How Disagreement about Social Costs leads to Inefficient Energy Productivity Investment*

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Abstract

Public energy productivity investment influences the amount of future energy consumption. If a present government expects its successor to value the social costs of fuel usage differently, this adds a strategic component to its investment considerations.

We analyze this governmental time-inconsistency situation as a sequential game. In particular, we show how the expectation of a more conservative party taking over makes a "green" government choose an investment level that is inefficient in that neither of the parties would prefer it to the investment level of a permanent green government. Under some circumstances, the opposition would even prefer the government to stay in power for sure: The gain of avoiding strategic investment then outweighs the loss of not being able to regulate energy consumption. We also analyze welfare gains of binding agreements.

Keywords: political economics, environmental policy, energy policy, energy efficiency, rebound effects, energy externalities, strategic investment, sequential games, time-inconsistency

JEL: Q48; Q58; Q55; D72

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1 Introduction

What determines a government's investment in energy productivity?[1] If a social planner was in power, the answer would be simple: She would equalize the investment's marginal cost and the marginal gain in social welfare, knowing that for every level of productivity, social welfare in turn will be maximized by an amount of energy that equalizes its marginal benefits and its marginal social costs. If energy usage has high social costs – due to externalities or due to high private costs – then the planner will invest more in case higher productivity reduces fuel consumption, whereas she will be less investment-prone if productivity stimulates fuel use.[2]

However, in a democracy the governing party has to take into account that the successive government's preferences towards environmental policy might be different from its own. In this article we show how such a valuation disagreement may lead to inefficient investment: If higher energy productivity reduces fuel use, a "green" government will not only invest more than the "conservative" – less environmentalist – opposition would want, but in anticipation of a conservative successor government it expands investment even further away from both parties' preferred investment levels.

The reason for this inefficiency is the dual role that public energy productivity investment plays in a democracy. First, it raises future benefits from energy usage, no matter which government will rule then. Second, it influences the amount of energy usage the future government will choose. If the next government is expected to have different preferences than the current one concerning the valuation of energy's social costs – for example because it assesses the cost of energy externalities differently –, then the second effect of energy productivity adds a strategic motive to the investment considerations. This leads to inefficient investment to the effect that even the conservative opposition would prefer a green "dictator's" investment level to that of a green

[1] In this article, "energy productivity" describes the technical rate with which physical energy sources translate into economically useful energy or "energy services" providing utility – for example, higher energy productivity means that a given amount of fuel or electricity produces more hours of a warm living room. The literature uses the terms "energy efficiency", "engineering efficiency" (Brookes, 2000), "technical" or "thermal efficiency" (Sorrell and Dimitropoulos, 2008). We use "productivity" instead of "efficiency" to avoid confusion as we classify the amount of investment as efficient or inefficient.

[2] Both reactions are possible depending on demand elasticities – see Saunders (2000) and other texts on "rebound effects" mentioned below.

government that expects to be voted out. In extreme cases, the opposition would even prefer the government to stay in power to the chance of being elected instead and then being able to regulate the economy's amount of energy consumption.

The economic intuition behind the green government's behavior is that it reckons that its successor will undervalue social costs (e.g., the successor disregards the importance of external effects) and so it has to trade off abating them against staying with its originally preferred amount of investment. This makes it rational for the government to overinvest today. Thus, the additional investment costs can be interpreted as a damage containment premium. In other words, the time-inconsistency of the government's preferences leads to strategic overinvestment.

Having so far concentrated on a green government that *over*invests if higher productivity reduces fuel use, the question might arise whether a conservative government similarly *under*invests. As we will show in our analysis, the opposite is true: Anticipating a greener successor also leads a conservative government to invest more than it would if it had a guarantee to be re-elected – if the conservatives cannot directly raise future fuel use, then they can at least, via higher productivity, raise the benefits from the fuel use their successor chooses. So both parties overinvest if they are going to lose power. Nonetheless, the conservative party will still not invest as much as the greens would want, which makes the inefficiency less clear: Even though there is a strategic motive that influences investment, democracy at least leads to a middle course.

