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Migration and human capital in an endogenous fertility model

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Abstract. How do high and low skilled migration affect fertility and human capital in migrants' origin countries? This question is analyzed within an overlapping generations model where parents choose the number of high and low skilled children they would like to have. Individuals migrate with a certain probability and remit to their parents. It is shown that a brain drain induces parents to have more high and less low educated children. Under certain conditions fertility may either rise or decline due to a brain drain. Low skilled emigration leads to reversed results, while the overall impact on human capital of either type of migration remains ambiguous. Subsequently, the model is calibrated on a developing economy. It is found that increased high skilled emigration reduces fertility and fosters human capital accumulation, while low skilled emigration induces higher population growth and a lower level of education.

Keywords: Migration, human capital, fertility.
JEL Classification: F22, J13, J24.

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1 Introduction

At the world level, the number of international migrants rose from 76 million in 1960 to 175 million in 2000, but considering population growth the world share of migrants remained quite stable (2.5% in 1960 to 2.9% in 2000). Nevertheless, by making countries increasingly interdependent, globalization, rising income inequality, enhanced transportation technology, deacreasing tansportation costs, and stronger demographic disparities between developed and developing countries play in favor of a reinforcement of international migration in the next decades. Moreover, the fact that developed countries are ever more attractive for workers from developing regions is documented by the share of international migrants in developed countries that rose from 4% in 1970 to 8% in 2000 (see UN 2003 and IOM 2005). This is even more true for skilled emigration which is expected to be increasingly important, since immigration policies in migrants' host countries tend to be more and more skilled-biased. Docquier and Marfouk (2006) report that between 1990 and 2000, the augmentation in the number of skilled immigrants in OECD countries was about 64%, while it was only about half as much for unskilled immigrants. Moreover, most of these additional migrants originated from developing countries. The exodus of skilled workers from developing countries is however feared to have severe consequences on already poor economies, since it deprives them from their most talented labor force.

While the early theoretical literature of the 60s pointed out that a brain drain has basically no impact on migrants' origin countries and should not be a cause for worry (Grubel and Scott, 1966), during the 70s economists, and foremost Bhagwati and Hamada (1974), stressed that skilled emigration induces a negative externality on sending countries and that "there *is* a loss to those left behind". In recent years, economists took a fresh look at the issue and highlighted a range of positive side-effects of skilled emigration. One major beneficial externality of a brain drain is that it induces greater incentives for individuals to educate because of a higher expected skill premium. Then, if the newly educated individuals outweigh the ones leaving the country, human capital at origin is enhanced compared to a situation without a brain drain (Mountford, 1997; Stark et al., 1997; Beine et al., 2001), which may act as a substitute for educational subsidies (Stark and Wang, 2002).[1] However these migration models take population as constant and do

[1] In an extensive survey, Docquier (2006) describes the different positive externalities linked to skilled

not analyse fertility decisions. In fact, an important literature shows that the decisions parents face in terms of fertility and of investment in the education of their children are central for a country's economic development, see for instance Becker and Barro (1988) as well as de la Croix and Doepke (2003). Since the quality-quantity trade-off in terms of children influences human capital formation, it is crucial for a country's economic growth and it seems straightforward to study the impact of emigration within an endogenous fertility model. To our knowledge, only the migration model of Chen (2006) features endogenous fertility, but restricted to the brain drain issue. He analyzes the difference between public and private funded education systems in a model where agents have an average human capital level and a stochastic probability to emigrate. Our study differs in terms of the aim and of the framework used.

This paper analyzes how high and low skilled emigration shape parents' fertility choices and thus human capital formation. Contrarily to most endogenous fertility models, individuals do not decide upon the total number of their children and their education level (or investment in their education), but directly about how many low and high skilled children they would like to have. This is also a major contrast to Chen (2006) and allows us to explicitely introduce skill heterogeneity among agents in our overlapping generations (OLG) model. Also the end of their childhood, individuals migrate with a certain probability and remit to their retired parents. This is another distinct feature from Chen, since remitting behavior may influence the expected return of raising and/or educating children and thus adults' fertility decisions. It is shown that a brain drain induces parents to have more high and less low educated children, but may either raise or reduce fertility (total number of their offspring). A *necessary* condition to experience a decline in fertility due to skilled emigration is that a parent's (relative) cost must be higher than her (relative) expected utility gain from raising a high educated child. In contrast, a *sufficient* condition to have a higher fertility due to a brain drain is that this condition is reversed i.e. the cost of raising a high educated child is smaller than the gain. Low skilled emigration leads to reversed results: less high and more low skilled children. Finally, the impact of migration on human capital is ambiguous.

To provide more concrete findings, the model is calibrated on the Phillipines, which is an economy open to migration and experiencing large inflows of remittances. It is

emigration.

found that increased high skilled emigration reduces fertility and fosters human capital accumulation, while low skilled emigration induces higher population growth and a lower level of education. More precisely, a permanent increase of 10% in emigration flows is simulated. When the additional emigrants are high skilled (low skilled), the share of high skilled in the work force changes from 22.2% to 28.4% (to 21.2%) and the annual population growth from 1.98% to 1.36% (to 2.1%).

The paper is organized as follows. Section 2 introduces the model and explains the theoretical effects of increased emigration. The illustration on the Phillipines economy is presented in section 3. Section 4 concludes.

2 The Economic Model

We consider an overlapping generation economy where individuals live for 3 periods: childhood, adulthood and old age. Each individual has one parent, which creates the connection between generations. Individuals have either a low (superscript l) or a high education level (superscript h). Higher education is costly, while lower education is offered for free by the society.[2]. During their childhood, individuals who attend school do not work, whether they obtain higher education or not. Also, agents work only in their adulthood and earn a wage that depends on their education level. High skilled adults earn a wage w^h, while low skilled ones a wage w^l with $w^h > w^l$.

We consider a small open economy where capital is perfectly mobile, which implies a fixed international interest rate R^*. Also, both high and low skilled wages are exogenous and constant. Both low and high skilled labor in this small open economy can emigrate to an advanced economy and earn a higher salary, w^{*i} $(i = h, l)$, which is exogenously given with $w^{*i} > w^i$. Finally, we assume that emigration is not large enough to affect the economy of the destination country.

[2]For instance, individuals with a college degree could be considered as high skilled and individuals without a college degree as low skilled. Then education after high school would be costly, while educatio below college level would be free.

2.1 Individual behavior

All decisions are made by the individual during her adulthood. Thus at time t, each adult with education level i cares about her own old age consumption D^i_{t+1} and about the expected income of her children, V^i_{t+1}. It is assumed that individuals consume only when old. Thus there is no arbitrage opportunity for consumption, which is purchased through savings and remittances. The individual also cares about the return from her "education investment", that is, the expected income of her children V^i_{t+1}, which represents the altruistic component in the utility. Moreover, an adult chooses how many low (n^i_t) and high skilled children (m^i_t) she would like to have.

At the beginning of their adulthood, individuals with education level i can emigrate with a probability p^i, $i = h, l$ to a more advanced economy. Hence the expected income of a child with education level $i = h, l$ is

$$\overline{w}^i = (1 - p^i)w^i + p^i w^{*i}, \ i = h, l. \tag{1}$$

Raising one child takes time fraction $\phi \in (0, 1)$ of an adult's time and high skilled children induce an additional cost for their education x. Therefore savings, S^i_{t+1}, result from an adult's labor earnings minus raising and educational costs of her children,

$$S^i_{t+1} = R^*[w^i(1 - \phi(n^i_t + m^i_t))] - xm^i_t, \tag{2}$$

where in the following we normalize the fixed constant interest rate R^* to 1.

It is assumed that all children care about their parents and remit a proportion of their (foreign) income to their parents. Therefore for a parent of education i expected transfers, Ω^i, from her high and low skilled children are given by

$$\Omega^i_{t+1} = T^i_{t+1} + Z^i_{t+1} = \theta^h \overline{w}^h m^i_t + \theta^l \overline{w}^l n^i_t, \tag{3}$$

which comprise not only money transmitted by adults staying in the home country to their parents, $T^i_t = (1 - p^l)\theta^l w^l n^i + (1 - p^h)\theta^h w^h m^i$, but also remittances, Z, defined as $Z^i_t = p^l \theta^l w^{*l} n^i + p^h \theta^h w^{*h} m^i$. Then $\theta^i (> 0)$ is the propensity to transfer money to her parents for an individual with education level i (or to remit for a migrant with education level i).

Lifetime consumption writes as follows

$$D_{t+1}^i = S_{t+1}^i + \Omega_{t+1}^i. \tag{4}$$

The utility function of an individual who is an adult with education level i at time t is then given by:

$$U(D_{t+1}^i, V_{t+1}^i) = \ln(D_{t+1}^i) + \ln(V_{t+1}^i), \tag{5}$$

and

$$V_{t+1}^i = \alpha(n^i)^\epsilon \overline{w}^l + (1-\alpha)(m^i)^\epsilon \overline{w}^h.$$

A part from the fact that we explicity introduce heterogeneity among the types of children, the non-linear term in V_{t+1}^i is similar to Becker and Barro (1988); Barro and Becker (1989); Doepke (2005), with $\alpha \in (0,1)$ measuring the weight given to low skilled children and $\epsilon \in (0,1)$ playing the role of the elasticity of the utility to any type of children. As mentioned by Barro and Becker (1989), this form of the altruism term means that, for a given expected income per child $\overline{w}^i$, "parental utility $U(\cdot)$ increases, but at a diminishing rate, with the number of children" (here n^i and m^i).

