligned}$$

Let $(A \setminus C)^1 = \left\{a_1^-, \ldots, a_{u_1}^-\right\}$, $(A \setminus C)^2 = \left\{a_{u_1+1}^-, \ldots, a_u^-\right\}$, $(B \setminus D)^1 = \left\{b_1^-, \ldots, b_{u_1}^-\right\}$, $(B \setminus D)^2 = \left\{b_{u_1+1}^-, \ldots, b_{v_2}^-\right\}$, $(B \setminus D)^3 = \left\{b_{v_2+1}^-, \ldots, b_v^-\right\}$. Note that, by hypothesis, $\left|(B \setminus D)^2\right| + \left|(B \setminus D)^3\right| > \left|(A \setminus C)^2\right|$. We will consider two cases:

(1.1) $\left|(A \setminus C)^2\right| > \left|(B \setminus D)^2\right|$, and

(1.2) $\left|(A \setminus C)^2\right| \leq \left|(B \setminus D)^2\right|$.

(1.1) Let $\left|(A \setminus C)^2\right| > \left|(B \setminus D)^2\right|$. Consider $\left\{b_{v_2+1}^-, \ldots, b_u^-\right\} \subset (B \setminus D)^3$ and let $(A \setminus C)^2_{v_2} = \left\{a_{u_1+1}^-, \ldots, a_{v_2}^-\right\}$, $(A \setminus C)^2_v = \left\{a_{v_2+1}^-, \ldots, a_u^-\right\}$. By Theorem 2, $\left(\left((A \setminus C) \cup (B \setminus D)^2\right) \setminus (A \setminus C)^2_{v_2}, C\right) \sim (A \setminus C, C)$. Let

$$B^* = \left((A \setminus C) \cup (B \setminus D)^2 \cup \left\{b^-_{v_2+1}, \ldots, b^-_u\right\}\right) \setminus \left((A \setminus C)^2_{v_2} \cup (A \setminus C)^2_v\right).$$

Again by Theorem 2, $\left(B^*, \left(C \cup (A \setminus C)^2_v\right) \setminus \left\{b^-_{v_2+1}, \ldots, b^-_u\right\}\right) \sim (A \setminus C, C)$. By transitivity, $\left(B^*, \left(C \cup (A \setminus C)^2_v\right) \setminus \left\{b^-_{v_2+1}, \ldots, b^-_u\right\}\right) \succ (B \setminus D, D)$. Note that, by construction, $B^* \subset B \setminus D$. Thus, by ROB2 repeatedly applied, $\left(B^*, \left(C \cup (A \setminus C)^2_v\right) \setminus \left\{b^-_{v_2+1}, \ldots, b^-_u\right\}\right) \succ ((B \setminus D) \cup (B \cap D), D)$. Given that the first choice situation is indifferent to $(A \setminus C, C)$, by transitivity, $(A \setminus C, C) \succ (B, D)$. By Lemma 1 $(A, C) \succ (A \setminus C, C)$. Again by transitivity, $(A, C) \succ (B, D)$.

(1.2) Let $\left|(A \setminus C)^2\right| \leq \left|(B \setminus D)^2\right|$. Consider $\left\{b^-_{u_1+1}, \ldots, b^-_u\right\} \subseteq (B \setminus D)^2$. Then $\left(\left((A \setminus C)^1 \cup \left\{b^-_{u_1+1}, \ldots, b^-_u\right\}\right) \setminus (A \setminus C)^2, C\right) \sim (A \setminus C, C)$ by Theorem 2. By transitivity, $\left(\left((A \setminus C)^1 \cup \left\{b^-_{u_1+1}, \ldots, b^-_u\right\}\right) \setminus (A \setminus C)^2, C\right) \succ (B \setminus D, D)$. By ROB2 $\left(\left((A \setminus C)^1 \cup \left\{b^-_{u_1+1}, \ldots, b^-_u\right\}\right) \setminus (A \setminus C)^2, C\right) \succ (B, D)$. Given that $\left(\left((A \setminus C)^1 \cup \left\{b^-_{u_1+1}, \ldots, b^-_u\right\}\right) \setminus (A \setminus C)^2, C\right) \sim (A \setminus C, C)$, by transitivity, $(A \setminus C, C) \succ (B \setminus D, D)$.

