The Election of a World Champion[1]

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Abstract. This paper examines the mechanisms by which a World Champion is chosen in the Formula One Championship. Furthermore it is analysed whether there is a best method to do this. For this purpose we will discuss the methods used by the Fédération Internationale de l'Automobile (FIA) since the founding of the World Championship in 1950. We show how the election of a method affects the Formula One contest. We then give insight whether there is a best method to select a World Champion or not. We therefore discuss Arrow's Impossible Theorem with respect to this sports contest. Moreover we simulated several seasons and compared different scoring vectors with respect to indicators that might be important for viewer's demand.

Keywords: Social Choice, Aggregation rules, Ranking, Sport, Formula One

JEL-classification: D71 (Social Choice), L83 (Sports), C63 (Simulation Modelling)

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1 Introduction

The Formula One World Championship is a car race series with a long tradition. It started in 1950 and became one of the biggest sports events worldwide. During the last sixty years there were several rule changes regarding the organization of the competition between drivers and teams and with respect to the determination of the world champion. In the beginning of 2009 the Fédération Internationale de l´Automobile (FIA), which organizes the Championship, intended to change, among other things, the rules for choosing the Formula One World Champion.[3] This was not the first time the FIA modified the rules. Since 1950, there were actually four major changes. However the negative reaction to the suggested rule change was quite intense. The intention of the FIA was to modify the selection of the world champion via a scoring function and to install a rule where only the numbers of victories are decisive. The announcement of this concept started a public debate after some newspapers noted the fact that several of the past world championships would have had different outcomes if the new rules would had been employed.[4] The following protests were successful and the FIA first postponed the introduction of the new rules to 2010 and then skipped the plan altogether. Instead the FIA introduced a modification of the traditional scoring vectors.

However there are several questions that need to be answered: Why had the protest been so intense? How much would the rule change have mattered? The central question, though, is: Is there a best way to determine a world champion and if there is how does it look like?

There are several studies that did research on the topic Formula One. Kipker (2002) and Krauskopf, Langen and Bünger (2010) did research in television viewer's demand and Mastromarco and Runkel (2004) examined the relationship of rule changes and competitive balance. Furthermore Stadelmann and Eichenberger (2008) wanted to find the best driver of all time. But as far as we know there are no studies examining different ways of the world champion determination.

To analyse these question we examine in section 2 different scoring vectors and within a simple model we show in how many cases different scoring vectors would nevertheless produce the same world champion. To analyse whether there is a best way to determine the best driver we examine in section 3 the Arrow Theorem and apply it to the Formula One competition. We show that with some simple assumptions the only possible method in this sport contest is to select the world champion via a scoring function. In section 4 we analyse some aspects besides deciding the world championship that might have relevance for choosing an appropriate scoring vector. We analyse the rule

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[3] Further changes dealt with budget limits and technical limitations.

[4] Of course it is too simple to review a rule change by comparing what this rule would have done in the past without anticipating a behavioural change of the actors. This is common economic knowledge since Robert Lucas (1976).

change of 2010 and give some insight on how a best scoring vector could look like. The fifth section concludes.

2 Scoring vectors of the Formula One

A scoring function assigns for every alternative, depending on the place in a single ranking, a specific number of points and is aggregating several rankings to one final ranking by ordering via the total points of each alternative. This basic scheme can be modified in many ways. In the history of the Formula One World Championship six different scoring functions or more precisely scoring vectors were used.[5]

Table 1 shows all the vectors that were used, labeled with the year they were established. With the proposed but now rejected rules everything would have stayed the same except for the determination of the first place in the final ranking. For this purpose only the first places in each race would have been taken into account. This can be interpreted in terms of social choice as a simple plurality vote (Gaertner 2006) and the vector would look like a2010fia.[6] It is important to notice that the suggested vector should only determine the world champion. All the other places in the final ranking would have been determined by a2003. One reason for this could have been that a simple plurality vote in some cases does not result in a complete transitive order. This paradox was first described by Condorcet (1785) and occurred in the Formula One in 2002 (Soares et al. (2005)).