Thus, combining the above informations, each adult is facing the following problem

$$\max_{n^i,m^i} U^i = \max_{n^i,m^i}\{\ln(D_{t+1}^i) + \ln(V_{t+1}^i)\}, \qquad i = l, h, \tag{6}$$

subject to (4) and which consists into the maximization of her lifetime utility by choosing the number of low (n^i) and high skilled children (m^i).

2.2 Solving the individual problem

In appendix, we show that the first order condition of U^i with respect to n_t^i is

$$\frac{\phi w_t^i - \theta^l \overline{w}_{t+1}^l}{D_{t+1}^i} = \frac{\alpha \overline{w}_{t+1}^l \epsilon (n_t^i)^{\epsilon-1}}{V_{t+1}^i}, \tag{7}$$

which states that the net marginal cost of raising a low skilled child, $\phi w_t^i - \theta^l \overline{w}_{t+1}^l$ (cost minus expected transfers), in terms of consumption, should equal the marginal utility gain from a low skilled child's expected income, in terms of the future value of total children (V). If this equality does not hold, raising children is either too costly (it is then optimal to have no children), or not costly enough (then individuals choose to have more and more children).

Similarly, the first order condition of U^i with respect to m_t^i shows that

$$\frac{\phi w_t^i + x - \theta^h \overline{w}_{t+1}^h}{D_{t+1}^i} = \frac{(1-\alpha)\overline{w}_{t+1}^h \epsilon (m_t^i)^{\epsilon-1}}{V_{t+1}^i}, \tag{8}$$

which reads that the net marginal cost of educating one child in terms of consumption (left hand side) should be equal to the marginal benefit from educating a child.

The second order conditions of the agents' maximization problem are satisfied. Therefore the solutions from (7) and (8) are optimal for the household problem.

It is easy to see that in (7) and (8), both the right hand sides are positive, implying that the left hand sides are positive also. These are necessary conditions for the existence of interior solutions and it is assumed that, in what follows, these conditions always hold.

Assumption 1. The following conditions are supposed to always hold (for $i = l, h$ and $\forall t$),

$$\phi w_t^i > \theta^l \overline{w}_{t+1}^l,$$

$$\phi w_t^i + x > \theta^h \overline{w}_{t+1}^h.$$

Assumption 1 guarantees that raising children is expensive, otherwise parents will have as many children as they can; at the same time, educating children is also costly, otherwise all children will get higher education.

Combining these two equations (see appendix), we obtain explicit solutions for m and n, which are put forward in the following proposition.

Proposition 2. *Under Assumption 1 we have*

$$m_t^i = \frac{\epsilon(1-\alpha)\overline{w}_{t+1}^h w_t^i}{(1+\epsilon)\left[\phi w_t^i + x - \theta^h \overline{w}_{t+1}^h\right]\left[\alpha \overline{w}_{t+1}^l \sigma_{n,m}^i + (1-\alpha)\overline{w}_{t+1}^h\right]} \tag{9}$$

and

$$n_t^i = (\sigma_{n,m}^i)^{\frac{1}{\epsilon}} m_t^i, \tag{10}$$

where

$$\sigma_{n,m}^i = \left(\frac{B_t}{A_t^i}\right)^{\frac{\epsilon}{1-\epsilon}}, \quad \text{with} \;\; A_t^i = \frac{\phi w_t^i - \theta^l \overline{w}_{t+1}^l}{\phi w_t^i + x - \theta^h \overline{w}_{t+1}^h}, \;\; B_t = \frac{\alpha \overline{w}_{t+1}^l}{(1-\alpha)\overline{w}_{t+1}^h}. \tag{11}$$

In fact A_t^i represents the ratio of net costs of raising a low to a high educated child (see (7) and (8)), while B_t is the ratio of the contribution of a low educated child to a high educated child in parental utility. Also, if ϵ is the elasticity of the utility to any type of children, then $\sigma_{n,m}^i$ can be considered as the elasticity of substitution between high and low educated children in each household.

Given the explicit expression of m^i and n^i, we can study the change in these two choice variables with respect to a change in p^h. In the appendix we prove the following proposition.

Proposition 3. *Under assumption 1 the number of high educated children is an increasing function of the skilled migration probability p^h, while number of low educated children is a decreasing function of p^h. Mathematically, we have*

$$\frac{\partial m_t^i}{\partial p^h} > 0, \;\; \frac{\partial n_t^i}{\partial p^h} < 0, \;\; \forall t, \; i = l, h.$$

The intuition of this proposition is very clear: a brain drain would lead to a trade-off between high and low skilled children which is in favor of an increase in the number of the former. However, the impact of a rise in p^h on the total number of children, $n_t^i + m_t^i$, is not so clear. Nevertheless, we have the following results by combining equations (28), (29), and (30) in appendix.

Proposition 4. *Assume Assumption 1 holds.*

(i) The effect of p^h on fertility, $n_t^i + m_t^i$, is ambiguous.

(ii) One necessary condition for a decline in fertility, $\partial(n_t^i + m_t^i)/\partial p^h < 0$, is

$$\frac{m_t^i\left(\phi w^i + x - \theta^h \overline{w}^h\right)}{(m_t^i + n_t^i)\left(\phi w^i - \theta^l \overline{w}^l\right)} > \frac{\epsilon(1-\alpha)\overline{w}^h}{\alpha \overline{w}^l \sigma_{n,m}^i + (1-\alpha)\overline{w}^h}, \tag{12}$$

where the right hand side is increasing in $\overline{w}^h$.

(iii) Furthermore, the other direction of the above inequality offers a sufficient condition to have an increase in the total number of children following a rise in p^h.

The above proposition can be commented as follows. It is almost that a rise in the skilled probability to emigrate p^h leads to an ambiguous effect on the total number of children since the number of low educated children decreases and the one of high educated children increases (point (i) in proposition 4).

However the necessary condition (12) under point (ii) of proposition 4 delivers some insights on when a brain drain leads to a decline in fertility. First, notice that the right hand side of the necessary condition is the ratio of a parent's utility value from a high skilled child's expected income, $(1-\alpha)\overline{w}^h$, to a parent's utility from an "average" child, $\alpha\overline{w}^l\sigma_{n,m}^i + (1-\alpha)\overline{w}^h$, by taking into account the two elasticities, ϵ and $\sigma_{n,m}^i$. Secondly, the left hand side of the necessary condition stands for the ratio of net education costs of all high educated children to net raising costs of low skilled children applied to all children (recall (8) and (7)). Hence, the necessary condition can be understood in a quite intuitive way: a brain drain leads parents to have less children if the relative cost of raising a high skilled child is higher than its relative marginal gain. More precisely, if the ratio of educational to raising costs (LHS) is higher than the ratio of the marginal utility gain from a high skilled child to the one from an "average" child (RHS). Two factors strenghten the inequality in (12). Having a large share of high educated to total children $\frac{m^i}{n^i+m^i}$, which hinges on parents' choices, and/or facing elevated exogenous education fees x contribute in having too high relative raising costs of high skilled children.

The interpretation of the sufficient condition in point (iii) is now straightforward. If the relative raising costs of high skilled children are too low, then a brain drain induces that the number of additional high educated children dominates the reduction in the number of low educated children. Thus fertility increases after a rise in p^h.

Finally, due to the symmetry of the effects of p^l and p^h on n^i and m^i, the same calculations lead us to the following corollary

Corollary 1. *Assume Assumption 1 holds. Then m_t^i is decreasing and n_t^i is increasing in p^l, while the effect on the total number of children is ambiguous.*

2.3 The impact of a brain drain on human capital

From the previous section, we know that parents choose to have more high and less low skilled children, which acts positively on the formation of human capital. However, a brain drain means also that more high skilled people leave. Thus which effect dominates? Human capital at time t, denoted by H_t, can be defined as the share of high educated labor in the total active labor force. That is

$$H_t = \frac{N_t^h}{N_t^h + N_t^l}, \tag{13}$$

where N_t^h and N_t^l are respectively the high and low skilled active labor forces at time t, defined as

$$N_t^l = (1 - p^l)(N_{t-1}^l n_{t-1}^l + N_{t-1}^h n_{t-1}^h), \tag{14}$$

$$N_t^h = (1 - p^h)(N_{t-1}^l m_{t-1}^l + N_{t-1}^h m_{t-1}^h). \tag{15}$$

Thus $N_t^l(N_t^h)$ are the low (high) skilled individuals born at time $t-1$ from both low skilled family and high skilled family and staying in their home country.

Therefore in order to study the effect of a change of p^h on human capital, that is $\frac{\partial H_t}{\partial p^h}$, we only need to study the effect of p^h on $\frac{1}{H_t} = 1 + \frac{N_t^l}{N_t^h}$.

