# Income Tax Evasion in a Society of Heterogeneous Agents – Evidence from an Agent-based Model

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We analyze the evolution and extent of income tax evasion under alternative governmental policies in an agent-based model with heterogeneous agents. A novel aspect of our modeling is the use of an exponential utility function, which allows us to assume rather realistic audit probabilities and to yield more realistic results with respect to the extent of tax evasion. Further, the introduction of lapse of time effects constitutes another novel aspect of our model. Among other things, the model allows for assessing the impact of alternative policies on tax evasion. Subject to the model features, we find that ethical norms and lapse of time effects reduce the extent of tax evasion particularly strong.

Keywords:
income tax evasion, heterogeneous population, lapse of time, ethical behavior, agent-based models,

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## 1 Introduction

In this paper we develop an agent-based income tax evasion model that allows for simulating large populations of heterogeneous agents. Heterogeneity refers not only to individually different income levels and risk preferences, but also to four behaviorally different agent types of which one type is in line with Allingham and Sandmo (1972). Among other things, a novel aspect of our model consists of using an exponential utility function for specifying the expected utility of the Allingham and Sandmo type. Our results show that this specification yields far more realistic behavior patterns for such tax payers than specifications based on Cobb-Douglas utility functions. Moreover, the specification allows us to incorporate lapse of time effects on tax evasion, which constitutes another novel aspect of our model.

The remainder of this paper is organized as follows. In the next section we briefly discuss agent-based tax evasion models. In section three we introduce our model. Simulation scenarios and results are presented in section four. The last section concludes.

## 2 Background

Agent-based models are gaining some popularity in economics since the 1990s (Tesfatsion and Judd 2006). However, the first agent-based tax evasion models did not appear before the 2000s. This notwithstanding, we already observe two rather distinct developments in the literature.

First, there are a number of agent-based tax evasion models designed by economists, for example, Mittone and Patelli (2000), Davis et al. (2003), Korobow et al. (2007) and Bloomquist (2004, 2009), of which some are summarized and compared by Bloomquist (2006). In addition, a second branch has emerged from a new research area of physics, which is called econophysics or sociophysics. Overviews are provided by Schulz (2003) and Kulakowski and Nawojczyk (2008). Tax evasion papers falling into this latter domain have generated some interesting results. For example, Zaklan et al. (2008: 5858) find that even very small levels of enforcement are sufficient to establish almost full tax compliance and Zaklan et al. (2009: 12) conclude that regardless of how strong group influence may be, enforcement always works to enhance tax compliance.

In general, agent-based tax evasion models that fall into the econophysics domain, for example, papers by Zaklan et al. (2009, 2008), Lima (2009), Lima and Zaklan (2008), all rest on a model of ferromagnetism. The latter was originally developed by the German physicist Ernst Ising in 1925 and Brush (1967) extended and generalized the Ising model, which is today a standard model in statistical mechanics. In agent-based tax evasion models of the econophysics type the Ising model is used to mimic conditional cooperation among agents (Zaklan et al. 2009). Yet, the actual patterns and levels of tax evasion in these models depend on two additional factors: the network structure of society and the tax enforcement mechanism. The network structure is implemented by alternative lattice types and tax enforcement consists of the two economic standard parameters audit probability and penalty rate.

Since the latter two factors are common for every agent-based tax evasion model (Bloomquist 2006), it follows that the role of the Ising model is the essential factor that makes agent-based tax evasion models of econophysics type different from those designed by economists. It must be emphasized, however, that the role of the Ising model goes much beyond that of providing a basis for conditional cooperation. In fact, the Ising model, or more precisely, the role of temperature in the Ising model, is a driving force for the results obtained from these models. The value of temperature essentially determines to which degree agents take decisions more or less autonomously. For very low temperatures agents hardly decide autonomously, but copy the behavior of others in their relevant social network or group. Furthermore, there exists a critical temperature that is important for regime switches, i.e., switches in the order of the whole system. To put this differently, the exogenously determined variable ‘temperature’ influences all agents of society simultaneously. Does such a variable really exist in real human societies and, if so, can we give an economic interpretation of this process? Apparently, up to now there is no convincing explanation or re-interpretation of ‘temperature’ in economic and social systems (Schulz 2003).

Therefore, in this paper we refrain from using the Ising model. Rather, we develop an agent-based model that is based on a simple lattice structure and rule based type interaction in a population of heterogeneous agents.

## 3 The Model

In this section we begin with a brief overview regarding some general aspects of the model design. The following subsection deals with a detailed description of the behavioral types and their decision rules. On the technical side the model is set up in MATLAB (version 7.8.0. R2009a) and codes are available upon request.

### *3.1 Model Design*

In both agent-based tax evasion models designed by economists (Bloomquist 2006) and in theoretical approaches to tax evasion (Prinz 2010) it becomes more and more popular to consider behaviorally different types of agents to capture more realistic images of real societies. We follow this route and consider four behavioral types. In particular, '*maximizing a-type agents*' who show an expected utility maximizing behavior in line with the Allingham and Sandmo (1972) model, '*imitating b-type agents*' who copy successful tax evasion behavior, if they observe it within their social network, '*ethical c-type agents*' who always declare their true income due to certain behavioral norms such as Kantian behavior, patriotism and the like, and finally '*random d-type agents*' who wish to declare their true income but may make mistakes within a neighborhood around their true income, for example, because of the complexities of tax law. We assume that agent type shares are *a priori* given and in principle constant over all periods, but the initial distribution of these type shares can be freely selected as a parameter of the model.