However, both characterizations only apply if productivity is "energy-saving", that is, it reduces fuel use. By contrast, in situations where productivity increases fuel use instead of reducing it – so-called "backfire" – the behavior is mirrored: A government anticipating a successor with different preferences will *under*invest, no matter whether the successor will be greener or more conservative.

In this article, we analyze the choice of energy productivity investment as a sequential game of two parties acting in two periods: The government in the first period may invest in energy productivity, whereas the second-period government chooses the level of the country's fuel usage, taking energy productivity as given.[3]

[3]To focus on the strategic behavior, it is deliberately left exogenous which party is in power. One possible justification may be that parties not only differ in policy preferences, but, for example, in politicians'

We will proceed as follows. Section 2 provides a short discussion of related literature. In Section 3, we lay out the model by introducing terms, objective functions, basic optimality conditions, and some simplifications and restrictions we use. In Section 4 we discuss the solution of our model by first showing how a government chooses productivity investment when it knows it will stay in power, then deriving the investment of a government that knows it will be replaced. We show that due to the strategic behavior, a conservative opposition in some cases is even better off if the green government stays in power instead of knowing it will be replaced. We then show potential improvements through binding agreements. Finally, Section 5 summarizes, discusses the findings and suggests directions for future research.

2 Literature

Limiting primary-energy consumption to reduce externalities – both local pollutants like SO_2 and greenhouse gases – is a topic of intensive political debate. In this context, raising energy productivity is sometimes promoted as a "free-lunch" policy which makes everybody better off and reduces energy consumption at the same time. This dubious idea has been widely discussed under the headings of "Jevons' Paradox" (cf. Alcott, 2005), "Rebound effects", or "Khazzoom-Brookes postulate" (cf. Saunders, 1992, 2000, or Herring, 1999) or the term "backfire" used above – our analysis adds a political-economy dimension to the productivity-policy skepticism.

Our analysis is also closely related to a branch of the political-economy literature, namely to the literature analyzing time-inconsistency problems of governmental actions, and the most directly comparable articles are those explaining "strategic debt accumulation" (cf. Persson and Svensson, 1989, and Alesina and Tabellini, 1990, or Persson and Tabellini, 2000 for a survey of that literature). In these models, governments accumulate an inefficiently high level of debt with the objective of binding the hands of the expected next government. However, as Persson and Svensson (1989) point out, any state variable that influences a successor's decisions can play a similar

popularity: If one popular candidate has brought her party to power, but has lost popularity afterwards, this may make the party anticipate a defeat in the next election.

role. In this article, we examine the effects of considering the level of energy productivity as such a state variable. Knowing how strategic considerations influence energy policy is getting increasingly important as politics is strongly concerned with reducing fuel use, as views on the appropriate level of internalization differ, and as political interest groups try to drag policy into their preferred direction.

This idea of binding a future government has been applied to environmental issues by Marsiliani and Renström (2000) who analyze tax-earmarking, that is, dedicating receipts of taxes to certain kinds of government expenditures. As another instrument for overcoming time-inconsistency, Abrego and Perroni (2002) analyze investment subsidies for pollution abatement in detail. Brunner et al. (2012) survey these articles and other literature on commitment problems in climate policy.

In our model, time-inconsistency in the strict sense does not arise: If the government's preferences stay the same, it will not want to change plans. Therefore, heterogenous preferences are central to the model, so that it relates to other models of conflict of interest in an environmental-policy context. An example is Brandt (2004) who, among other things, applies the theory of interest groups of Becker (1983) to environmental policy choice. Another example is Aidt (1998), who uses the Grossman and Helpman (1994) common agency framework for environmental policy. However, as we do not model interest groups, the political structure of our model is more similar to pure partisan models.

Finally, even though in our model it is possible to interpret the parties as one representing "true" social costs and the other some wrong understanding of social costs, we do not have to. If, for example, some people discount the future so much that they regard climate policy as unnecessary, while others regard it as important (Kirchgässner and Schneider, 2003), then even under certainty about the effects of climate change the degree of "externality" of the first group's behavior depends on preferences only.