Case I. If p^h varies at time t and there is no perfect foresight, then all N_{t-1}^l, n_{t-1}^l, N_{t-1}^h, n_{t-1}^h, m_{t-1}^l and m_{t-1}^h are independent of a change in p^h. Therefore, it follows that

$$\frac{\partial}{\partial p^h}\left(\frac{N_t^l}{N_t^h}\right) = \left(\frac{N_t^l}{N_t^h}\right)\frac{1}{(1-p^h)} > 0,$$

that is,

$$\frac{\partial}{\partial p^h}\left(\frac{1}{H_t}\right) > 0.$$

As a result, we have

$$\frac{\partial H_t}{\partial p^h} < 0, \tag{16}$$

which means that if there is no information about a policy change concerning in p^h, then parents are not prepared for it and will not send more children to obtain higher education following a brain drain. The result is that more high skilled workers emigrate without inducing any additional formation of human capital.

Case II. There is perfect foresight and parents are prepared for the change in p^h that happens in the next period. Imagine that at time $t+1$, p^h increases. Direct calculation shows

$$\frac{\partial}{\partial p^h}\left(\frac{N^l_{t+1}}{N^h_{t+1}}\right) = G(n,m)\frac{(1-p^l)}{(1-p^h)^2} + \frac{(1-p^l)}{(1-p^h)}\frac{\partial G(n,m)}{\partial p^h}, \tag{17}$$

where

$$G(n,m) = \frac{N^l_t n^l_t + N^h_t n^h_t}{N^l_t m^l_t + N^h_t m^h_t}.$$

We know that both m^l and m^h (n^l and n^h) are increasing (decreasing) in terms of p^h. Thus a higher p^h will lead to a rise in the denominator and to a reduction in the numerator, while N^l_t and N^h_t are decided at time $t-1$ and will thus not be affected by a change in p^h happening at time $t+1$. Hence we obtain

$$\frac{\partial G(n,m)}{\partial p^h} < 0.$$

To conclude, the first term on the right hand side of (17) is positive and represents the ex post loss of human capital due to a brain drain, while the second term on the RHS stands for the ex ante stimulation of human capital due to a brain drain. Since these two effects also depend on the population size N^l_t and N^h_t, it is open to question whether at the end a brain drain results in a brain loss or in a brain gain within our endogenous fertility model. A calibration of our model on a situation of a typical developing country open to labor mobility may give us a specific answer.

3 Numerical Analysis

In this section we provide a numerical illustration to analyze the effects of increased emigration on fertility and human capital. Higher migration can be due to the fact that destination countries adopt more liberal immigration policies. Since immigration policies tend to be more and more skilled-biased, we first focus on the effects of higher *high* skilled emigration. Consecutively, we compare the findings with a situation of increased *low* skilled migration.

3.1 Calibration

Our model is calibrated to depict a typical situation of South-North migration and as such the parameter of our model are adjusted to match the economy of the Philippines (to be the migrants' origin country). This choice seems appropriate since international migration and large flows of remittances are notorious characteristics of the Philippine economy for several decades now (see the IMF study of Burgess and Haksar, 2005). The foreign country of the model, is represented by a combination of OECD countries, where the importance of each of them is weighted by the share of Filipino emigrants they host (see below). The initial steady state is assumed to correspond to 2000 data. The values of parameters and exogenous variables are reported in table 1 and chosen as follows.

Table 1: Parameter values for the Philippines

$\phi = 0.15$	$\epsilon = 0.5$	$w^l = 1$	$w^h = 5.022$	$p^h = 0.086$	$p^l = 0.043$
$\alpha = 0.62$	$\theta^l = 0.1$	$w^{*l} = 1.96$	$w^{*h} = 29.29$	$x_t^l = 0.92$	$R^* = 1.806$

According to Haveman and Wolfe (1995) parents spend around 15% of their time raising children, which enables us to set the raising cost parameter ϕ to 0.15. Also, following Rosenzweig (2006) the wage of a high skilled worker in the Philippines is 5.022 times larger than the one of a low skilled. Thus if w^l is set to 1, w^h equals 5.022. Since one period is considered to be 20 years, the interest factor is set to $R^* = 1.806$ which corresponds to an annual interest rate of 3%.

A next step is to choose the probabilities to emigrate, p^h and p^l, which are not directly observable. However, Docquier and Marfouk (2006) document that 67% of the Filipinos living in OECD in 2000 are skilled, thus we can set $p^h = 2\ p^l$. Also, since one period lasts 20 years, it can be considered that the number of migrants in the OECD in 2000 reported by these authors represents the number of emigrants during one period in our model, meaning that 1'678'735 Filipinos go abroad. [3]. If the number of migrants can be written as $p^l N^l + p^h N^h$ then taking N^l and N^h from Docquier and Marfouk, we have that $p^l = 0.043094295$ and $p^h = 0.08618859$.

For the remaining exogenous variables no data are available. To start with, the parameter ϵ in the "altruistic" argument of the utility function is set to 0.5, but will be subject to several robustness checks in a later section. Remaining variables are set in order to match four main characteristics of the Philippine economy. Let us now describe this procedure. First, we know from Docquier and Marfouk (2006), which themselves rely on the data of Barro and Lee (2001), that in 2000 the ratio of the low-to-high skilled labor force, $1/h$ $(= N^l/N^h)$, amounts to 3.5045. This value is met by fixing the education costs of a child to $x_t^l = 0.917045$ and by the plausible assumption that $x^h = x^l$. Second, if we consider one period to be 20 years, then population growth in our model equals $g = 1.481$, implying that $\alpha = 0.621093$. Moreover, we can consider the wage differential between the Phillipines and the OECD to be similar to the per capita GDP differential. According to the World Development Indicators WDI (2003), average per capita GDP between 1999-2004 was \$3'991 in the Philippines and \$34'268 in the OECD (PPP, constant 2000 international \$), thus 7.98 times higher in the OECD.[4] If average domestic wage is defined as $\widehat{w} = (w^h + 1/hw^l)/(1 + 1/h)$ and average foreign wage $\widehat{w}^* = (w^{*h} + 1/h^* w^{*l})/(1 + 1/h^*)$, then the average wage difference $\omega = \widehat{w}^*/\widehat{w}$ equals

[3]This number is not exaggerated, because when considering also temporary residents (42%) and irregular migrants (21%) together with permanent residents (37%), the number of Filipinos living and working overseas was estimated to be around 7.58 million in 2002 with an increase of 1 million since 1996. This number is equivalent to almost one quarter of the domestic labor force (Burgess and Haksar, 2005; Castro, 2006)

[4]According to Docquier and Marfouk, migrants from the Phillipines living in the OECD in 2000 were distributed as follows: United States (69.31%), Canada (11.41%), Australia (4.65%), Japan (4.56%), Italy (2.44%), United Kingdom (2.07%), Germany (0.75%), Korea (0.72%), Spain (0.67%), New Zealand (0.51%), Austria (0.45%), Switzerland (0.43%), Netherlands (0.34%), Greece (0.29%), France (0.28%), Norway (0.25%), Sweden (0.23%), Ireland (0.21%), Denmark (0.15%), Belgium (0.13%), Iceland (0.04%), Mexico (0.04%), Finland (0.037%), Czech Republic (0.0014%), Hungary (0.001%), Slovakia (0.0001%).

7.98. Relying on the same sources as for the domestic economy and applying the same weights for the distribution of migrants among OECD countries as for GDP per capita, the average ratio of low-to-high skilled labor force in the OECD, $1/h^*$, was 1.096703272 and the skill premium, w^{*h}/w^{*l}, 13.78465156. Then to match the average wage difference, w^{*h} is required to be 29.2902, while $w^{*l} = 13.78\ w^{*h}$. Finally, we need to set the propensities to remit θ^l and θ^h. While skilled migrants remit a larger amount than low educated migrants, recent research claims taht their propensity to remit is lower than the one of low skilled migrants, see Faini (2007) and Nimii et al. (2008). In our central scenario it is assumed that the propensity to remit of the skilled is 50% as much as the low skilled one and thus $\theta^h = 0.5\ \theta^l$. This assumption will be subject to robustness checks. Based on Fund staff estimates and on the World Bank, indicate that remittances in percentage of GDP amount to 9.4%. If we define GDP, Y, by the sum of incomes from labor and savings, then $Y_t = N_t^h w_t^h + N^l w_t^l + (R^* - 1)(N_{t-1}^h s_{t-1}^h + N_{t-1}^l s_{t-1}^l)$ and the total amount of remittances in one period, Λ, by $\Lambda_t = N_{t-1}^h Z_t^h + N_{t-1}^l Z_t^l$. Then $\Lambda_t/Y_t = 0.094$ implies that $\theta^l = 0.103657$.[5]