The number of agents *N* and the number of tax relevant periods *T* are also parameters of the model. According to Bloomquist (2006) and Zaklan et al. (2009), agent-based tax evasion models designed by economists usually deal with a range of just 1,000 agents, whereas models falling into the econophysics domain use about 1,000,000 or more agents. Our model runs with 150,000 agents and we consider 40 tax relevant periods (tax years), because such a time span may well represent the average for which agents may have to pay taxes in the real world. The lattice structure which we assume is of a simple ring world type (see Epstein and Axtell 1996). Essentially, this means that the agent population is represented by a row vector of 150,000 agents, where each agent has a certain visibility into one direction. For example, each agent may observe the behavior of one (four, eight, …)

of his/her neighbor(s) to the left. The row vector can be interpreted as a ring world, if the first agent on the left hand side of the row vector can observe the behavior of the last agent on the right hand side of the row vector. Each agent is endowed with an individual taxable income *W*, which may differ among agents, and has to declare in every tax relevant period an income *X* to the tax authorities, which is the decision variable. Agents are faced with three tax evasion relevant parameters, which are well known from Allingham and Sandmo (1972), in particular, the tax rate on declared income $\theta$, the tax rate on undeclared income $\pi$ and the objective audit probability *p*. In addition, we introduce a new parameter, which is the tax law complexity parameter $\gamma$, where higher values indicate a more complex tax law. All four parameters are directly or indirectly set by the government via the tax authorities. Therefore, to simulate tax policy changes, these four parameters may change only after an election year, say every fourth year, but are constant in all other periods (see Table 1).

### *3.2 Behavioral Types*

*Maximizing a-type agents*: Following Allingham and Sandmo (1972) again, we assume that maximizing a-types are risk-adverse and, therefore, have a concave utility function. In particular, we specify the utility function *U* of the *i*-th a-type as an exponential function (Kirkwood 2004), which implies a constant absolute risk aversion (CARA),

$$U_i(ATPI_i) = 1 - e^{-\lambda_i(ATPI_i)}, \tag{1}$$

with $i = 1, \ldots, N$. In (1) $ATPI_i$ denotes the after tax and penalty net income and $\lambda_i$ denotes an individual, randomly allotted risk parameter, which is uniformly distributed on the interval [0; 1], where $\lambda = 0$ indicates a preference for risk-neutrality and $\lambda = 1$ indicates a high preference for risk-aversion. Thus, a-type agents are heterogeneous with respect to their risk preferences. As noted, agents declare in every tax relevant period an income $X_i$ to the tax authorities. If they are not detected they retain an after tax and penalty net income according to (2),

$$ATPI_i = W_i - \theta X_i, \tag{2}$$

However, if they are subject to an audit, the tax authorities learn their true income $W_i$ and, therefore, their extent of tax evasion. In this case the after tax and penalty net income is determined according to (3),

$$ATPI_i = W_i - \theta X_i - \pi(W_i - X_i). \tag{3}$$

To simplify notation, in the following we refrain from using the index $i$, unless where strictly necessary. By definition we have $\theta < \pi$ and, thus, the excess of $\pi$ over $\theta$ essentially represents a penalty for tax evasion. Moreover, we distinguish between an objective audit probability $p$, which is determined by the government and a subjective audit probability $p_s$, which is relevant for the individual decision of the agent. Typically we assume $p = p_s$, but if an audit has taken place and a penalty was due, we assume instead that maximizing a-type agents disregard the objective audit probability $p$ in some following periods. In particular, they raise their subjective audit probability to $p_s = 1$ in the first period after the audit and then gradually decrease their subjective audit probability in each of the following periods by using their updating parameter $\delta = -0.20$ until the objective audit probability $p$ is reached again, with $p = p_s$.

Thus, in line with Allingham and Sandmo (1972), a-type agents maximize their expected utility according to (4),

$$EU(W, X) = (1 - p_S) \cdot \left(1 - e^{-\lambda \cdot (W - \theta X)}\right) + p_S \cdot \left(1 - e^{-\lambda \cdot (W - \theta X - \pi \cdot (W - X))}\right), \tag{4}$$

By taking the first order derivative with respect to $X$ and by taking the two constrains mentioned by Allingham and Sandmo (1972, eq. 5 and 6) into account, we obtain the necessary and sufficient condition for an inner solution, given our specification,

$$\frac{\theta}{\theta + (-\theta + \pi) \cdot e^{\lambda \pi W}} < p_S < \frac{\theta}{\pi}, \tag{5}$$

The inner solution is obtained by setting the first order derivative equal to zero and rearranging,

$$X = W - \frac{ln\left(\frac{(1-p_S)\cdot\theta}{p_S\cdot(-\theta+\pi)}\right)}{\lambda\pi} . \tag{6}$$