3 The model

3.1 Agents, objectives, and temporal structure

We analyze the behavior of two political parties in two periods. A party's (or its members') utility equals its valuation of society's total welfare,[4] which is derived from energy services' social costs and benefits depending on fuel consumption and energy productivity investment. In each period the governing party determines the value of one variable: The first-period government chooses investment which determines the level of energy productivity one period later, whereas the second-period government chooses fuel consumption. The second-period optimal choice depends on the level of productivity.

Productivity investment in turn is chosen taking account of its impact on fuel consumption: If the second government's perception of the social costs of fuel usage differs from the first's, then the level of productivity plays a strategic role as it may drag the future government's behavior nearer to the first government's optimum. Our aim is to deduce to what extent strategic investment of the first period's government influences each party's utility.

Our model's parties are indexed by $i \in \{x, y\}$. They do not discount, so party utility W^i is the unweighted sum of period welfare w_t^i for the time periods $t \in \{1, 2\}$:

$$W^i = \sum_{t=1}^{2} w_t^i. \tag{3.1}$$

The first period is the investment period in which a non-negative magnitude of investment expenditure T is chosen. It is financed by a lump-sum tax. So period-1 welfare w_1^i is given by:

$$w_1^i = w_1 = -T \qquad\qquad i \in \{x, y\}, \tag{3.2}$$

which is independent of party preferences: Both parties value the costs of investment alike.

[4] The analysis can be understood as discussing the behavior referring to one sector or to a whole economy.

In the second period, the economy uses a non-negative amount of a homogeneous fuel F. Depending on the level of energy productivity, fuel consumption generates an amount of energy services E: $E(F, \tau) \equiv \tau \cdot F$. These energy services produce benefits $B(E)$ for the economy. We assume positive and diminishing marginal benefits of energy services ($\partial B/\partial E > 0$, $\partial^2 B/\partial E^2 < 0$) and the usual requirements for an interior solution ($\lim_{E\to 0} \partial B/\partial E = \infty$, $\lim_{E\to\infty} \partial B/\partial E = 0$).[5]

Both parties value benefits alike. In contrast, the parties differ in the perception of fuel consumption costs. The cost function is given as $K^i(F) \equiv \kappa_i \cdot Z(F)$, where marginal costs are globally non-negative and finite for finite values of F, and the function is convex ($\infty > Z \geq 0$ for $\infty > F \geq 0$, $Z > 0$ for $F > 0$, $\partial Z/\partial F \geq 0$, $\partial^2 Z/\partial F^2 \geq 0$). So the perception of costs differs between the parties depending on the difference in a weight parameter $\kappa_i > 0$. For example, the setting could be that one party values private costs only while the other one values both private costs and external effects.

Party i's valuation of period 2 welfare equals the difference between benefits and perceived costs:

$$w_2^i = B(E) - \kappa_i \cdot Z(F) \qquad\qquad i \in \{x, y\}\,. \tag{3.3}$$

Finally, the periods are connected as period-2 productivity τ is a function of period-1 investment. We write $T \equiv T(\tau)$ which determines investment cost as depending on the desired level of energy productivity. We assume the investment cost function to have the same properties as the fuel usage cost function ($\infty > T \geq 0$ for $\infty > \tau \geq 0$, $T > 0$ for $\tau > 0$, $\partial T/\partial \tau \geq 0$, $\partial^2 T/\partial \tau^2 \geq 0$).

As a consequence of the different perception of period-2 welfare, overall utility W^i also differs between the parties exactly by the w_2^i difference. It is now given by:

$$W^i(F, \tau) = \underbrace{-T(\tau)}_{w_1} + \underbrace{B(E) - \kappa_i \cdot Z(F)}_{w_2^i} \qquad\qquad i \in \{x, y\}\,. \tag{3.4}$$

[5] The τ, F, E concepts are borrowed from the energy efficiency literature; the variables follow Saunders (2008) who calls τ an "engineering efficiency parameter". The way productivity augments the energy services derived from a given amount of fuel use reflects the way energy productivity is commonly understood: For example, with better insulation, a certain amount of heating oil will produce a warm living-room for a longer time.