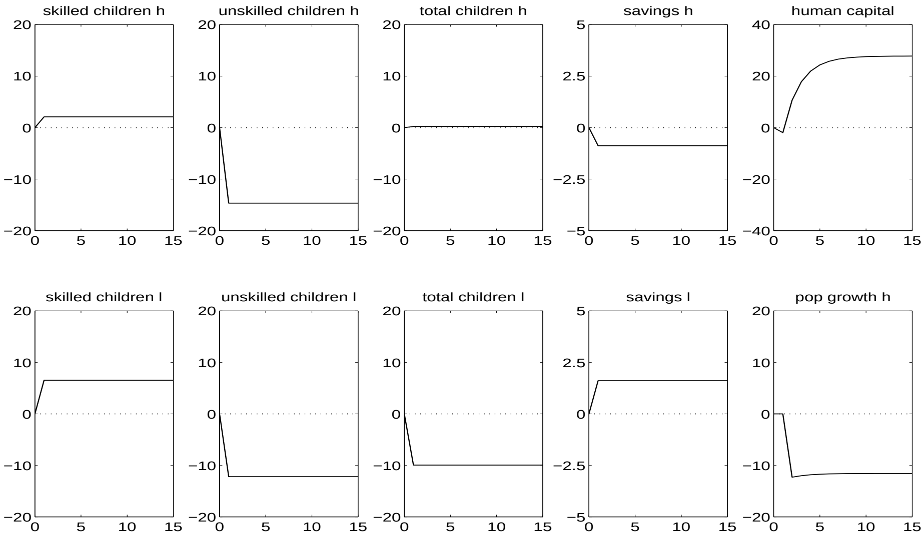
3.2 Results

We analyze the effects of a permanent increase of 10% in emigration flows, which means that an additional 164 thousand migrants are leaving the Phillipines at each period with respect to the baseline. Two scenarios are compared. Under the *high* skilled emigration scenario, additional migrants are all skilled and thus p^h rises from 0.086 to 0.109. Conversely, under the *low* skilled emigration, additional migrants are low skilled and p^l changes from 0.043 to 0.05.

Figure 1 shows how the choices of the households are influenced by the adoption of increased high skilled emigration. As expected from our theoretical results, households choose to finance higher education to a larger number of children and to raise less low skilled children (columns 1 and 2). While theoretically the effect of p^h on total children was ambiguous, we can see now from column 3, that low skilled parents would prefer to

[5] According to aggregate data on remittances from the International Monetary Fund (IMF 2007) remittances amount to $7876 million in 2003. Moreover a more recent report of the WorldBank (2006) indicates that the remittances share of GDP in the Philippines would even amount to 13.5% (see World Bank, 2006, p.90, Figure 4.1).

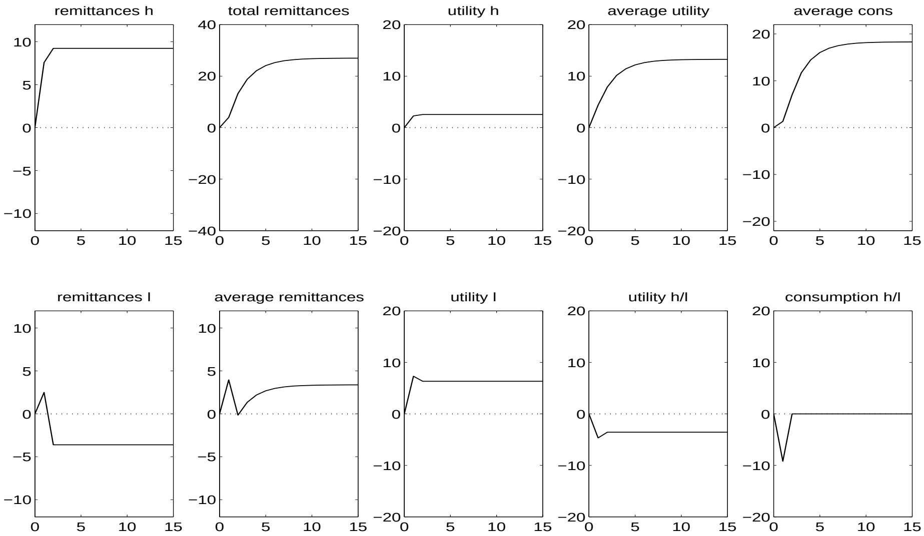
Figure 1: Impact of increased *high* skilled emigration on households' decisions



Values display percentage changes with respect to the baseline.
"l" refers to low and "h" to high skilled individuals.

have less children, while high skilled raise slighty more children. Thus an increase in the probability to emigrate reduces fertility. What about human capital? The effect of a brain drain on human capital H is (slightly) negative in the short run (when the policy is adopted). However the additional children having obtained higher education thanks to the new policy, will add to the high skilled labor force and more than compensate for the departing high educated workers. Moreover, we can see that the growth rate of the high skilled population initially declines, because of the departure of skilled workers in the first period. Shorthly after, it augments since both types of parents opt for more skilled children. This short term rise happens only for the growth rate of the high skilled population (the one of the low skilled is not shown). In the long run, the growth rate of the high and low skilled populations are the same and stabilize at a lower level compared to the baseline. A doubling of the migration flows in which addtional emigrants are all highly educated leads, in the long run, to a 27.80% rise in human capital (i.e. H rises from 22.2% to 28.4%) and to a 8.47% decrease in population growth rate (which means that the annual population growth rate declines from 1.98% to 1.36%).

Figure 2: Impact of increased *high* skilled emigration on welfare



Values display percentage changes with respect to the baseline.
"l" refers to low and "h" to high skilled individuals.

Figure 2 points at the impact on other economic indicators,for instance, at remittances per high (Z^h) and per low skilled parent/receiver (Z^l), total remittances (Λ) and average remittances per receiver ($\overline{Z}$) defined as

$$\Lambda_t = N^h_{t-1} Z^h_t + N^l_{t-1} Z^l_t,$$

$$\overline{\Omega}_t = \frac{\Lambda_t}{(N^h_{t-1} + N^l_{t-1})}.$$

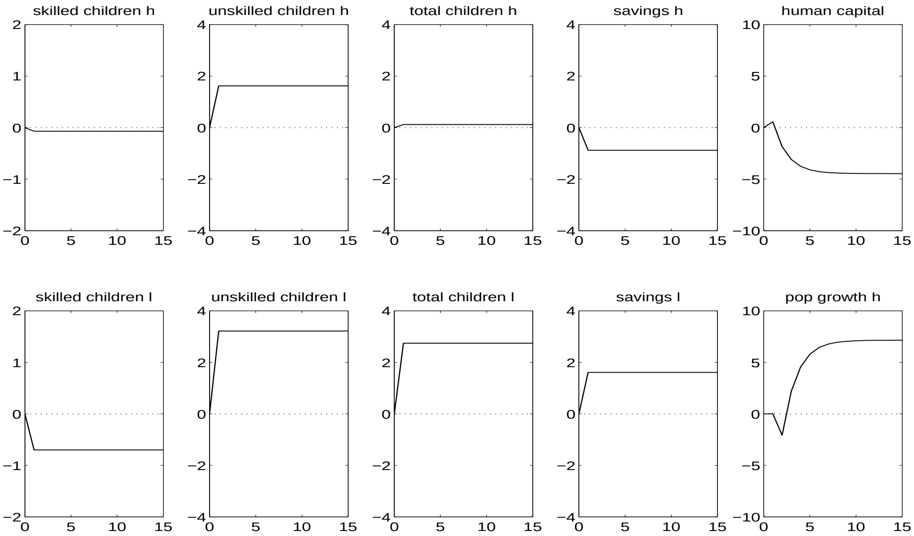
We also look at the impact on average utility ($\overline{U}$) and average utility from consumption ($\overline{\Psi}$):

$$\overline{U}_t = \frac{N^h_{t-1} U^h_t + N^l_{t-1} U^l_t}{N^h_{t-1} + N^l_{t-1}},$$

$$\overline{\Psi}_t = \frac{N^h_{t-1} ln(D^h_t) + N^l_{t-1} ln(D^l_t)}{N^h_{t-1} + N^l_{t-1}}.$$

Moreover, the ratio of the high-to-low skilled utilities (Ξ) or of high-to-low skilled utilities

Figure 3: Impact of increased *low* skilled emigration on households' decisions



Values display percentage changes with respect to the baseline.
"l" refers to low and "h" to high skilled individuals.