According to (5) a-types declare their full income, $X = W$, if $\pi \cdot p_s > \theta$ holds, they declare no income at all, $X = 0$, if $p_s$ falls below the lower limit of the interval and they declare part of their income if $p_s$ falls into the interval, that is, they declare $X$ according to (6). However, the upper limit of the interval is generally determined by $\theta$ and $\pi$ and holds for every a-type agent alike, whereas the lower limit of the interval depends also on $W$ and $\lambda$, which we have both individualized. Therefore, the length of the interval in (5) may differ for every a-type agent. For example, assume that we have $\theta = 0.2$, $\pi = 0.3$ and that there are two agents $i = 1, 2$, with $\lambda_1 = 0.05$, $W_1 = 10$ and $\lambda_2 = 0.5$, $W_2 = 50$. Agent 1 has a low preference for risk-aversion and a low income and his interval according to (5) is: $0.63 < p_s < 0.67$, whereas agent 2 has both a higher preference for risk-aversion and a higher income, so that his interval amounts to: $0.0011 < p_s < 0.67$. Hence, agent 2 will declare part of his income even for very low audit probabilities of less than one percent. In fact, such rather long intervals result from our specification of the utility function and help to make the model more realistic.

*Imitating b-type agents*: They have a visibility parameter $v$, which indicates the size of their social network, that is, the number of agents they observe. We assume a social network size of $\nu = 4$, as in Zaklan et al. (2009: 4). Agents confine in all agents that belong to their social network and, therefore, the extent of tax evasion is known within each social network. Thus, in period $t$ imitating b-type agents declare the mean of the voluntarily declared income, $X_{i,t}$, observed within their social network during the previous tax period $t-1$, with $t = 2, \ldots, T$, which is calculated by,

$$X_{i,t} = \frac{1}{\nu} \sum_{j=i-\nu}^{i-1} \frac{X_{j,t-1}}{W_{j,t-1}} \cdot W_{i,t} . \tag{7}$$

Note, however, that b-types declare $X_{i,t}$ according to (7) only if tax evasion was on average successful within the social network they observe. That is, they first check whether or not the following condition is fulfilled,

$$\frac{1}{\nu}\sum_{j=i-\nu}^{i-1}\frac{ATPI_{j,t-1}}{W_{j,t-1}} > 1-\theta_{t-1}. \tag{8}$$

If condition (8) does not hold, imitating b-type agents use their default option and declare their true income or, in other words, mutate to a c-type. Further, if b-type agents are audited and a penalty is due, they also mutate to a c-type and declare their true income for a limited number of tax periods $\tau$ and in each simulation we have set $\tau = 4$. Thereafter, they return to their imitating behavior and may again evade taxes. Hence, if a-type or b-type agents have been found guilty of tax evasion both types may refrain from tax evasion for some periods. For a-types this period is endogenously determined by (5) and $\delta$. In contrast, it is exogenously determined for b-types by $\tau$, as in Zaklan et al. (2008). Also, the behavior pattern as such may be explained by psychological reasons (see Kirchler 2007 for an overview) or by ethical concerns.

*Ethical c-type agents*: They are motivated by some ethical behavioral norms and, therefore, declare their true income $X = W$ irrespectively of the consequences and never change this decision in any period of the model.

*Random d-type agents*: In principle, like c-type agents, d-type agents wish to declare their true income, but are faced with a complexity parameter $\gamma$, which indicates the complexities of the tax law. Given these complexities, random d-type agents may make unintended mistakes with respect to declaring their true income and, therefore, the decision variable $X$ is assumed to be normal distributed with expectation $\mu = W$ and standard deviation $\sigma = \gamma \cdot W$. Hence, random d-types may declare less than their true income or more, but on average they declare their true income correctly. Note, however, that imitating b-types may copy d-type behavior, in particular, if the d-types accidently evade taxes.

## 4 Simulation scenarios

In the following we describe the simulation scenarios which we run and present some results. In each simulation, we consider a fixed b-type share of 35 percent and a fixed d-type share of 15 percent. The d-type share is to some extent in line with Andreoni et al. (1998), who claim that about seven percent of U.S. households overpaid their taxes in 1988. For simplicity, we assume that about the same percentage share has accidently underpaid their taxes, which gives the d-type share of about 15 percent. Furthermore, the a-type share is raised in steps of ten percentage points from zero to 50 percent at the expense of the c-type share and *vice versa*. This gives six different type distributions ranging from zero percent a-types and 50 percent c-types to 50 percent a-types and zero percent c-types, each with fixed shares of b-types (35 percent) and d-types (15 percent). We run each of these six type distributions a 100 times and with the same parameter set, because this procedure allows us to assess the impact of ethical behavior on tax evasion under *ceteris paribus* conditions.

### *4.1 First Scenario*

In each run of this scenario taxable income *W* is uniformly distributed on the integer interval [0; 100], randomly allotted in the first period and then held constant with respect to all future periods. To simulate changes in tax policies of the government under *ceteris paribus* conditions we allow for just one parameter change every fourth period. The relevant parameter set is given in Table 1, where changes are denoted in bold.