For any productivity level τ and for any fuel consumption F, the marginal utility of fuel, $\partial W^i/\partial F$, is lower for a higher κ_i: A "greener" party always takes costs into account whose existence (or relevance) a more "conservative" party does not acknowledge. Therefore, in our analysis these characterizations are relative: If $\kappa_x > \kappa_y$, then the x party is "green" and the y party is "conservative" while for $\kappa_x < \kappa_y$ the opposite is true.

The governments' choice variables are productivity (investment) τ in the first period and fuel use F in the second. The temporal structure reflects that the effects of investment are more long-run than governmental energy regulation. That we do not allow productivity de-investment in the second period can be justified by understanding productivity as representing either a stock of technological knowledge or specific capital goods.

3.2 Preferred investment in period 1

Even though the investment choice of the period-1 government depends on its impact on period-2 behavior, we will first derive the amount of investment that the period-1 government would choose if period-2 fuel consumption was exogenously given. This amount will later serve as a benchmark for efficient investment.

To maximize welfare as defined by equation (3.4), the period-1 government chooses τ according to the following first-order condition:

$$-\frac{\partial T(\tau^*)}{\partial \tau} + \frac{\partial B(E^*)}{\partial E} \cdot F = 0. \tag{3.5}$$

Here, τ^* denotes the level fulfilling this equation, while $E^* \equiv \tau^* \cdot F$ is the level of energy services derived from choosing productivity investment optimally when fuel consumption F is given. This condition defines the first-period government's preferred investment τ^* as a function of fuel consumption:

$$\tau^* \equiv \tau^*(F). \tag{3.6}$$

The government balances marginal investment costs in period 1 on the one hand and marginal benefits gained from augmenting the energy services of fuel consumption in period 2 on the other. As fuel consumption and, therefore, its costs are exogenously given by assumption, the productivity level preferred by the parties does not depend on their valuation parameter κ_i. To put it another way, in period 1 both parties would agree on τ^* if fuel consumption was exogenous. Given our assumptions about the functions, the second-order condition is always fulfilled.

How is τ^* affected by the amount of fuel consumption? To find out, we differentiate the first-order condition (3.5) with respect to τ and F, substitute the first-order condition again and rearrange, which yields the following elasticity:

$$\omega \equiv \frac{\partial \tau^*/\tau^*}{\partial F/F} = \frac{1-\beta(E^*)}{\beta(E^*)+\theta(\tau^*)} \tag{3.7}$$

where $\beta \equiv -E \cdot \left(\partial^2 B/\partial E^2\right) / \left(\partial B/\partial E\right)$ is the negative of the elasticity of marginal benefits with respect to energy services and $\theta \equiv \tau \cdot \left(\partial^2 T/\partial \tau^2\right) / \left(\partial T/\partial \tau\right)$ is the elasticity of marginal investment costs with respect to the productivity level. Our assumption of positive and diminishing marginal benefits of energy services and a convex cost function imply $\beta > 0$ and $\theta \geq 0$.

ω can be positive or negative depending on the sign of $1-\beta$. β is large if the benefit function shows strongly diminishing marginal benefits, so β represents the strength of satiation effects. If $1-\beta < 0$, the elasticity is negative: A larger amount of fuel consumption then reduces marginal benefits of productivity. Also, relatively strongly diminishing marginal benefits (a large β) or a relatively steeply rising marginal investment cost curve (a large θ) imply that the positive effects of raising τ are small; therefore, they make the denominator large and thus lower the elasticity.