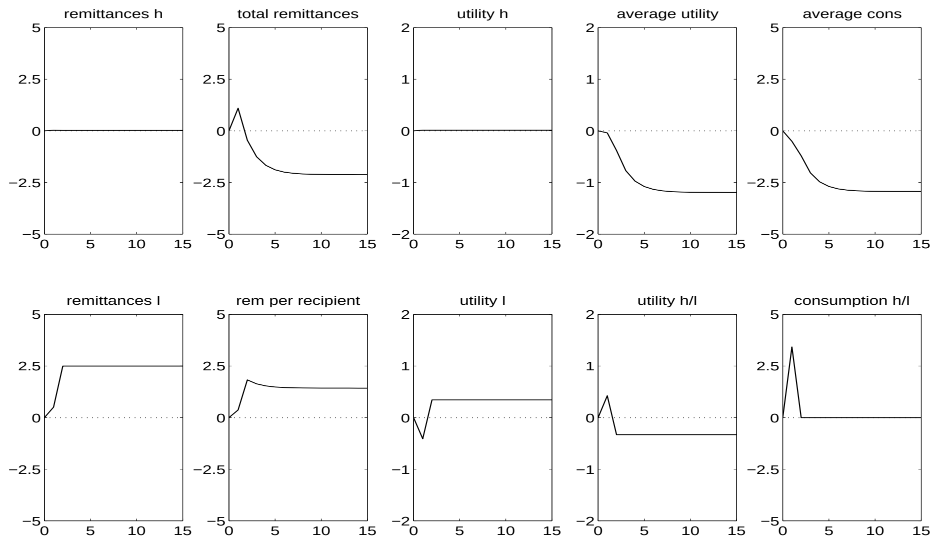
from consumption (Π) can be considered as indicators of inter-household inequality:

$$\Xi_t = \frac{U_t^h}{U_t^l},$$

$$\Pi_t = \frac{ln(D_t^h)}{ln(D_t^l)}.$$

While total remittances and average remittances received rise in the long run (column 2), average remittances received by each skill group behave differently (column 1). Obviously in the first period average remittances for both skill groups rise when more individuals leave the country. However, in the long run remittances for low skilled individuals are decreased, because the remittances received by their additional high skilled children do not compensate for the remittances foregone by raising less low skilled children. In contrast, skilled parents benefit from higher per capita remittances. In column 3, skilled emigration has only a slight impact on average utility of high skilled individuals but raises considerably the one of low skilled ones. Then average per capita utility will rise and the welfare of low compared to high skilled individuals will improve (column 4).

Figure 4: Impact of increased *low* skilled emigration on welfare



Values display percentage changes with respect to the baseline.
"l" refers to low and "h" to high skilled individuals.

These latter results are explained by the "altruistic" component of the utility. In fact, if we consider welfare to be measured only by the consumption part of the utility, $ln(D^i)$, then average utility per skill group will have only a temporary impact. The ratio of high-to-low skilled utilities from consumption, Π_t, will decline in the first period (bottom graph in column 5) because utility from consumption of a high skilled individual, $ln(D^h)$, decreases more than the utility from consumption of low skilled individuals. Finally, average utility from consumption, $\overline{\Psi}_t$, rises also in the long run because more and more people become high skilled and enjoy a higher utility.

Figure 3 depicts the effects on households' fertility decisions when additional migrants are low skilled. From the theoretical analysis, we know that the choices on the number of high and low skilled children are upturned compared to a brain drain. Such a policy will also lead to an increase in fertility (column 3). Moreover, the impact on fertility and human capital is not only reversed, but also of much smaller magnitude than under high skilled emigration. Increased unskilled emigration induces, in the long run, a drop of

4.46% in human capital (H goes from 22.2% to 21.2%) and a rise of 7.15% in population growth (the annual growth rate changes from 1.98 to 2.1%). Figure 4 shows the effect of such a policy on welfare indicators. Similarly to a brain drain, higher unskilled emigration leads to more remittances. But the rise is less strong in the long run than with a brain drain. The reason is that because of the low-skilled biased emigration policy and because parents choose to finance higher education to less children, there are less high skilled emigrants, who remit higher amounts. It can also be observed that in contrast to the brain drain scenario, low skilled parents benefit on average from higher remittances. In both scenarios, the utility of low skilled individuals rises in absolute terms (bottom graph in column 3) and relatively to skilled individuals (bottom graph in column 4).

3.3 Robustness

Are the above findings consistent with migrants' remittances behavior and with the choice of ϵ? Figure 5 reports the impact of high skilled emigration on human capital formation and population growth when low skilled migrants have a higher propensity to remit (i.e. the central scenario when $\theta^h = 0.5\theta^l$), when both types of individuals have equal propensities to remit ($\theta^h = \theta^l$) and when no remittances are sent back ($\Lambda = 0$).[6] The effects on human capital and population growth are robust under these different scenarios. When high skilled remit in the same propensity as low skilled, more remittances are sent back (see table 2) and thus the incentives to send more children to get education are higher. It results that human capital is more improved than in the benchmark. However, in the absence of remittances, human capital is nevertheless enhanced (even though less than in the other two scenarios), because parents are altruistic and prefer having more high skilled children because these ones enjoy a higher expected wage. In terms of population growth, the scenario in which both high and low skilled remit in the same way has a less reducing impact than the benchmark. The reason is that since high skilled migrants remit more, the number of skilled children is further stimulated and the decrease in population growth is dampened (see table 2).When there is no perspective of remittances, low skilled children are relatively more ineresting in the "no remittances" scenario than in the other

[6]For each alternative baseline, the different exogenous variables are recalibrated to meet the characteristics of the Philippine economy.

two scenarios. Then the decline in the number of low skilled children is less important and the effect on population growth reduced.

Figure 5: Impact of *skilled* emigration under alternative behaviors to remit

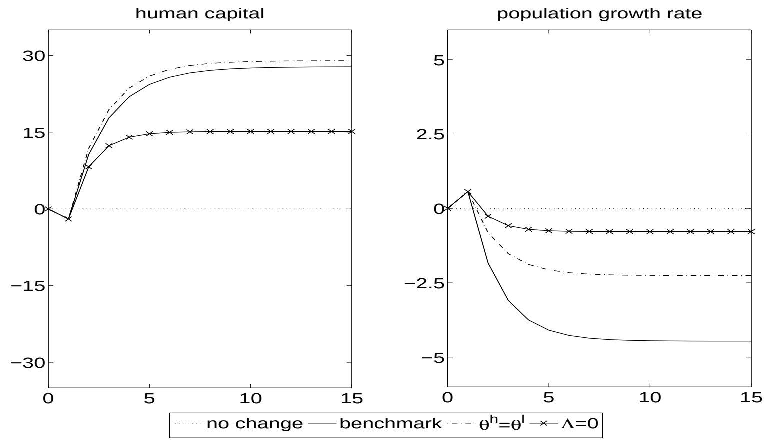


Figure 6: Impact of *unskilled* emigration under alternative behaviors to remit

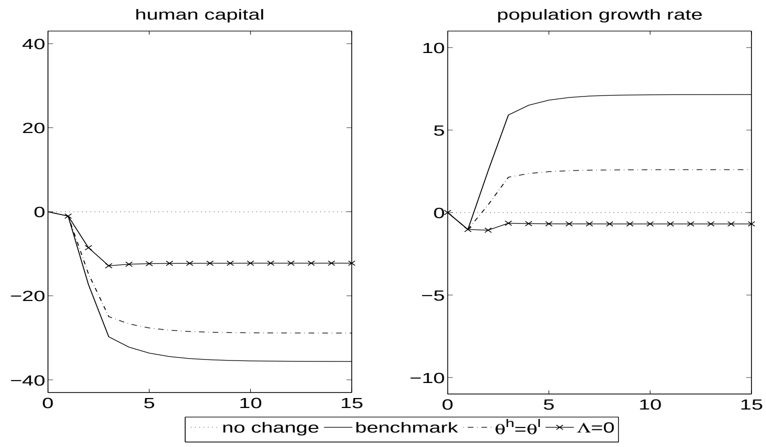


Figure 6 shows the effect of increased low skilled emigration. The scenario with the same remitting behavior for high and low skilled leads to an inferior reduction in human capital than the benchmark. Under the latter scenario low skilled remit more than when

$\theta^h = \theta^l$, and thus parents react stronger to a rise in p^l. This also explains the higher increase in population growth. The absence of remittances leads to a slight reduction in population growth, because low skilled parents do not react by a strong increase in the number of low skilled children, since these ones do not repay them with any remittances. Finally, figures 7 and 8 in appendix show that the results of larger emigration on human capital formation and population growth are robust to a choice of ϵ (for ϵ equal to 0.25 and 0.75).

4 Conclusion

An endogenous fertility model with overlapping generations is introduced, where parents choose the number of low and high educated children they would like to raise. We analyze the impact of high and low skilled emigration on parents' fertility choices and on human capital. It is shown that a brain drain induces parents to support higher education of a larger number of their children and to raise less low skilled ones. Furthermore, a necessary condition to see a decline in the total number of children is that the relative cost of financing children's higher education is larger than its expected gain. Low skilled emigration leads to contrary results. The impact of either type of emigration on human capital is ambiguous.

Finally, the model is calibrated on the Phillipines to provide some quantitative results. We simulate an increase of 10% in emigration flows. When these additional migrants are high skilled, human capital is enhanced in the long run (increase of 27.8% in the share of high skilled individuals) and population growth experiences a slow down (from 1.98% to 1.36% annual growth). Alternatively, when the "new" emigrants are low skilled, the impact is reversed and of a lower magnitude: the level of human capital is exacerbated (drop of 4.46%) and population growth stimulated (from 1.98% to 2.1%).

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A Appendix

A.1 Appendix A: Proof of Proposition 2

The explicit solutions for n^i and m^i are obtained in two steps. We first compute the linear relationship between n^i and m^i, and find the explicit solution m^i.