Table 1: Scoring vectors in the Formula One

Scoring vector	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
a1950	8	6	4	3	2	0	0	0	0	0
a1960	8	6	4	3	2	1	0	0	0	0
a1961	9	6	4	3	2	1	0	0	0	0
a1991	10	6	4	3	2	1	0	0	0	0
a2003	10	8	6	5	4	3	2	1	0	0
a2010	25	18	15	12	10	8	6	4	2	1
a2010fia	1	0	0	0	0	0	0	0	0	0
a2010fota	12	9	7	5	4	3	2	1	0	0

It is interesting, especially when changing from one scoring vector to another, in how many cases different scoring vectors deliver different aggregations. It is trivial that different scoring vectors can lead to different rankings, except if they are monotone transformations of each other. Saari (1984) has shown how many different rankings can be generated out of the same profile by using different

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[5] Until 1991 it was usual that not all results were counted. Until 1959 the driver with the fastest lap in a race earned an addition point. As we are interested only in the difference the vectors make we are subtracting from these facts.

[6] The Formula One Teams Association (FOTA) suggested the a2010fota vector as an alternative for the a2010fia in the discussion of 2009.

scoring functions. Some further remarks on this topic in the Formula One came from Kladroba (2000). In the following we are examining only in how many cases different scoring vectors are leading to the same world champions applied to the same season outcome.

For a small number of different outcomes this is quite easy and the simpler problems can be solved by using pen and paper. But in the case of the Formula One the numbers are far too big. With m drivers and n races in the season there are $(m!)^n$ possible different outcomes. By only examining the p places where points can be earned the number slightly reduces to $\dfrac{m!}{((m-p)!)^n}$. Taking the values of the 2009 season with $m=20, n=17$ and $p=8$ we get approximately $9,96*10^{164}$ different possible outcomes and this is far too great to calculate.

To measure the differences between two scoring vectors we therefore tested them with random samples. First we created a random outcome of one race and by doing this n times, we got a random season. Then we used the two scoring vectors we wanted to compare to determine the world champion for that simulated season. We simulated 1000 seasons determining the fraction of concordant world champions and repeated this 1000 times. We implicitly assume that every season outcome has the same possibility, i.e. all drivers are equally strong. We did this to give insight in all theoretical possible outcomes, not to give a realistic model.

Table 2: Descriptive Statistics of the comparison of a2003 and a2010fia

| Mean	Median	Max	Min	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Observations
0.529	0.529	0.584	0.476	0.016	-0.047	3.052	0.483	1000

According to Table 2, which shows the descriptive statistics of the comparison between the vectors a2003 and a2010fia, the results are normally distributed. This is also confirmed by the significant low value of the Jacque-Bera test. The central limit theorem states that the mean of a sufficiently large number of independent random variables each with finite mean and variance will be approximately normally distributed. So our sample is sufficiently big enough and in Table 3 we are only reporting the means of our observations.

Table 3: Comparison of the vectors used in the Formula 1

	a1960	a1961	a1991	a2003	a2010	a2010fia
a1950	0.926*	0.894	0.849	0.766	0.741	0.542
a1960	1	0.928*	0.872	0.809	0.775	0.555
a1961		1	0.941*	0.772	0.759	0.616
a1991			1	0.7301*	0.731	0.661
a2003				1	0.908*	0.529
a2010					1	0.367
a2010fia						1

Comparing all the vectors ever used in the Formula One there are several interesting properties. First of all, the values marked with an asterisk in Table 3 are the ones where a switch in the Formula One rules actually occurred. It is obvious that the rules suggested for 2010 would mark the biggest change in the history of the Formula One. Only in 52.9% the vector a2003 and a2010fia deliver the same world champion. The second biggest change in the history was in 2003 with 73% accordance to the previous year's vector. The vector actually introduced in the beginning of 2010 has accordance with the previous vector of 90.8%. The changes before 2003 were not only small in the degree of the vector change but also in the effects these changes have onto the world champion decision.