**Table 1: Governmental Tax Policy Changes**

| **Period** | | **1** | **5** | **9** | **13** | **17** | **21** | **25** | **29** | **33** | **37** | **40** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Audit prob. | $\boldsymbol{p}$ | 0.01 | **0.03** | 0.03 | 0.03 | 0.03 | 0.03 | **0.04** | **0.05** | 0.05 | 0.05 | 0.05 |
| Tax rate | $\boldsymbol{\theta}$ | 0.20 | 0.20 | 0.20 | **0.30** | 0.30 | **0.40** | 0.40 | 0.40 | 0.40 | **0.30** | 0.30 |
| Penalty rate | $\boldsymbol{\pi}$ | 0.30 | 0.30 | **0.45** | 0.45 | 0.45 | 0.45 | 0.45 | 0.45 | **0.50** | 0.50 | 0.50 |
| Complexity | $\boldsymbol{\gamma}$ | 0.10 | 0.10 | 0.10 | 0.10 | **0.20** | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |

*Note: Period denotes the period in which a change takes place, which is every fourth period and after an election. The changing parameter is denoted in bold. All figures are denoted in percent.*

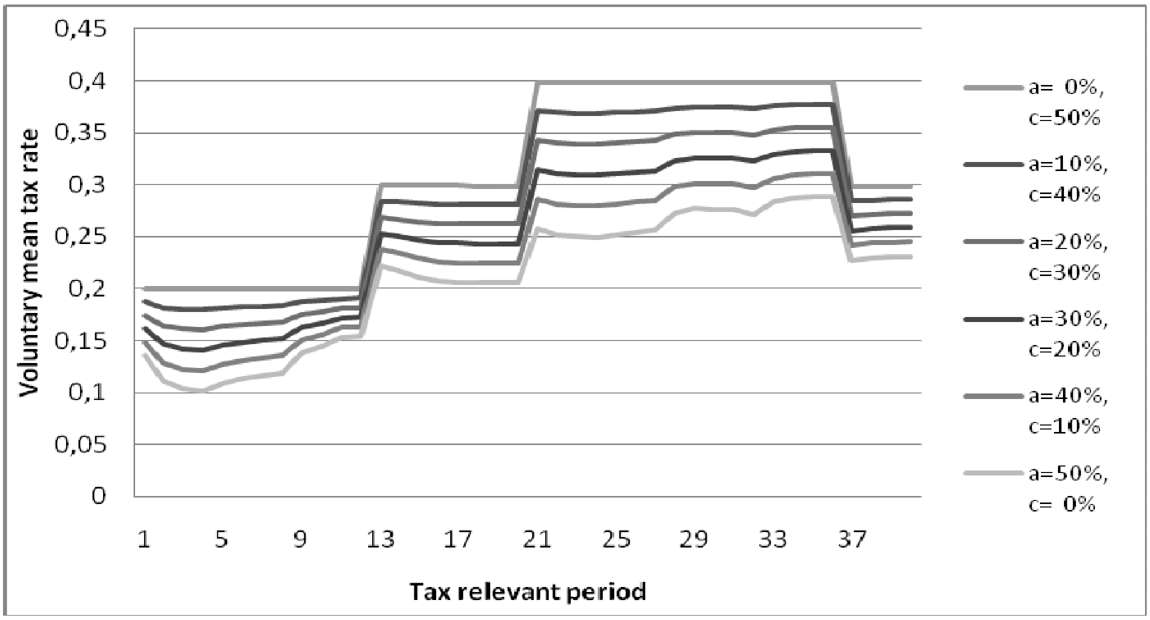
The parameter set shown in Table 1 may be interpreted as reflecting either different governments or policy changes of the ruling government. The set is characterized by comparatively low values for all four parameters during the first four tax periods. In the following eight periods (5 to 12) the government tries to fight tax evasion by raising the audit probability and the tax rate on undeclared income. However, in periods 13 to 24 the government not only raises tax rates on declared income, but also the complexity of the tax system. During the following twelve periods (25 to 36) the government again tries to actively fight tax evasion by raising the audit probability and the tax rate on undeclared income. Finally, during the last four tax periods (37 to 40) the government eventually fights income tax evasion by lowering the tax rate on declared income.

Moreover, in periods ten and 27, we audit an additional ten percent of a-type and b-type agents to simulate an exogenous shock. For example, that the tax authorities receive a CD-ROM with incriminating, personalized bank information, which is tax evasion relevant. Essentially, such a scenario resembles the 2008 Liechtenstein or the 2010 Swiss CD tax affairs in Germany. Figure 1 shows the voluntary mean tax rate, $\theta \cdot \Sigma X_i / \Sigma W_i$, based on 100 runs, each with 150,000 agents, over 40 tax periods (years) for six different type distributions.

In Figure 1, the first line from above represents the voluntary mean tax rate for type distribution: $a$= 0%, $b$=35%, $c$=50%, $d$=15%. This line may be used as a benchmark because there is almost no tax evasion since all types declare their full income either directly or on average, so that the voluntary mean tax rate coincides with the official tax rate on declared income, $\theta$. However, in period 17 the change of tax complexity parameter $\gamma$ (see Table 1) leads to a very slight knick, due to unintended tax evasion of random d-type agents and some b-type imitation of this behavior. In contrast, the five remaining lines all represent some extent of tax evasion, which is measured as the vertical difference between these lines and the benchmark. Thus, the last line from above, representing type distribution: $a$=50%, $b$=35%, $c$=0%, $d$=15%, shows the highest extent of tax evasion.

This result clearly indicates that ethically motivated behavior is an effective tool for fighting tax evasion. Ethical education and related means that foster tax law compliance should, therefore, play a more prominent role in the debate on fighting tax evasion.