3.3 Preferred fuel consumption in period 2

The period-2 government chooses fuel consumption F. We denote the government by $\lambda \in \{x, y\}$ so that we can, for example, analyze in later sections what different impact it has on party x's utility whether party x or party y is in power ($i = x, \lambda = x$

or $i = x, \lambda = y$). The λ government's optimal choice depends on the τ, which is predetermined, and its valuation of fuel use costs, κ_λ. To maximize its period-2 welfare as given by equation (3.3), the government chooses F to fulfill the first-order condition

$$\frac{\partial B\left(E_\lambda\right)}{\partial E} \cdot \tau - \kappa_\lambda \cdot \frac{\partial Z\left(F_\lambda\right)}{\partial F} = 0, \tag{3.8}$$

where F_λ denotes the level fulfilling this equation and $E_\lambda \equiv \tau \cdot F_\lambda$ is the level of energy services derived from choosing fuel consumption optimally when productivity is given. This condition defines F_λ as a function of fuel use cost valuation and productivity:

$$F_\lambda \equiv F_\lambda(\kappa_\lambda, \tau). \tag{3.9}$$

Given our assumptions about the functions, the second-order condition is always fulfilled.

We would expect that a "greener" government chooses less fuel consumption for any level of productivity which implies that the elasticity of fuel consumption with respect to the valuation parameter is negative. We derive the elasticity by differentiating the period-2 government's optimality condition (3.8) for a given level of τ, substituting the optimality condition again and rearranging:

$$\xi_\lambda \equiv \frac{\partial F_\lambda / F_\lambda}{\partial \kappa_\lambda / \kappa_\lambda} = -\frac{1}{\beta(E_\lambda) + \varphi(F_\lambda)}, \tag{3.10}$$

where the minus confirms the intuition. β is defined as it was in the preceding subsection and $\varphi \equiv F \cdot \left(\partial^2 Z / \partial F^2\right) / \left(\partial Z / \partial F\right)$ is the elasticity of marginal fuel usage costs. Our assumption of a convex cost function implies $\varphi \geq 0$, and as pointed out above $\beta > 0$ so that ξ_λ is always negative. Strongly diminishing marginal benefits or a relatively steeply rising marginal fuel usage cost curve lower the elasticity.

The dependence of F_λ on τ, by contrast, is just as ambiguous as the dependence of τ^* on F was shown to be in the preceding subsection. Differentiating (3.8) for a given cost valuation parameter κ_λ and rearranging yields the elasticity of fuel use with respect to productivity,

$$\eta_\lambda \equiv \frac{\partial F_\lambda / F_\lambda}{\partial \tau / \tau} = \frac{1 - \beta(E_\lambda)}{\beta(E_\lambda) + \varphi(F_\lambda)}. \tag{3.11}$$

Like ω in the preceding subsection, this elasticity can be positive or negative depending on the sign of $1 - \beta$. The reason is parallel as well: If $\beta > 1$, then satiation effects of increased energy productivity are so strong that marginal benefits of fuel use are reduced so that optimal fuel consumption is reduced as well.

3.4 Terminology and simplifying restrictions

Following from the way we have specified the benefit function, the effects of a change in fuel consumption on productivity's marginal benefits and the effects of a change in productivity on marginal benefits of fuel use are parallel. The impact of the latter relation has been discussed in the energy economics literature under the label of "rebound effects". Loosely following Saunders (2008), we label cases of negative elasticities $\eta_\lambda < 0$, $\omega < 0$ as "energy-saving" and cases of positive elasticities $\eta_\lambda > 0$, $\omega > 0$ as "backfire".[6] Though we analyze both cases in the following, we assume that for a given economy, β is bounded away from unity either from below ($1 > \beta > 0$) or from above ($\beta > 1$), so we will discuss energy-saving benefit functions and backfire benefit functions separately. The functions τ^* and F_λ are thus assumed to be strictly monotonic and invertible. As the sign of η_λ and the sign of ω both depend on the sign of $1 - \beta$, we apply the terms "backfire" and "energy-saving" to both the fuel consumption function F_λ and the preferred investment function τ^*.