**Figure 1: Accordance of a2003 and a2010fia depending on the number of drivers and races**
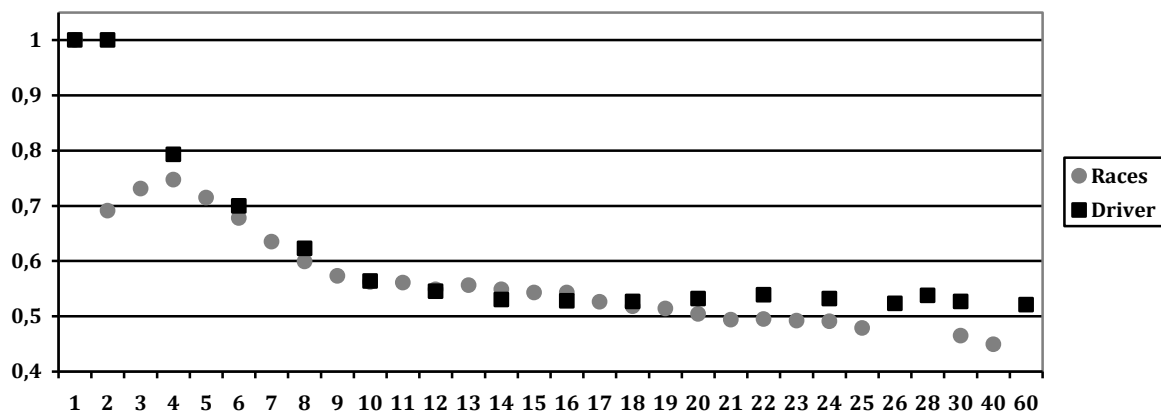
As the numbers of drivers or races per season differ from time to time we tested the differences of the a2003 and a2010fia for alternative numbers of drivers and races. We can see in Figure 1 that for two drivers the vectors do not differ at all, which is not surprising considering that ties are decided with the respective other vector. After this the compliance steadily declines to a value of around 0.5 and with more than 11 drivers does not seem to alter anymore.

The picture is slightly different for alternative numbers of races. For one race the value is of course 1, has a second peak at 4 and after this is constantly going down. Taking a Formula One season with more than four races for granted we can say that more races mean more different results with these two vectors.

Looking at the discussion of introducing a new point system at the beginning of the 2009 season it was common sense to strengthen the importance of the first place. The FOTA suggested a vector with a raise of 2 points for the first place. This is not as radical as making only victories count but

raises the question of how many additional points for the first place in the a2003 are necessary to obtain the same result as the a2010fia.

Figure 2: Accordance of a2010fia and modified versions of a2003
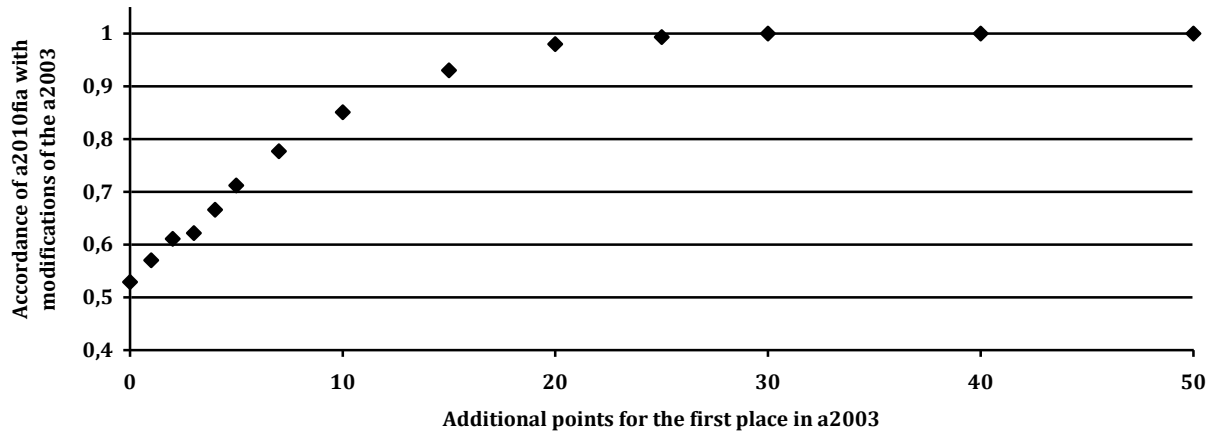
We compared the a2010fia and a2003 and gave additional points for the first place in the a2003. In Figure 2 we can of course see an increase which converts to 1, which means total conformity. It reaches total conformity with 30 additional points for the first place, hence 40 points in total. However also with 10 points, only twice as much as in a2003 amount, we get conformity of around 85%. The discussion of 2009 seems less dramatic from this perspective.