**Figure 1: Voluntary Mean Tax Rate of Scenario 1**




*Note: Seen from above the six lines denote the mean tax rate due to voluntary tax declarations ($\theta \cdot \Sigma X_i / \Sigma W_i$), subject to the relevant type distribution and the prevailing parameter set. Parameter values controlled by the government are provided in Table 1. All remaining parameter values are fixed for the 40 tax periods and assume the following values: a-type subjective audit probability updating $\delta$ = -*0.20, *b-type visibility $\nu = 4$, b-type mutation period $\tau = 4$.*

Moreover, inspection of Figure 1 also allows for assessing the effectiveness of governmental tax policies described in Table 1, subject to the underlying model features. For simplicity, however, in the following we just consider the last line from above, where tax evasion is highest. To begin with, consider period one. Here tax evasion amounts to about ((20-13.5)/20=) 32.5 percent, which is calculated from (($\theta$·100-VMTR·100))/ $\theta$·100=), where VMTR denotes voluntary mean tax rate as displayed in Figure 1. The extent of tax evasion can be explained on the one hand because a-types, who represent 50 percent of the population, declare either part of their income or nothing at all, as demonstrated in section 3.2, and, on the other hand, because b-types use their default option and declare their full income whereas d-types declare their full income on average.

In period two, the extent of tax evasion is driven by two effects that have an opposite influence. First, b-types begin to imitate successful tax evasion behavior shown by a-types in period one, which leads to more tax evasion. Second, those a-types who were audited in the first period and paid a penalty now update their subjective audit probability to $p_s = 1$. Therefore, they declare their true income in period two. Inspection of Figure 1 indicates that the first effect dominates from period two to four, leading to more tax evasion.

From period four to twelve the voluntary mean tax rate increases from almost ten percent to about 15.5 percent. This development is due to various reasons. First, as of round two audited b-types, who paid a penalty, declare their true income for the next four periods due to $\tau = 4$, so that the share of b-type agents who declare their true income accumulates over four periods. Second, essentially the same is true for audited a-type agents, who paid a penalty. However, their period for declaring their full income is endogenously determined. In periods two to four, a-types declare their full income if $p_s > 0.67$, according to (5). Hence, if they update their subjective audit probability to $p_s = 1$ in the first period after the audit and then decrease it each following period by $\delta = -0.20$ until $p = p_s$ is reached again, they declare their full income for two after audit periods because of $1 > 0.67$ and $0.8 > 0.67$. Besides they declare part of their income or no income at all as of the third after audit period, depending on their individual interval length according to (5). Third, the increase of the objective audit probability $p$ in period five leads *ceteris paribus* to a higher number of audited a-types and b-types, and, therefore, to more agents who update their subjective audit probability and pay their full taxes. Fourth, the increase of the tax rate on undeclared income $\pi$ (penalty rate) leads *ceteris paribus* to a different interval according to (5) and, therefore, may change the number of periods for which a-types declare their income partly or fully. For example, with respect to periods nine to twelve the upper limit of the interval amounts to 0.44 and, therefore, a-types would declare their full income for three after audit periods, rather than for two as in previous periods. Fifth, the number of periods for which a-types declare their income fully or partly is also important for the type interaction between a-types and b-types, because a lower (higher) extent of a-type tax evasion leads to less (more) b-type tax evasion.

During periods 13 to 24 we do not observe any continuous increase in the mean voluntary tax rate and this is because both the objective audit probability and the tax rate on undeclared income remain constant. Rather, because the government increases the tax rate on declared income $\theta$, *ceteris paribus* tax evasion increases again. For example, in period four the extent of tax evasion amounts to ((20-

10)/20=) 50 percent, then falls to ((20-15.5)/20=) 22.5 percent in period twelve, but increases to ((30-21)/30=) 30 percent in period 16 and to ((40-25)/40=) 37.5 percent in period 24. Among other things, the increase in tax evasion is due to a reduction of the number of after audit periods during which a-types declare their income and because of an amplification of this process by imitating b-types.

During periods 25 to 36 the government fights income tax evasion again by raising the objective audit probability and the tax rate on undeclared income, which yields a rising voluntary mean tax rate and reduces tax evasion to ((40-25.5)/40=) 36.25 percent in period 27, to ((40-27)/40=) 32.5 percent in period 32, and to ((40-29)/40=) 27.5 percent in period 36.

Finally, during periods 37 to 40 the government pursues a different tax policy and fights tax evasion by reducing the tax rate on declared income, while maintaining comparatively high audit and penalty rates, which *ceteris paribus* yields ((30-23)/30=) 23.33 percent tax evasion in period 40. Also, the two exogenous shocks in periods ten and 27 contribute to the reduction of tax evasion. In fact, tax evasion is reduced by 4.5 percentage points from period ten to eleven and by four percentage points from period 27 to 28. Also, given the parameter set, audited and punished a-type agents will declare their full income in two periods after the first shock, but in only one period after the second shock.