[6]For a further discussion of consumption reactions on productivity changes in a consumer-demand context, see Wirl (1997). Saunders (2000, 2008) defines the "rebound" of an economy as $R \equiv 1 + \eta$. A rebound of R means that $(1 - R) \cdot 100\%$ of technological productivity gains are translated into actual fuel conservation. Saunders (2008) distinguishes five cases: $R > 1$ is "backfire", $R = 1$ is "full rebound", $0 < R < 1$ is "partial rebound", $R = 0$ is "zero rebound" and $R < 0$ is "super-conservation". In our setting, the rebound is given by

$$R = 1 + \frac{1 - \beta}{\beta + \varphi} = \frac{1 + \varphi}{\beta + \varphi}.$$

and it must be positive; so "zero rebound" and "super-conservation" cannot occur. Also note that R is the elasticity of energy services with respect to productivity, so in our setting higher productivity always translates into more energy services used.

For the cost functions we assume:

$$\theta \geq 1, \varphi \geq 1. \tag{3.12}$$

As ω is bounded by $1/\theta$ and -1 and η_λ is bounded by $1/\varphi$ and -1, this assumption ensures $1 > \omega > -1$ and $1 > \eta_\lambda > -1$. We later show that these assumptions are sufficient (but a little stricter than necessary) for guaranteeing an interior solution in the two-period optimization, at least if the governments of both periods have equal preferences.

In the following section, we derive productivity investment τ in the different cases of a party being in power both in period 1 and period 2, and of a party being voted out after period 1. For brevity we call the first case, without further implications, a "dictatorship", and the second a "democracy". The democracy will be further split into first analyzing the time-inconsistent (or myopic or not subgame-perfect) case where the period-1 government invests as if it would stay in power and then analyzing the time-consistent case where the period-1 government anticipates the behavior of the period-2 government. Alternatively, the time-consistent result can be interpreted as representing a situation in which the period-1 government knows that it will be replaced whereas the time-inconsistent result implies that the government is replaced by surprise.

For the rest of the analysis, we standardize notation: x is the government party of period-1, so y is the period-1 opposition. Thus, $\kappa_x < \kappa_y$ means that the period-1 government is more conservative than its opposition whereas $\kappa_x > \kappa_y$ means that it is greener. Because the governing party of period 2 is denoted by λ, $\lambda = x$ defines a dictator, while in a democracy we have $\lambda = y$.

4 The government's investment behavior

4.1 The investment choice of a dictator

A dictator chooses a productivity level that is simultaneously consistent with optimal productivity given fuel consumption, $\tau^*(F)$, and the fuel consumption level preferred

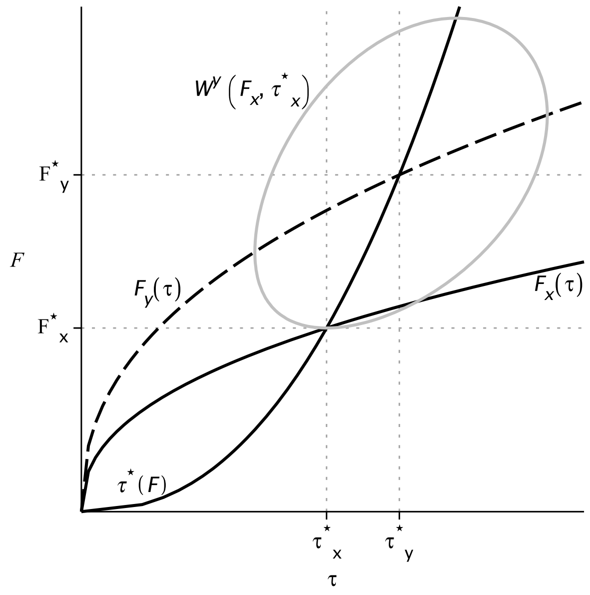

Figure 1: Investment determination in a dictatorship

by the x party given productivity, $F_x(\tau)$. The solid curves in Figure 1 show example $\tau^*(F), F_x(\tau)$ functions.[7] As the slopes of both curves are positive, the diagram represents a backfire case.[8] The curves intersect twice, but given our assumptions about benefit and cost functions, only a $\tau > 0$ solution can represent a utility maximum of the x party. We call this level τ^*_x with a corresponding level of fuel consumption F^*_x.