3 Some Background from Social Choice Theory

After showing the differences the vectors of the Formula One have made in the election of a world champion we now try to identify a best method. Since there are several parallels between the determination of a winner in a Formula One season and the aggregation of preferences, social choice theory may provide some insights.[7] Aggregating the outcomes of all races into the determination of a world champion is similar to the aggregation of different preferences into the choice of a collective action.

Arrow (1951) mentioned four preferable conditions an aggregation function should comply with and he furthermore showed that no aggregation rule could simultaneously meet all four requirements. The four conditions are the unrestricted domain condition, the weak Pareto principle, the independence of irrelevant alternatives and non-dictatorship. The unrestricted domain condition re-

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[7] For this we are subtracting from the fact that there might be changes throughout the season like strategy, driver´s form etc..

quests that no individual preference ordering should be excluded *a priori*. If all individuals prefer one alternative with respect to another, the weak Pareto principle implies that this preference should also hold for the aggregated preference. The independence of irrelevant alternatives declares that the comparison of two alternatives should only depend on these two alternatives. The non-dictatorship condition demands that no individual should be able to determine the result of the preference ordering no matter how the other individual's preferences look like.

These conditions are to be applied to a Formula One season. To have a "fair" championship there should not be the possibility of excluding any theoretical possible outcome of the championship *a priori*. This is certainly true for the championship. If the non-dictatorship condition was violated there would be one, and only one, race that determines the season's ranking making all other races irrelevant. This can not be in the interest of the FIA so this condition should not be violated. In fact an even stronger version of non-dictatorship holds, called anonymity (Gaertner 2006). Anonymity means, that all races in the season are equally important for the final ranking. It is easy to see that also the weak Pareto principle should be satisfied. As there are only strict preferences in each race with the weak Pareto principle, a driver who is always better than another driver should also be better in the final ranking.[8] In combination with the other conditions the weak Pareto principle leads to neutrality. This means that by changing the positions of any two drivers in every race the same change will occur in the championship ranking. At first sight it should be clear that if these conditions are not met an aggregation rule would neglect fundamental intuitions about fairness in a sports contest. As shown by Arrow no rule can satisfy all conditions mentioned above. Therefore, all feasible aggregation rules employed by the Formula One will violate the independence of irrelevant alternatives. The comparison of two drivers in the final ranking will depend on the performance of the other drivers.

Apart from the conditions mentioned by Arrow, an aggregation rule in sport competition should also satisfy the condition of consistency. That means that drivers who are in the best set of every subset of the season, should also be in the best set of the whole season. Considering these fundamental rules, Young (1975) stated the characteristics of the aggregation rule: "A social choice function is anonymous, neutral and consistent if and only if it is a scoring function". If it is not possible to satisfy all preferable conditions, a more or less pronounced balance between these conditions should be found. By declaring some conditions to be more important than others, the only stable solution to

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[8] This might not apply for those ranks in the races where zero points are earned. This can be disregarded when only the top positions in the championship are considered.

aggregate the results of the races is a scoring function. And as we have seen, since the founding of the Formula One championship, this has been exactly the way the FIA aggregated the results.

4 Is there a best scoring vector?

The Arrow Theorem shows that the only adequate method is to aggregate the results via a scoring function. This leaves the question for the appropriate scoring vector. There are endless different possibilities how a scoring vector can be constructed. Some remarks for scoring vectors in sports contests came from Petigk (1990). In the Formula One six different scoring vectors were used. To answer the question whether there is a best scoring vector we first need an aim the scoring vector should be best in. The first but not the only goal is of course the selection of the season's best driver. But even the definition of the best driver depends highly on normative considerations. Is a constant driver better than a driver with a higher variance in places but more top results? Furthermore the selection of the best driver might be the most important but not the only objective the scoring vector should accomplish. The FIA, as the organizer of the world championship, could have a lot of alternative aims. We assume that the FIA is an organisation with the goal of profit maximisation. In the end the revenues the FIA can generate are highly depending on spectators' interest. Spectators' interest is generating direct revenues and indirect via commercial revenues. So we think the objective goal of the FIA is the maximisation of spectators' interest. A lot of the variables which are used to describe spectators' interest are based on the concepts of fairness and suspense or uncertainty of outcome (e.g. Simmons (2009)).