By Lemma 1 $(A, C) \succ (A \setminus C, C)$, and again by transitivity, $(A, C) \succ (B, D)$.

(2) Let $A \setminus C = \left\{a^-_1, \ldots, a^-_u\right\}$, $B \setminus D = \left\{b^-_1, \ldots, b^-_u\right\}$, $A \cap C = \left\{a^+_1, \ldots, a^+_r\right\}$, and $B \cap D = \left\{b^+_1, \ldots, b^+_s\right\}$, $r > s$. By Theorem 2, $\left((A \setminus C) \cup \left\{a^+_1, \ldots, a^+_s\right\}, C\right) \sim (B, D)$. By Lemma 1 we have that $(A, C) \succ \left((A \setminus C) \cup \left\{a^+_1, \ldots, a^+_s\right\}, C\right)$. Again by transitivity, $(A, C) \succ (B, D)$.

(3) By Theorem 2, $(A, C) \sim (B, D)$. ■

Theorem 5 *$\succsim \in \mathcal{P}$ satisfy EC, IND, SM, and DI if and only if $\succsim = \succsim^4$.*

We will prove first the following two lemmas.

Lemma 4 *Let $\succsim \in \mathcal{P}$ satisfy DI, and let $(A, C), (B, C) \in \mathcal{X} \times \mathcal{X}_\emptyset$ be such* *that $B = A \cup E$ with $|E \cap C| = |E \setminus C|$. Then $(A, C) \sim (B, C)$.*

Proof of Lemma 4. Take (A, C) and (B, C) as above. Then $A \cup (E \cap C) = B \setminus (E \setminus C)$. Hence, by reflexivity, $(A \cup (E \cap C), C) \sim (B \setminus (E \setminus C), C)$. Applying $|E \cap C| = |E \setminus C|$-times DI results in $(A, C) \sim (B, C)$. ■

Lemma 5 *Let $\succsim \in \mathcal{P}$ satisfy IND, SM, and DI. Then it also satisfies SA.*

Proof of Lemma 5. We have to prove that $(\{x\}, C) \succ (\{x, y\}, C)$ for any $x \in X$ and any $y \in X \setminus C$. If $C \neq \{x\}$, then, there exists $z \in C$, $z \neq x$. By reflexivity, $(\{z\}, C) \sim (\{z\}, C)$. Applying DI results in $(\emptyset, C) \sim (\{z, y\}, C)$, and by SM we have $(\{z, y\}, C) \succ (\{y\}, C)$. By transitivity, $(\emptyset, C) \succ (\{y\}, C)$, and by IND, $(\{x\}, C) \succ (\{x, y\}, C)$. If $C = \{x\}$, by SM, $(\{x, y\}, C) \succ (\{y\}, C)$. By IND, $(\{x\}, C) \succ (\emptyset, C)$. On the other hand, by reflexivity, $(\{x\}, C) \sim (\{x\}, C)$, and by DI, $(\emptyset, C) \sim (\{x, y\}, C)$. Thus, by transitivity, $(\{x\}, C) \succ (\{x, y\}, C)$. ■

Corollary 1 *Let $\succsim \in \mathcal{P}$ satisfy IND, SM, and DI. Then the statements in Lemma 1 and Lemma 3 hold.*

Proof of Theorem 5[2]. It can be easily checked that $\succsim^4$ satisfies the four axioms. Suppose now that $\succsim \in \mathcal{P}$ satisfies EC, IND, SM, and DI. We have to prove that

(1) $|A \cap C| - |A \setminus C| > |B \cap D| - |B \setminus D|$ implies $(A, C) \succ (B, D)$, and

(2) $|A \cap C| - |A \setminus C| = |B \cap D| - |B \setminus D|$ implies $(A, C) \sim (B, D)$.