Step 1. The relationship between n^i and m^i

By substituting teh equation of $\overline{w}^i$ into the utility function and the ones of S^i_{t+1} and Ω^i_{t+1} into D^i_{t+1}, we are facing the following optimization problem

$$\max_{n^i,m^i} U^i_t = \max_{n^i,m^i} \left[\ln(D^i_{t+1}) + \ln(V^i_{t+1})\right],$$

with

$$D^i_{t+1} = \left[w^i_t\left(1-\phi(n^i_t+m^i_t)\right) - xm^i_t\right] + \left[\overline{w}^h_{t+1}\theta^h m^i_t + \overline{w}^l_{t+1}\theta^l n^i_t\right], \tag{18}$$

and

$$V^i_{t+1} = \overline{w}^l_{t+1}\alpha(n^i_t)^\epsilon + \overline{w}^h_{t+1}(1-\alpha)(m^i_t)^\epsilon, \tag{19}$$

First order condition of U^i with respect to n^i_t reads

$$\frac{-\phi w^i_t + \theta^l \overline{w}^l_{t+1}}{D^i_{t+1}} + \frac{\alpha \overline{w}^l_{t+1}\epsilon(n^i_t)^{\epsilon-1}}{V^i_{t+1}} = 0,$$

which is equivalent to

$$\frac{\phi w^i_t - \theta^l \overline{w}^l_{t+1}}{D^i_{t+1}} = \frac{\alpha \overline{w}^l_{t+1}\epsilon(n^i_t)^{\epsilon-1}}{V^i_{t+1}}. \tag{20}$$

Similarly, the first order condition of U^i with respect to m^i_t shows

$$\frac{-\phi w^i_t - x + \theta^h \overline{w}^h_{t+1}}{D^i_{t+1}} + \frac{(1-\alpha)\overline{w}^h_{t+1}\epsilon(m^i_t)^{\epsilon-1}}{V^i_{t+1}} = 0,$$

which is the same as

$$\frac{\phi w^i_t + x - \theta^h \overline{w}^h_{t+1}}{D^i_{t+1}} = \frac{(1-\alpha)\overline{w}^h_{t+1}\epsilon(m^i_t)^{\epsilon-1}}{V^i_{t+1}}. \tag{21}$$

Dividing (7) by (8), we obtain

$$\frac{\phi w_t^i - \theta^l \overline{w}_{t+1}^l}{\phi w_t^i + x - \theta^h \overline{w}_{t+1}^h} = \frac{\alpha \overline{w}_{t+1}^l (n_t^i)^{\epsilon-1}}{(1-\alpha)\overline{w}_{t+1}^h (m_t^i)^{\epsilon-1}}.$$

Denote

$$A_t^i = \frac{\phi w_t^i - \theta^l \overline{w}_{t+1}^l}{\phi w_t^i + x - \theta^h \overline{w}_{t+1}^h}, \quad B_t = \frac{\alpha \overline{w}_{t+1}^l}{(1-\alpha)\overline{w}_{t+1}^h}.$$

Hence, we obtain

$$n_t^i = \left(\frac{B_t}{A_t^i}\right)^{\frac{1}{1-\epsilon}} m_t^i, \text{ or } (n_t^i)^{\epsilon-1} = \left(\frac{A_t^i}{B_t}\right)(m_t^i)^{\epsilon-1}. \tag{22}$$

Step 2. Obtaining m^i

By rewriting (7) as follows

$$\left(\phi w_t^i - \theta^l \overline{w}_{t+1}^l\right) V_{t+1}^i = \alpha \epsilon \overline{w}_{t+1}^l (n_t^i)^{\epsilon-1} D_{t+1}^i,$$

using (18) and (19), and rearranging the terms, yields

$$\lambda \alpha (1+\epsilon)\overline{w}_{t+1}^l (n_t^i)^{\epsilon} = \alpha \epsilon \overline{w}_{t+1}^l (n_t^i)^{\epsilon-1}\left[w_t^i - \Gamma_1^i m_t^i\right] - \lambda(1-\alpha)\overline{w}_{t+1}^h (m_t^i)^{\epsilon}. \tag{23}$$

with $\lambda = \phi w_t^i - \theta^l \overline{w}_{t+1}^l$ and $\Gamma_1^i = \phi w_t^i + x - \theta^h \overline{w}_{t+1}^h$.

When substituting (22) into the right hand side of (23) and after rearranging the terms, we obtain

$$\alpha(1+\epsilon)\overline{w}_{t+1}^l (n_t^i)^{\epsilon} = \frac{(1-\alpha)\epsilon \overline{w}_{t+1}^h}{\phi w_t^i + x - \theta^h \overline{w}_{t+1}^h} w_t^i (m_t^i)^{\epsilon-1} - (1-\alpha)(1+\epsilon)\overline{w}_{t+1}^h (m_t^i)^{\epsilon}. \tag{24}$$

Using (22) again and rearranging terms, yields

$$(1+\epsilon)\left(\alpha \overline{w}_{t+1}^l \sigma_{n,m}^i + (1-\alpha)\overline{w}_{t+1}^h\right) m_t^i \overline{w}_{t+1}^h = \frac{\epsilon(1-\alpha)\overline{w}_{t+1}^h}{\phi w_t^i + x - \theta^h \overline{w}_{t+1}^h} w_t^i, \tag{25}$$

where $\sigma^i_{n,m} = \left(\frac{B_t}{A^i_t}\right)^{\frac{\epsilon}{1-\epsilon}}$.

Hence m^i_t can be explicitly rewritten as

$$m^i_t = \frac{\epsilon(1-\alpha)\overline{w}^h_{t+1} w^i_t}{(1+\epsilon)\left[\phi w^i_t + x - \theta^h \overline{w}^h_{t+1}\right]\left[\alpha \overline{w}^l_{t+1}\sigma^i_{n,m} + (1-\alpha)\overline{w}^h_{t+1}\right]}.$$

Finally, due to (22) and (9), we have

$$n^i_t = (\sigma^i_{n,m})^{\frac{1}{\epsilon}} m^i_t.$$

Appendix B: Proof of Proposition 3

This proof can be established in three steps. In step 1, the effect of p^h on the elasticity of substitution $\sigma^i_{n,m}$ is computed. Step 2 shows that m^i is an increasing function of p^h, while step 3 demonstrates that n^i is decreasing in p^h.

Step 1. Elasticity $\sigma^i_{n,m}$ is decreasing in p^h.

Taking logarithm of $\sigma^i_{n,m} = \left(\frac{B_t}{A^i_t}\right)^{\frac{\epsilon}{1-\epsilon}}$, it follows

$$\ln(\sigma^i_{n,m}) = \frac{\epsilon}{1-\epsilon}\ln\left(\frac{B_t}{A^i_t}\right).$$

Thus

$$\frac{1}{\sigma^i_{n,m}}\frac{\partial \sigma^i_{n,m}}{\partial p^h} = \frac{\epsilon}{1-\epsilon}\frac{A^i_t}{B_t}\frac{\partial}{\partial p^h}\left(\frac{B_t}{A^i_t}\right),$$

and

$$sign\left(\frac{\partial \sigma^i_{n,m}}{\partial p^h}\right) = sign\left(\frac{\partial}{\partial p^h}\left(\frac{B_t}{A^i_t}\right)\right),$$

due to the fact that $\frac{\epsilon}{1-\epsilon} > 0$ and $\frac{A^i_t}{B_t} > 0$.

From the definition of B_t and A^i_t, we have that

$$\frac{B_t}{A^i_t} = \frac{\alpha \overline{w}^l_{t+1}}{(1-\alpha)(\phi w^i_t - \theta^l \overline{w}^l_{t+1})}\left(\frac{\phi w^i_t + x}{\overline{w}^h_{t+1}} - \theta^h\right).$$

By the definition of $\overline{w}^h_{t+1}$, it follows

$$\frac{\partial}{\partial p^h}\left(\frac{B_t}{A^i_t}\right) = \frac{\alpha \overline{w}^l_{t+1}}{(1-\alpha)(\phi w^i_t - \theta^l \overline{w}^l_{t+1})}\left[-\frac{\phi w^i_t + x}{(\overline{w}^h_{t+1})^2}\left(w^{*h} - w^h\right)\right] < 0,$$

where we use the first order condition (or **Assumption 1**)

$$\phi w^i_t + x > \overline{w}^h_{t+1}, \;\; \phi w^i_t > \overline{w}^l_{t+1}.$$

Therefore $\sigma^i_{n,m}$ is decreasing in terms of p^h, or

$$\frac{\partial \sigma^i_{n,m}}{\partial p^h} < 0.$$

Step 2. m^i **is increasing in** p^h**.**

Denoting

$$\Gamma^i_1 = \phi w^i_t + x - \theta^h \overline{w}^h_{t+1}, \;\; \Gamma^i_2 = \alpha \overline{w}^l_{t+1}\sigma^i_{n,m} + (1-\alpha)\overline{w}^h_{t+1}, \;\; \Gamma^i = \Gamma^i_1\Gamma^i_2,$$

and directly taking the derivative of (9) with respect to p^h, yields

$$\begin{aligned}\frac{\partial m^i_t}{\partial p^h} &= \frac{\epsilon(1-\alpha)w^i_t}{(1+\epsilon)\Gamma^2}\left[\Gamma^i \frac{\partial \overline{w}^h_{t+1}}{\partial p^h} - \Gamma^i_2 \overline{w}^h_{t+1}(-\theta^h)\frac{\partial \overline{w}^h_{t+1}}{\partial p^h}\right. \\ &\left. - \Gamma^i_1 \overline{w}^h_{t+1}\left(\alpha \overline{w}^l_{t+1}\frac{\partial \sigma^i_{n,m}}{\partial p^h} + (1-\alpha)\frac{\partial \overline{w}^h_{t+1}}{\partial p^h}\right)\right].\end{aligned}$$