By contrast, the dashed curve represents the fuel consumption the y party would prefer. For every τ we have $F_x < F_y$ – which implies that in the example the x party values fuel consumptions costs higher than the y party, $\kappa_x > \kappa_y$. If the y party ruled instead of the x party in both periods, it would choose τ^*_y. At F^*_y, τ^*_y, the y party's utility attains a maximum. Additionally, the y party's indifference curve for the productivity level and fuel consumption actually chosen by the x-party government, $W^y(F^*_x, \tau^*_x)$, is plotted in gray.

Another way of analyzing the optimization problem is to note that in period 1 the x-

[7]All example figures are generated for constant-β benefit functions, constant-θ investment cost functions and constant-φ fuel consumption cost functions, which makes ω, η and ξ constant as well: $B(E) = b \cdot E^{1-\beta}/(1-\beta)$, $T(\tau) = k \cdot \tau^{1+\theta}/(1+\theta)$, and $K^i(F) = \kappa_i \cdot F^{1+\varphi}/(1+\varphi)$.

[8]Even if we had not assumed monotonic functions above, the two curves' slopes would have to be both positive or both negative in their intersection point: The sign of $1-\beta$ determines both slopes' signs.

party government knows that in period 2, it will choose fuel consumption as determined by the $F_x(\tau)$ curve; so this curve represents the period-1 government's feasible set of F, τ combinations. Substituting $F_x(\tau)$ into the x party's utility function given by (3.4) for $i = x$ and $F = F_x$ yields:

$$W^x(F_x(\tau), \tau) = \underbrace{-T(\tau)}_{w_1} + \underbrace{B(E_x) - \kappa_x \cdot Z(F_x)}_{w_2^x}. \tag{4.1}$$

This equation is the period-1 government's objective function. It bears investment costs in period 1 to raise welfare in period 2. The optimal investment choice has to fulfill the first-order condition $\partial W^x/\partial\tau = 0$:

$$-\frac{\partial T}{\partial \tau} + \frac{\partial B}{\partial E} \cdot \frac{\partial E_x}{\partial \tau} - \kappa_x \cdot \frac{\partial Z}{\partial F} \cdot \frac{\partial F_x}{\partial \tau} = 0. \tag{4.2}$$

We have $\partial E_x/\partial\tau = F_x + \tau \cdot \partial F_x/\partial\tau$. Substituting this and using the period-2 government's optimality condition (3.8) for $\lambda = x$, we derive:

$$\frac{\partial T(\tau_x^*)}{\partial \tau} = \frac{\partial B(E_x^*)}{\partial E} \cdot F_x^* \tag{4.3}$$

where τ_x^* is the level fulfilling this condition, $F_x^* \equiv F_x(\tau_x^*)$ and $E_x^* \equiv \tau_x^* \cdot F_x^*$.

The left-hand side of (4.3) shows marginal investment costs of productivity in period 1. The right-hand side shows marginal welfare gains of productivity in period 2 meaning marginal benefits of raising the energy services of optimally chosen fuel use. So (4.3) reflects the usual investment consideration of equalizing the investment's marginal costs and its marginal returns. As shown in Appendix A.1, for an energy-saving case the second-order condition always holds, while the assumptions in (3.12) are sufficient for the second-order condition to hold in a backfire case.

The optimality condition implicitly defines the optimal investment of an x-party dictator as a function of the party's cost valuation, $\tau_x^*(\kappa_x)$. To prepare the analysis of investment in a democracy, it is useful to derive whether a green or a conservative party would want more investment in period 1. We differentiate (4.2), varying κ_x and τ. After some rearrangements, this yields the following result.