Let $|A \cap C| = r$, $|B \cap D| = s$, $|A \setminus C| = u$, $|B \setminus D| = v$.

(1) Let $r - u > s - v$. We consider the following three possible cases:

(1.1) $r > u$ and $s > v$,

(1.2) $r > u$ and $s \leq v$,

[2] In what follows in this proof, for all $K \subseteq X$ and all $k \in \{1, \ldots, |K|\}$, we denote by $(K)_k$ any subset of K with k elements.

(1.3) $r \leq u$ and $s < v$.

(1.1) Let $r > u$ and $s > v$. By EC, $(\emptyset, C) \sim (\emptyset, D)$. By Lemma 4, $((A \cap C)_u \cup (A \setminus C), C) \sim (\emptyset, C)$. Also by Lemma 4, $((B \cap D)_v \cup (B \setminus D), D) \sim (\emptyset, D)$. Thus, by transitivity, $((A \cap C)_u \cup (A \setminus C), C) \sim ((B \cap D)_v \cup (B \setminus D), D)$. Given that $r-u > s-v$, by IND $(s-v)$-times, $\big((A \cap C)_{u+s-v} \cup (A \setminus C), C\big) \sim \big((B \cap D)_{v+s-v} \cup (B \setminus D), D\big)$, that is $\big((A \cap C)_{u+s-v} \cup (A \setminus C), C\big) \sim (B, D)$. By Lemma 1, $(A, C) \succ \big((A \cap C)_{u+s-v} \cup (A \setminus C), C\big)$, and by transitivity, $(A, C) \succ (B, D)$.

(1.2) Let $r > u$ and $s \leq v$. Like in case (1.1), by EC, Lemma 4 and transitivity we get $((A \cap C)_u \cup (A \setminus C), C) \sim ((B \cap D) \cup (B \setminus D)_s, D)$. By Lemma 1, $(A, C) \succ \big((A \cap C)_{u+s-v} \cup (A \setminus C), C\big)$, and by Lemma 3,

$$((B \cap D) \cup (B \setminus D)_s, D) \succ (B, D).$$

Thus, by transitivity, $(A, C) \succ (B, D)$.

(1.3) Let $r \leq u$ and $s < v$. As before, by EC, Lemma 4 and transitivity we get $((A \cap C) \cup (A \setminus C)_r, C) \sim ((B \cap D) \cup (B \setminus D)_s, D)$. Since $r - u > s - v$, then $u - r < v - s$. Then we can apply IND $(u - r)$-times obtaining $\big((A \cap C) \cup (A \setminus C)_{r+u-r}, C\big) \sim \big((B \cap D) \cup (B \setminus D)_{s+u-r}, D\big)$. That is, $(A, C) \sim \big((B \cap D) \cup (B \setminus D)_{s+u-r}, D\big)$. By Lemma 3,

$$\big((B \cap D) \cup (B \setminus D)_{s+u-r}, D\big) \succ (B, D).$$

Then, by transitivity, $(A, C) \succ (B, D)$.

(2) Let $r - u = s - v$. Assume, without loss of generality $r \geq s$ $(u \geq v)$.

If $r \geq u$ $(s \geq v)$, then, like in case (1), by EC, Lemma 4 and transitivity we get

$$((A \cap C)_u \cup (A \setminus C), C) \sim ((B \cap D)_v \cup (B \setminus D), D),$$

and by IND $(r - u)(= s - v)$-times, $(A, C) \sim (B, D)$.

If $r < u$ ($s < v$), then, by EC, Lemma 4 and transitivity we get

$$((A \cap C) \cup (A \setminus C)_r, C) \sim ((B \cap D) \cup (B \setminus D)_s, D),$$

and by IND $(u - r)(= v \geq s)$-times, $(A, C) \sim (B, D)$. ■

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