In the following we again simulate Formula One seasons and adopt different vectors to generate rankings. We then compare the results of these aggregations and show how the scoring vectors affect variables that are important for the viewer's demand. In contrast to the simulation in section 2 we now used different winning probabilities for the drivers. We assign for the drivers, who can also be interpreted as combinations of drivers and cars, units of talent. The talent units of one driver divided by the sum of all drivers' talent units is his winning probability. The complete assignment of talent units is a talent distribution. We used six different talent distributions to compare the vectors and the particular talent units per driver can be seen, along with the Gini coefficient of these distributions, in Table 4.

Table 4: Talent distributions

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Gini
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
2	4	3	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.204
3	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0.317
4	10	6	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.397
5	10	9	9	8	8	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.499
6	1024	512	256	128	64	32	16	8	4	2	1	1	1	1	1	1	1	1	1	1	0.844

In the first distribution every driver has one unit of talent which means that the possibility of every place is the same for every driver. In the second distribution driver 1 has four units of talent, driver 2 three units and so on. In the third distribution talent is constantly decreasing. The forth distribution is a more extreme version of the second distribution. In the fifth distribution we modelled three teams that are stronger than the rest and with a slight inequality in drivers' talent. In the sixth distribution, beginning with the tenth best driver, the drivers have the double amount of talent units compared to the next best driver.

A race simulation, for example with the second distribution, is calculated in the following way. We begin with the total talent population.[9] From this population a driver is chosen randomly, represented by one number. This is the winner of the first race. After this we removed all the other talent units of this driver in case there are any left. Then we are repeating this for all places until we have a whole race outcome. By repeating this *n* times we get a season existing of *n* races. For every value we present in the following, 1000 seasons were simulated

Next we have to choose variables that are proxies for viewer's attention. We assume that the longer a world championship is undecided the more suspense it has. Another cause for suspense might be the total amount of different champions over a number of seasons. For some viewers it could be important that the best driver becomes champion. The competitive balance or uncertainty of outcome is frequently used in the sport economics to describe viewer's attention (e.g. Quirk and Ford (1992)). Usually competitive balance is measured with the Gini coefficient. We compared different scoring vectors according to how they transform the unequal distributed talent into a point distribution at the end of the season. Our last variable is the number of cases, where the vector is not able to decide the world championship because of ties.

In Figure 3 we can see the average number of races after which the season is decided. For the most talent distributions (except the 6th distribution) the vectors which were actually used in the Formula One do not differ to a great extent. The a2010fia compared to the actually used vector delivers for

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[9] The vector in this case would be: (1,1,1,1,2,2,2,3,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20)

some distributions longer and for some other shorter seasons. It is noticeable that the Borda[10] vector is generating the highest number of undecided races for all distributions.