### *4.2 Second scenario*

Regarding the second scenario, we use exactly the same parameter values as in the preceding scenario. However, we now incorporate a lapse of time parameter, with $\alpha = 10$, as in the present German tax law. Hence, if an audit takes place the present period and up to ten preceding periods are subject to the audit and, therefore, maximizing a-type agents now need to consider several periods in their utility function. Thus, we need to modify the expected utility function (4) as suggested by Allingham and Sandmo (1972: eq. 23) and get,

$$EU(W,X) = (1-p_s)\cdot\left(1-e^{-\lambda\cdot(W-\theta X)}\right) + p_s\cdot\left(1-e^{-\lambda\cdot\left(-\sum_{k=t-\alpha}^{t-1}\pi_k\cdot(W_k-X_k)+W-\theta X-\pi\cdot(W-X)\right)}\right) \quad (9)$$

where $t$ denotes the period in which the audit takes place and $k$ denotes the period which is controlled due to the lapse of time parameter. The necessary and sufficient condition for an inner solution in the multi-period case is then given by,

$$\frac{\theta}{\theta+(-\theta+\pi)\cdot e^{\lambda\cdot\left(\sum_{k=t-\alpha}^{t-1}\pi_k\cdot(W_k-X_k)+\pi W\right)}}<p_s<\frac{\theta}{\theta+(-\theta+\pi)\cdot e^{\lambda\cdot\left(\sum_{k=t-\alpha}^{t-1}\pi_k\cdot(W_k-X_k)\right)}}, \tag{10}$$

and the inner solution itself is given by,

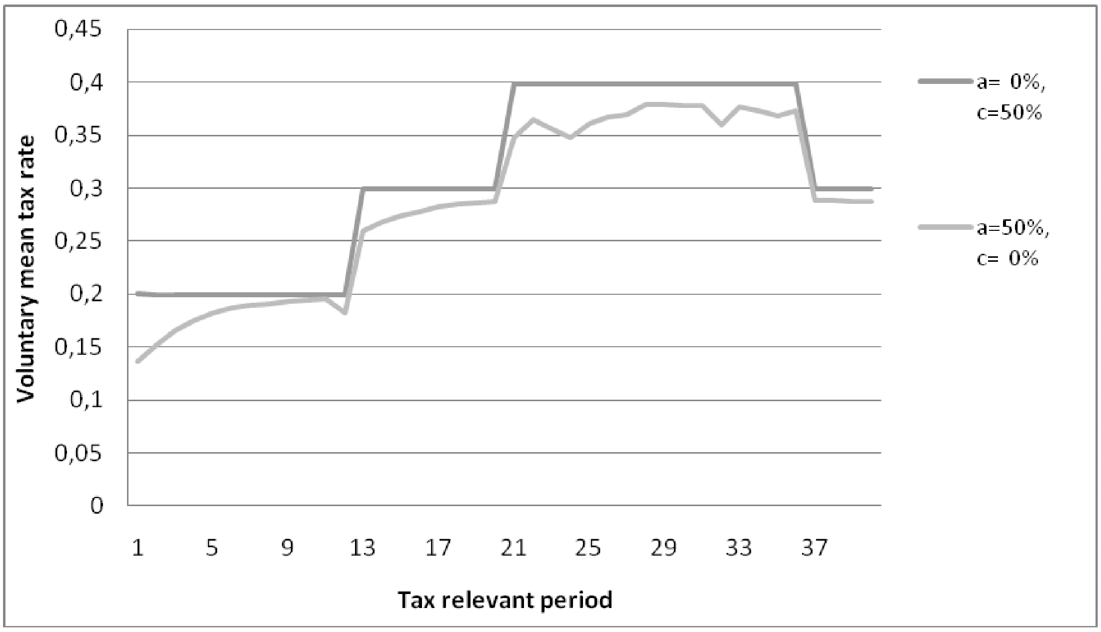
$$X=W+\sum_{k=t-\alpha}^{t-1}\frac{\pi_k}{\pi}\cdot(W_k-X_k)-\frac{ln\left(\frac{(1-p_s)\cdot\theta}{p_s\cdot(-\theta+\pi)}\right)}{\lambda\pi}. \tag{11}$$

A comparison of (10) with (5) shows that the upper limit of the interval is now also individualized and comparing (11) with (6) reveals that the middle term is added. Hence, the introduction of lapse of time effects leads to substantial changes with respect to the determinants of tax evasion behavior of maximizing a-types.

Figure 2 illustrates the impact of these behavioral changes with respect to the voluntary mean tax rate, but for matters of simplicity and readability we display the voluntary mean tax rate for only two type distributions. As in Figure 1, the first line from above represents the benchmark with zero percent a-types and 50 percent c-types and the last line from above represents the 50 percent a-types and zero percent c-types distribution. Thus, Figures 1 and 2 are comparable.

Comparison of Figures 1 and 2, last line from above, shows that the lapse of time effect leads to a substantial reduction of tax evasion over the entire time span, except in the first period where the extent of tax evasion is the same. The reason is that according to (9) the lapse of time effect induces maximizing a-types to take the extent of their own tax evasion in up to ten past periods into account. Therefore, as of period two, a-types may declare part of their income and after some additional periods they may even declare their true income, subject to the relevant parameter set shown in Table 1 and their individual income and risk parameters. Moreover, as the extent of a-type tax evasion is gradually reduced over time, this behavior pattern is copied by the b-types, which further reduces the extent of tax evasion.