Define

$$M^i = \Gamma^i + \Gamma^i_2 \theta^h \overline{w}^h_{t+1} - \Gamma^i_1(1-\alpha)\overline{w}^h_{t+1},$$

then $\frac{\partial m^i_t}{\partial p^h}$ can be rewritten as

$$\frac{\partial m^i_t}{\partial p^h} = \frac{\epsilon(1-\alpha)w^i_t}{(1+\epsilon)\Gamma^2}\left(M^i \frac{\partial \overline{w}^h_{t+1}}{\partial p^h} - \alpha \Gamma^i_1 \overline{w}^h_{t+1}\overline{w}^l_{t+1}\frac{\partial \sigma^i_{n,m}}{\partial p^h}\right).$$

In step 1, we prove that $\frac{\partial \sigma^i_{n,m}}{\partial p^h} < 0$, so the second terms in the right hand side is positive, and $\frac{\partial \overline{w}^h_{t+1}}{\partial p^h} = w^{*h} - w^h > 0$. Therefore, we only need to study the sign of M^i.

From the above definition, it follows

$$\begin{aligned} M^i &= \left(\phi w^i + x - \theta^h \overline{w}^h_{t+1}\right)\left(\alpha \overline{w}^l_{t+1}\sigma^i_{n,m} + (1-\alpha)\overline{w}^h_{t+1}\right) \\ &\quad + \theta^h \overline{w}^h_{t+1}\left(\alpha \overline{w}^l_{t+1}\sigma^i_{n,m} + (1-\alpha)\overline{w}^h_{t+1}\right) \\ &\quad - (1-\alpha)\overline{w}^h_{t+1}\left(\phi w^i + x - \theta^h \overline{w}^h_{t+1}\right) \\ &= \left(\phi w^i + x - \theta^h \overline{w}^h_{t+1}\right)\alpha \overline{w}^l_{t+1}\sigma^i_{n,m} + \theta^h \overline{w}^h_{t+1}\left(\alpha \overline{w}^l_{t+1}\sigma^i_{n,m} + (1-\alpha)\overline{w}^h_{t+1}\right) \\ &> 0. \end{aligned}$$

Therefore, we obtain

$$\frac{\partial m^i_t}{\partial p^h} > 0, \quad \forall t, \quad i = l, h. \tag{26}$$

Step 3. n^i_t is decreasing in p^h

(In the following, we omit the time subscript t.) Taking logarithm in (10), yields

$$\ln(n^i) = \frac{1}{\epsilon}\ln(\sigma^i_{n,m}) + \ln(m^i).$$

Hence direct calculation shows

$$\frac{1}{n^i}\frac{\partial n^i}{\partial p^h} = \frac{1}{\epsilon \sigma^i_{n,m}}\frac{\partial \sigma^i_{n,m}}{\partial p^h} + \frac{1}{m^i}\frac{\partial m^i}{\partial p^h}, \tag{27}$$

where the fist term is negative and the second term is positive, therefore we continue the analysis study to see which term dominates and determines the sign of $\frac{\partial n^i}{\partial p^h}$.

It is easy to check that

$$\begin{aligned} \frac{1}{m^i}\frac{\partial m^i}{\partial p^h} &= \frac{1}{\Gamma^i \overline{w^h}}\left[M^i(w^{*h} - w^h) - \Gamma^i_1 \alpha \overline{w^h w^l}\frac{\partial \sigma^i_{n,m}}{\partial p^h}\right] \\ &= \frac{M^i(w^{*h} - w^h)}{\Gamma^i \overline{w^h}} - \frac{\alpha \overline{w^l}}{\Gamma^i_2}\frac{\partial \sigma^i_{n,m}}{\partial p^h}. \end{aligned} \tag{28}$$

Substituting (28) into (27), yields

$$\frac{1}{n^i}\frac{\partial n^i}{\partial p^h} = \left(\frac{1}{\epsilon\sigma^i_{n,m}} - \frac{\alpha\overline{w^l}}{\Gamma^i_2}\right)\frac{\partial\sigma^i_{n,m}}{\partial p^h} + \frac{M^i(w^{*h} - w^h)}{\Gamma^i\overline{w^h}}. \tag{29}$$

Denote

$$\begin{aligned}
\Phi^i &= \frac{1}{\epsilon\sigma^i_{n,m}} - \frac{\alpha\overline{w^l}}{\Gamma^i_2} \\
&= \frac{1}{\epsilon\sigma^i_{n,m}\Gamma^i_2}\left(\Gamma^i_2 - \alpha\epsilon\overline{w^l}\sigma^i_{n,m}\right) \\
&= \frac{1}{\epsilon\sigma^i_{n,m}\Gamma^i_2}\left((1-\epsilon)\alpha\overline{w^l}\sigma^i_{n,m} + (1-\alpha)\overline{w^h}\right) \\
&> 0.
\end{aligned}$$

Recall that

$$\begin{aligned}
\frac{1}{\sigma^i_{n,m}}\frac{\partial\sigma^i_{n,m}}{\partial p^h} &= \frac{\epsilon}{(1-\epsilon)}\frac{A^i}{B}\frac{\partial}{\partial p^h}\left(\frac{B}{A^i}\right) \\
&= \frac{\epsilon}{(1-\epsilon)}\frac{A^i}{B}\frac{\alpha}{(1-\alpha)}\frac{\overline{w}^l}{(\phi w^i - \theta^l\overline{w}^l)}(-1)\frac{(\phi w^i + x)(w^{*h} - w^h)}{(\overline{w}^h)^2},
\end{aligned}$$

where

$$\frac{A^i}{B} = \frac{(\phi w^i - \theta^l\overline{w}^l)}{(\phi w^i + x - \theta^h\overline{w}^h)}\frac{(1-\alpha)\overline{w}^h}{\alpha\overline{w}^l}.$$

Hence

$$\frac{1}{\sigma^i_{n,m}}\frac{\partial\sigma^i_{n,m}}{\partial p^h} = -\frac{\epsilon}{(1-\epsilon)}\frac{(\phi w^i + x)(w^{*h} - w^h)}{(\phi w^i + x - \theta^h\overline{w}^h)\overline{w}^h}. \tag{30}$$

Substituting (30) into (29), it follows

$$\begin{aligned}
\frac{1}{n^i}\frac{\partial n^i}{\partial p^h} &= -\frac{(\phi w^i + x)(w^{*h} - w^h)(\Gamma^i_2 - \alpha\epsilon\sigma^i_{n,m}\overline{w^l})}{(1-\epsilon)\Gamma^i\overline{w^h}} + \frac{M^i(w^{*h} - w^h)}{\Gamma^i\overline{w^h}} \\
&= \frac{(w^{*h} - w^h)}{(1-\epsilon)\Gamma^i\overline{w^h}}\left[(1-\epsilon)M^i - (\phi w^i + x)(\Gamma^i_2 - \alpha\epsilon\sigma^i_{n,m}\overline{w^l})\right],
\end{aligned} \tag{31}$$

where

$$
\begin{aligned}
(\phi w^i + x)(\alpha\epsilon\sigma^i_{n,m}\overline{w^l} - \Gamma^i_2) &= (\phi w^i + x)\left(\alpha\epsilon\sigma^i_{n,m}\overline{w^l} - \alpha\sigma^i_{n,m} - (1-\alpha)\overline{w^h}\right) \\
&= -(\phi w^i + x)(1-\epsilon)\alpha\sigma^i_{n,m}\overline{w^l} - (\phi w^i + x)(1-\alpha)\overline{w^h},
\end{aligned}
$$

and

$$
\begin{aligned}
(1-\epsilon)M^i &= (1-\epsilon)\left[\Gamma^i_2 + \theta^h\Gamma^i_2\overline{w^h} - (1-\alpha)\Gamma^i_1\overline{w^h}\right] \\
&= (1-\epsilon)\alpha\sigma^i_{n,m}\Gamma^i_1\overline{w^l} + (1-\epsilon)\Gamma^i_2\theta^h\overline{w^h} \\
&= (1-\epsilon)\alpha\sigma^i_{n,m}\overline{w}^l\left[(\phi w^i + x) - \theta^h\overline{w^h}\right] + (1-\epsilon)\theta^h\overline{w^h}(\alpha\overline{w^l}\sigma^i_{n,m} + (1-\alpha)\overline{w^h}) \\
&= (\phi w^i + x)(1-\epsilon)\alpha\sigma^i_{n,m}\overline{w^l} + (1-\epsilon)(1-\alpha)\theta^h(\overline{w^h})^2.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\frac{1}{n^i}\frac{\partial n^i}{\partial p^h} &= \frac{(1-\alpha)(w^{*h} - w^h)}{(1-\epsilon)\Gamma^i}\left[(1-\epsilon)\theta^h w^h - (\phi w^i + x)\right] \\
&\begin{cases} > 0, & \text{if } (1-\epsilon)\theta^h w^h > (\phi w^i + x), \\ = 0, & \text{if } (1-\epsilon)\theta^h w^h = (\phi w^i + x), \\ < 0, & \text{if } (1-\epsilon)\theta^h w^h < (\phi w^i + x). \end{cases}
\end{aligned}
\tag{32}
$$

However, if Assumption 1 holds (i.e. the first order condition), then the first two cases in (32) are not possible, and as a result, we have

$$
\frac{\partial n^i_t}{\partial p^h} < 0, \quad \forall t, \quad i = l, h. \tag{33}
$$

This finishes the proof.