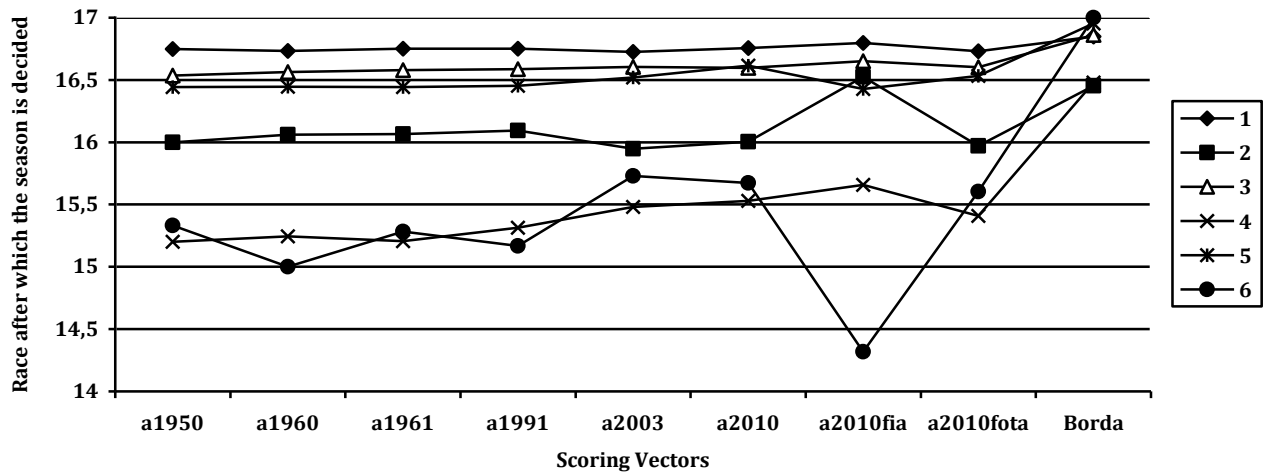**Figure 3: Average number of races after which a season is decided**

Figure 4 shows the number of different world champions the scoring vectors generate. For most of the cases the choice of a scoring vector does not matter as the number of different world champions does not change. Exceptions are the talent distributions 2 and 3 and the vector a2010fia. With both distributions the a2010 delivers more different world champions than the a2003 but less than the a2010fia.

Figure 4: Different Champions in 1000 seasons

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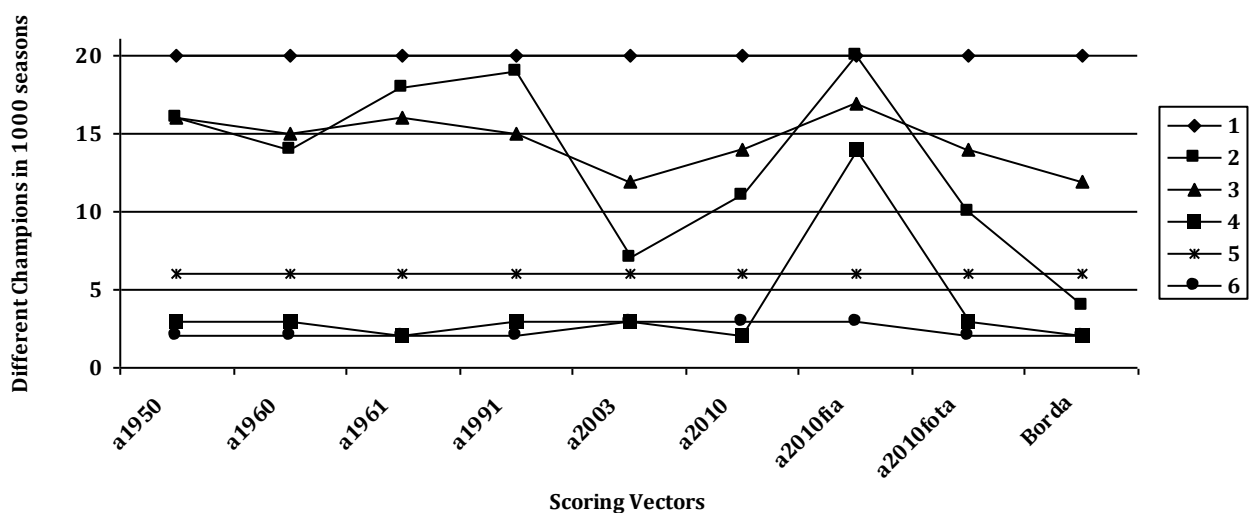
10 The vector is named after the French mathematician Jean-Charles de Borda. The vector assigns one point for the last place and for every better place always one point more.