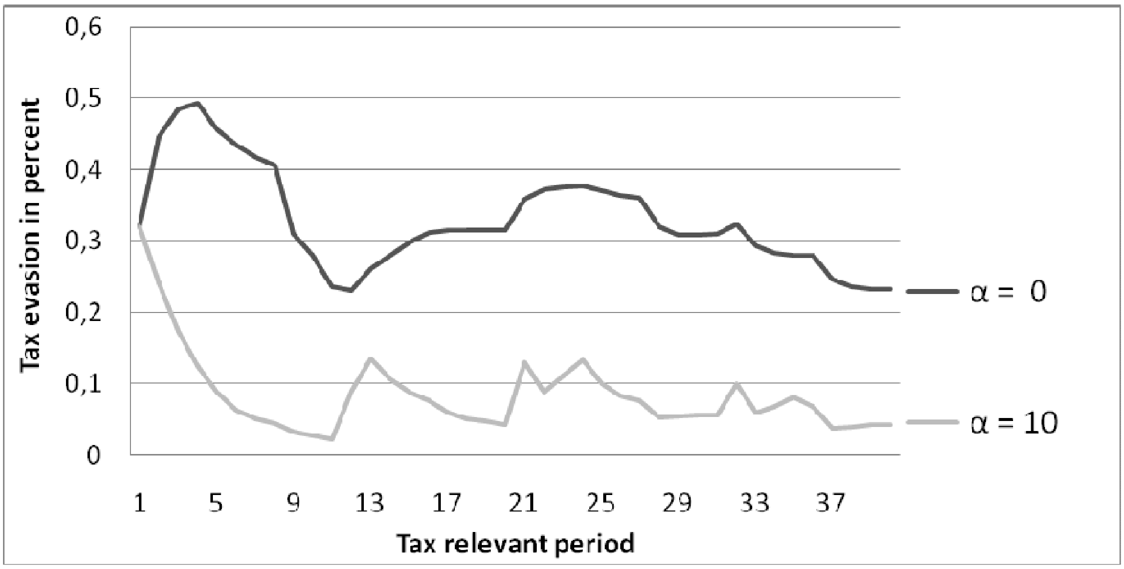
**Figure 2: Voluntary Mean Tax Rate of Scenario 2**




*Note: Seen from above the two lines denote the mean tax rate due to voluntary tax declarations ($\theta \cdot \Sigma X_i / \Sigma W_i$), subject to the relevant type distribution and the prevailing parameter set. Parameter values controlled by the government are provided in Table 1. All remaining parameter values are fixed for the 40 tax periods and assume the following values: a-type subjective audit probability updating $\delta$ = -*0.20, *b-type visibility $\nu = 4$, b-type mutation period $\tau = 4$.*

In fact, it follows from Figure 2 that the extent of tax evasion is reduced from 32.5 percent in period one to just ((20-19.5)/20=) 2.5 percent in period eleven. Thereafter, the decline of the voluntary mean tax rate (i.e., the increase in tax evasion) in period twelve is due to the fact that for a-types the initial period has dropped from the set of the relevant ten past periods. As for most a-types the initial period was characterized by tax evasion behavior, period twelve allows some a-types to evade taxes again. Like in the first scenario, the increase of $\theta$ in periods 13 and 21 leads *ceteris paribus* to more tax evasion, but with (30-27.5)/30=) 8.33 percent in period 16 and (40-35)/40=) 12.5 percent in period 24, the extent of tax evasion is much lower than in scenario one. Yet, in both scenarios the extent of tax evasion is *ceteris paribus* the highest where the spread between $\theta$ and $\pi$ is the lowest. Finally, by increasing this spread again, the government can eventually reduce the extent of tax evasion to just (30-28.75)/30=) 4.2 percent in period 40. Figure 3 illustrates the extent of tax evasion for the case of 50 percent a-types and no c-types of scenario 1 and 2 and, therefore, corresponds to the last line from above in Figures 1 and 2.

**Figure 3: Extent of Tax Evasion in Scenario 1 and 2**




*Note: The first line from above denotes the extent of tax evasion in percent with respect to the case of scenario 1 (α = 0), 50 percent a-types and no c-types (last line from above in Figure 1. Likewise, the second line from above denotes the extent of tax evasion in percent with respect to the case of scenario 2 (α = 10), 50 percent a-types and no c-types (last line from above in Figure 2).*

## 5 Concluding Remarks

In this paper we have simulated income tax evasion in a society of heterogeneous agents, where heterogeneity refers to different behavioral types, different individual risk-preferences and different taxable incomes. Within this society some agents maximize their expected utility via an exponential utility function. This modeling feature has important advantages. First, we can make rather realistic assumptions about the audit probabilities (Bloomquist 2006: 414). Second, we can incorporate lapse of time effects. To our best knowledge, this has not been done in any agent-based tax evasion model.

The simulation results presented in the preceding section by and large confirm the findings of previous theoretical, experimental and agent-based studies (e.g. Allingham and Sandmo 1972, Alm et al. 1992, Bloomquist 2006, Zaklan et al. 2009, 2008). In particular, raising (lowering) the audit probability or the tax rate on undeclared income or lowering (raising) the tax rate on declared income would *ceteris paribus* lead to less (more) tax evasion. However, *ceteris paribus* the magnitude of these effects may be rather small. In contrast, it turned out that the reduction of tax evasion by ethical behavior patterns and lapse of time effects may be rather substantial, subject to the underlying modeling features. Yet, if lapse of time effects are already incorporated, ethical behavior patterns seem to have much less influence. Besides, even at low audit rates, lapse of time effects may almost eliminate tax evasion, if the spread between the tax rate on declared and undeclared income is sufficiently high. In any case, our results call for the incorporation of long lapse of time periods. To this extent, the increase of the lapse of time period from five to ten years in Germany in 2009 should contribute to a substantial reduction of German income tax evasion during the foreseeable future.