Appendix C: Proof of Proposition 4 (case ii)

Denote the right hand side of (12) as

$$F(\overline{w}^h) = \frac{\epsilon(1-\alpha)\overline{w}^h}{\alpha\overline{w}^l\sigma^i_{n,m} + (1-\alpha)\overline{w}^h}.$$

And hence

$$\frac{1}{F(\overline{w}^h)} = \frac{\alpha\overline{w}^l\sigma^i_{n,m}}{\epsilon(1-\alpha)\overline{w}^h} + \frac{1}{\epsilon}.$$

Moreover

$$\begin{aligned}\frac{d}{d\overline{w}^h}\left(\frac{1}{F(\overline{w}^h)}\right) &= -\frac{1}{F^2(\overline{w}^h)}\frac{dF(\overline{w}^h)}{d\overline{w}^h} \\ &= \frac{\alpha\overline{w}^l}{\epsilon(1-\alpha)}\frac{\left(\overline{w}^h\frac{\partial\sigma^i_{n,m}}{\partial\overline{w}^h} - \sigma^i_{n,m}\right)}{\left(\overline{w}^h\right)^2},\end{aligned}$$

where, by the definition of $\sigma^i_{n,m}$ and by omitting subscript t for simplicity, leads to

$$\begin{aligned}\frac{\partial\sigma^i_{n,m}}{\partial\overline{w}^h} &= \frac{\epsilon}{1-\epsilon}\left(\frac{B}{A}\right)^{\frac{\epsilon}{1-\epsilon}-1}\frac{1}{A}\left[-\frac{B}{\overline{w}^h} - \frac{B\theta^h}{\phi w^i + x - \theta^h\overline{w}^h}\right] \\ &= -\frac{\epsilon}{1-\epsilon}\sigma^i_{n,m}\frac{\phi w^i + x}{(\phi w^i + x - \theta^h\overline{w}^h)\overline{w}^h} \\ &< 0,\end{aligned}$$

due to Assumption 1 and $0 < \epsilon < 1$.

Hence

$$\frac{d}{d\overline{w}^h}\left(\frac{1}{F(\overline{w}^h)}\right) < 0,$$

and

$$\frac{dF(\overline{w}^h)}{d\overline{w}^h} > 0.$$

The proof is finished.

Appendix D: Figures and tables of section 5.3.3

Figure 7: Impact of *high* skilled emigration for different values of ϵ

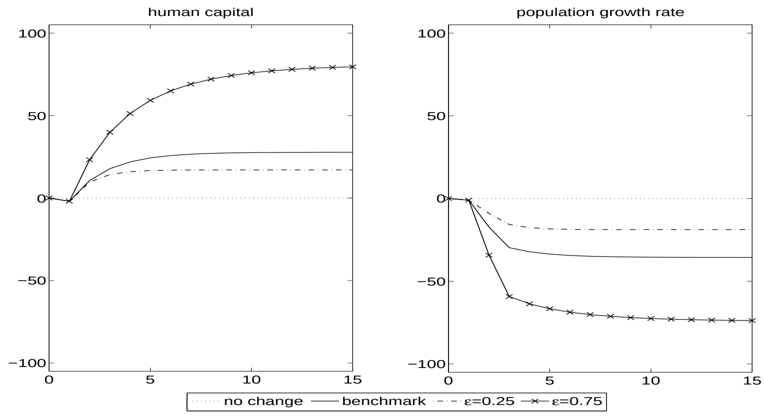

Figure 8: Impact of *low* skilled emigration for different values of ϵ

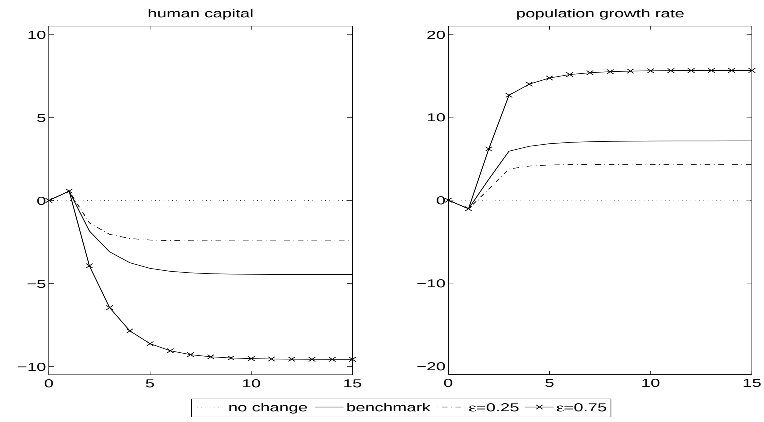

Table 2: Impact of an increase in p^h under different variants

Impact on household decisions	Variables	Benchmark	$\gamma^h = \gamma^l$	$\Lambda = 0$	$\epsilon = 0.25$	$\epsilon = 0.75$
High skilled children of high skilled parents	m^h	2.09	3.96	5.36	5.10	1.04
High skilled children of low skilled parents	m^l	6.53	10.60	9.09	10.28	8.54
Low skilled children of high skilled parents	n^h	-14.65	-14.23	-9.44	-8.07	-28.33
Low skilled children of low skilled parents	n^l	-12.20	-11.00	-6.23	-7.51	-24.12
Total children of high skilled parents	$m^h + n^h$	0.22	-0.28	-1.77	0.57	0.98
Total children of low skilled parents	$m^l + n^l$	-9.95	-8.40	-4.08	-4.77	-20.46
Savings of high skilled parents	s^h	-0.88	-1.49	0.00	-0.77	-0.82
Savings of low skilled parents	s^l	1.61	-1.27	0.00	-1.07	-0.02
Human capital	H	27.79	28.98	15.14	17.06	18.68
Growth rate of the population	g	-35.62	-28.87	-12.24	-18.81	-23.53
Impact on welfare						
Remittances per high skilled receiver	Z^h	9.23	11.01	0.00	10.50	8.96
Remittances per low skilled receiver	Z^l	-3.61	3.59	0.00	2.59	0.06
Total remittances	Λ	26.98	37.22	0.00	18.20	19.49
Average remittances	$\overline{Z}$	3.38	14.07	0.00	5.58	4.47
Average utility	$\overline{U}$	13.26	14.49	9.15	7.90	8.79
Ratio of utilities (high to low skilled)	Ξ	-3.56	-3.69	-2.71	-2.60	-2.72
Average utility from consumption	$\overline{\Psi}$	18.31	19.09	9.97	8.42	10.08
Ratio of utilities from consumption	Π	0.00	0.00	0.00	0.00	0.00

Table 3: Impact of an increase in p^l under different variants

Impact on household decisions	Variables	Benchmark	$\gamma^h = \gamma^l$	$\Lambda = 0$	$\epsilon = 0.25$	$\epsilon = 0.75$
High skilled children of high skilled parents	m^h	-0.07	-0.18	-0.53	-0.24	0.00
High skilled children of low skilled parents	m^l	-0.70	-0.78	-0.87	-0.53	-1.39
Low skilled children of high skilled parents	n^h	1.62	1.44	0.94	0.89	3.39
Low skilled children of low skilled parents	n^l	3.21	1.74	0.60	2.46	5.85
Total children of high skilled parents	$m^h + n^h$	0.12	0.20	0.18	0.15	0.01
Total children of low skilled parents	$m^l + n^l$	2.74	1.44	0.39	2.00	5.04
Savings of high skilled parents	s^h	0.00	0.01	0.00	0.00	0.00
Savings of low skilled parents	s^l	-1.11	-0.33	0.00	-0.73	-0.83
Human capital	H	-4.46	-2.26	-0.78	-2.44	-2.95
Growth rate of the population	g	7.15	2.60	-0.69	4.32	4.95
Impact on welfare						
Remittances per high skilled receiver	Z^h	0.02	-0.08	0.00	0.01	0.00
Remittances per low skilled receiver	Z^l	2.49	0.92	0.00	1.78	2.00
Total remittances	Λ	-2.12	-1.45	0.00	-0.84	-1.16
Average remittances	$\overline{Z}$	1.43	0.02	0.00	1.26	1.27
Average utility	$\overline{U}$	-1.19	-0.51	0.07	-0.36	-0.55
Ratio of utilities (high to low skilled)	Ξ	-0.33	-0.41	-0.54	-0.27	-0.28
Average utility from consumption	$\overline{\Psi}$	-2.94	-1.49	-0.51	-1.20	-1.59
Ratio of utilities from consumption	Π	0.00	0.00	0.00	0.00	0.00