Figure 5: How often is the best driver becoming world champion
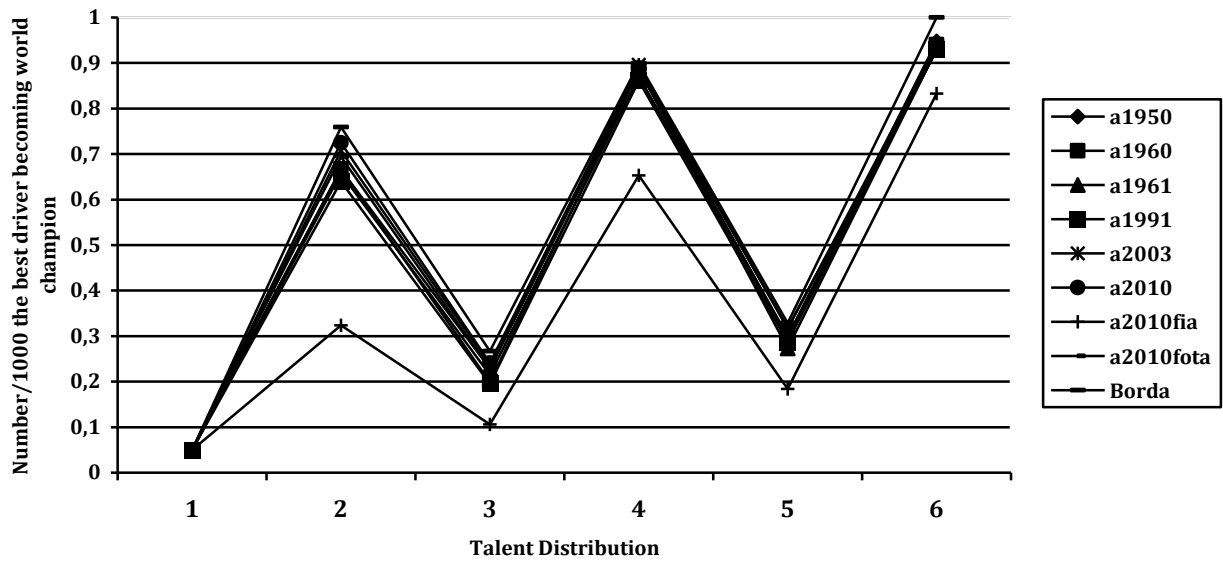
As mentioned above the definition of the best driver highly depends on normative considerations. In our simulations we therefore tested the possibility of the most talented driver becoming world champion. As we can see in Figure 5 nearly all scoring vectors deliver similar results. So in most cases the vectors are not determining the probability for the best driver becoming world champion. The one exception is the vector a2010fia. For all cases where there actually is a most talented driver the appliance of the vector a2010fia leads to a lower probability for the best driver becoming world champion.

In figure 6 the Gini coefficients of the different talent distributions were plotted against the Gini co-efficients of the point distribution in the final ranking. The graphs show how the vectors transform the inequality of talent into an unequal point distribution. On the 45° line the inequality of these two distributions is the same. We see that the highest inequality of scored points for every talent distri-bution, even with equally distributed talent, was produced by the a2010fia. The Borda scoring vec-tors has for every talent distribution the lowest corresponding Gini coefficient in point distribution. The second lowest coefficient for all talent coefficients has got vector a2010 and the third lowest a2003.