Finally, it would be of interest to further adjust the model to real world situations. For example, the initial income distribution could be adjusted to allow for distributions with alternative gini coefficients, the initial income could be allowed to change over time to account for individual pay increases and decreases, and the code could be adjusted to allow agents to die and to be born, so that the agent population becomes heterogeneous with respect to age. However, these tasks rather delineate a future research agenda.

**References**


Allingham, Michael G. and Agnar Sandmo, (1972), "Income Tax Evasion: A Theoretical Analysis", *Journal of Public Economics,* 1, 323-338.

Alm, James, Gary H. McClelland, and William D. Schulze, (1992), "Why Do People Pay Taxes?", *Journal of Public Economics*, 48, 21–38.

Andreoni, James, Brian Erard and Jonathan Feinstein, (1998), "Tax Compliance", *Journal of Economic Literature*, 36(2), 818-860.

Bloomquist, Kim M., (2004), "Modeling Taxpayers' Response to Compliance Improvement Alternatives." Paper presented at the Annual Conference of the North American Association for Computational Social and Organizational Sciences, Pittsburgh, PA.

Bloomquist, Kim M., (2006), "A Comparison of Agent-based Models of Income Tax Evasion," *Social Science Computer Review*, 24(4), 411-425.

Bloomquist, Kim M., (2009), "Tax Compliance as an Evolutionary Coordination Game: An Agent-based Approach," mimeo, paper presented at Shadow Economy, Tax Evasion and Social Norms, July 2009, University of Muenster, Germany.

Brush, Stephen G., (1967), "History of the Lenz-Ising Model," *Reviews of Modern Physics*, 39(4): 883-895.

Davis, Jon S., Gary Hecht and Jon D. Perkins, (2003), "Social Behaviors, Enforcement, and Tax Compliance Dynamics," *The Accounting Review*, 78(1), 39-69.

Epstein, Joshua M., Robert Axtell, (1996), *Growing Artificial Societies*, Cambridge (MA): The MIT Press.

Kirchler, Erich, (2007), *The Economic Psychology of Tax Behavior*. Cambridge: Cambridge University Press.

Kirkwood, Craig W., (2004), "Approximating Risk Aversion in Decision Analysis Applications," *Decision Analysis*, 1, March, 51-67.

Korobow, Adam, Chris Johnson and Robert Axtell, (2007), "An Agent-based Model of Tax Compliance with Social Networks", *National Tax Journal*, 60(3), 589-610.

Kulakowski, Krzysztof, (2008), "A Note on Temperature without Energy – A Social Example," arXiv: 0807.0711v1 [*physics: soc-ph*] 4 Jul 2008.

Kulakowski, Krzysztof and Maria Nawojczyk, (2008), "Sociophysics – An Astriding Science," arXiv: 0805.3886v1 [*physics: soc-ph*] 26 May 2008.

Lima, Francisco W.S., (2009), "Analysing and Controlling the Tax Evasion Dynamics via Majority-Vote Model," arXiv: 0910.1964v1 [*physics: soc-ph*] 11 Oct 2009.

Lima, Francisco W.S. and Georg Zaklan, (2008), "A Multi-agent-based Approach to Tax Morale," *International Journal of Modern Physics C: Computational Physics & Physical Computation*, 19(12): 1797-1808.

Mirowski, Philip, (1989), *More Heat than Light: Economics as Social Physics: Physics as Nature`s Economics*, New York: Cambridge University Press.

Mittone, Luigi and Paolo Patelli (2000), "Imitative Behaviour in Tax Evasion," in Francesco Luna and Benedikt Stefansson (eds.), *Economic Simulations in Swarm: Agent-based Modelling and Object Oriented Programming*, Dordrecht, Boston, London: Kluwer Academic Publishers, 133-158.

Prinz, Aloys, (2010), *A Moral Theory of Tax Evasion*, Working Paper, Muenster: University of Muenster, paper presented at the 2010 annual meeting of the European Public Choice Society.

Schulz, Michael, (2003), *Statistical Physics and Economics. Concepts, Tools, and Applications*. New York, Berlin, Heidelberg: Springer.

Stauffer, Dietrich, (2008), "Social Applications of Two-dimensional Ising Models," *American Journal of Physics*, 76(4&5), 470-473.

Tesfatsion, Leigh and Kenneth L. Judd, (eds.), (2006), *Handbook of Computational Economics*, Volume 2, Amsterdam: Elsevier.

Zaklan, Georg, Francesco W.S. Lima and, Frank Westerhoff (2008), "Controlling Tax Evasion Fluctuations," *Physica A: Statistical Mechanics and its Applications*, 387(23), 5857-5861.

Zaklan, Georg, Frank Westerhoff and Dietrich Stauffer, (2009), "Analysing Tax Evasion Dynamics via the Ising Model," *Journal of Economic Interaction and Coordination*, 4, 1-14.