Figure 6: The effect of scoring vectors on competitive balance
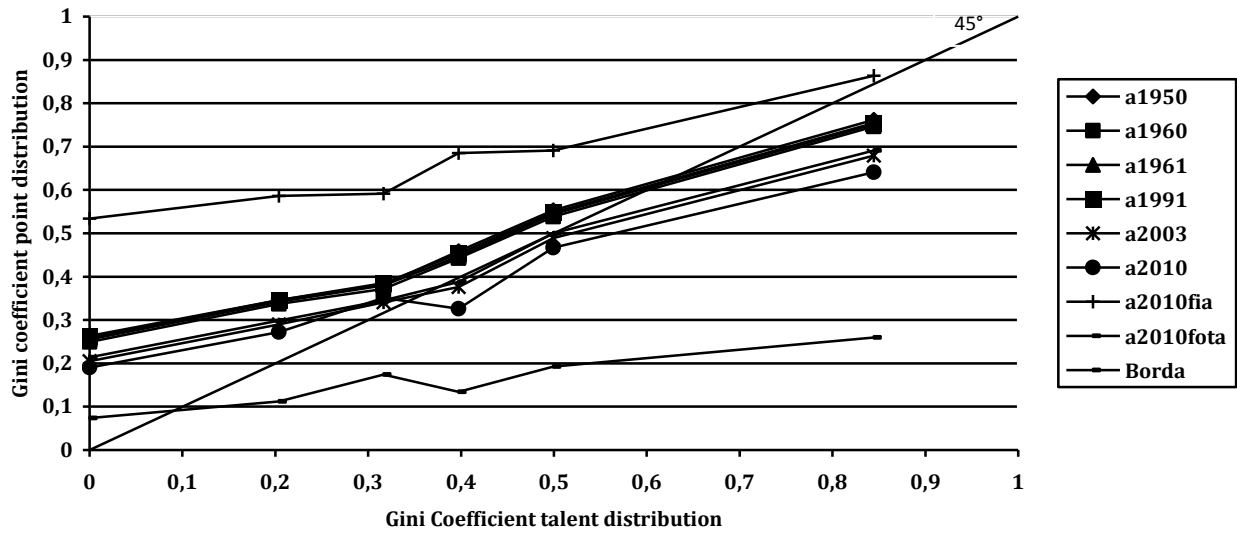
Every vector produces an unequal point distribution and the level of inequality depends, among other things, on the number of ranks the scoring vector is assigning point to. For this reason vector a2010fia, with only one point rank, always produces the highest inequality. The vectors which were used between 1960 and 2003 do not differ in the number of point ranks and produce nearly the same relation between the two Ginis. With the a2003, a2010 and the Borda vector the equality in the point distribution is rising with the number of point ranks.

Table 5 provides the fraction of 1000 simulated seasons a vector was not able to determine a world champion because ties occurred. The a2010fia has always got the highest fraction whereas the Borda has got the lowest in most of the cases. Comparing the a2003 and a2010 the latter is better in deciding world championships.

Table 5: Fraction of cases the vector is unable to decide the world championship

	a1950	a1960	a1961	a1991	a2003	a2010	a2010fia	a2010fota	Borda
1	0.082	0.071	0.085	0.074	0.061	0.028	0.485	0.056	0.034
2	0.029	0.017	0.028	0.015	0.021	0.007	0.306	0.017	0.012
3	0.055	0.054	0.043	0.037	0.038	0.021	0.363	0.032	0.023
4	0.014	0.016	0.022	0.016	0.008	0.006	0.161	0.009	0.012
5	0.032	0.046	0.047	0.036	0.044	0.019	0.304	0.029	0.049
6	0.007	0.010	0.013	0.003	0.010	0.005	0.062	0.007	0.012

5 Conclusions

In this paper we have shown that the introduction of a majority vote in the election of a Formula One world championship would indeed have been the biggest change in the history of different scoring systems in the Formula One. We have shown with the Arrow Theorem that the only possible method to select a world champion in the Formula One is by aggregating the races via a scoring function. The questions whether there is a best scoring vector or not cannot be answered in general because it highly depends on normative considerations. We assumed that the FIA is a profit maximizer and therefore wants to maximise the viewer's demand. Thus we compared different scoring vectors for different talent distributions according to variables that are considered to be important for viewer's demand.

The vector the FIA introduced in 2010 does not affect the average real season; the one suggested at first would have had a bigger impact for this variable. The new vector also does not change the probability of the most talented driver becoming world champion. But for most distributions the vector actually used in 2010 raises the number of different world champions. It also produces for all talent distributions a lower inequality in the point distribution. Altogether the introduction of the new vector seems to be a slight improvement in the variables we choose to analyse.

In general we can say that vectors with a higher number of point ranks generate longer undecided seasons. This leaves less interesting races at the end of the season. These vectors also generate lower inequality in the point distributions and thus a higher competitive balance. On the other hand fewer point ranks make the success of an underdog more realistic. Thus such vectors generate a higher number of different champions. To establish a best scoring vector more research on the specifics of viewer's demand is needed. Future studies should also examine the suspense in the races or in other words the fight for better positions in a race and this effect on viewer's demand. Our study shows that the decision for a longer or shorter point vector has positive as well as negative effects on viewer's demand. Therefore empirical studies of the importance of different factors for the viewer's demand may provide the solution for a best scoring vector.

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