Proposition 1. *The elasticity of the level of productivity investment that an x party dictator chooses in period 1, τ_x^*, with respect to its fuel use cost valuation parameter κ_x is given as follows:*

$$\psi_x \equiv \frac{\partial \tau_x^* / \tau_x^*}{\partial \kappa_x / \kappa_x} = -\xi_x \cdot \left(\eta_x - \frac{1}{\omega} \right)^{-1}, \tag{4.4}$$

where ξ_x, η_x and ω are evaluated at τ_x^*.

Proof: See Appendix A.2.

We know that $-\xi_x$ is positive. As shown in Appendix A.1, $\eta_x - \omega^{-1}$ is positive in an energy-saving situation and negative in a backfire situation. So as we should intuitively expect, preferred investment levels of both parties depend positively on their energy usage cost valuation if investment helps saving fuel (and vice versa).

A larger ξ_x means a higher sensitivity of fuel consumption with respect to κ_x so that a change of κ_x has a stronger effect. A large positive η_x means that fuel consumption expands a lot in reaction to higher productivity, which, due to rising marginal costs, limits the reaction of τ_x^*; a large positive ω implies that the optimal productivity reacts strongly to the reduced fuel consumption implied by a larger κ_x. The opposite reasoning applies to energy-saving functions.

However, for τ_x^* to be considered optimal by the x party, it obviously does not matter whether the x party really is in power in the first period – it only matters that it is in the second. More generally, $\tau_\lambda^*(\kappa_\lambda)$ is the level of investment the period-2 λ-party government finds optimal in period 1. From this follows proposition 2:

Proposition 2. *The elasticity of the level of productivity investment that the λ party considers optimal if it expects to be in power in period 2, τ_λ^*, with respect to its fuel use cost valuation parameter κ_λ is given as follows:*

$$\psi_\lambda \equiv \frac{\partial \tau_\lambda^* / \tau_\lambda^*}{\partial \kappa_\lambda / \kappa_\lambda} = -\xi_\lambda \cdot \left(\eta_\lambda - \frac{1}{\omega} \right)^{-1}, \tag{4.5}$$

where ξ_λ, η_λ and ω are evaluated at τ_λ^*.

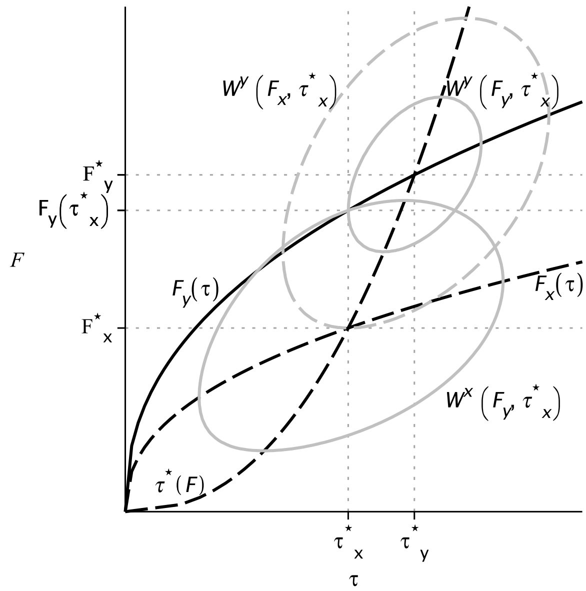

Figure 2: Investment determination in a democracy: Time-inconsistent investment

4.2 The investment choice in a democracy

Time-inconsistent investment We now suppose that the y party comes to power in period 2: $\lambda = y$, so that $\kappa_\lambda = \kappa_y$. If the investment level was still τ^*_x, the y party would be better off now and the x party would be worse off: being able to control fuel consumption must improve a party's situation if productivity is given.

We show an example in Figure 2. As assumed, the intersection of $F_x(\tau)$ and $\tau^*(F)$ still determines the investment level, τ^*_x – these curves are now mapped in a dashed mode as their intersection determines the effective F, τ combination only indirectly (they determine τ but not F). The amount of fuel consumption is chosen by the y-party government in period 2 and so the relevant curve for F is given by $F_y(\tau)$. With $F_y(\tau^*_x)$, the y party is able to reach a higher indifference curve, while the x party is drawn off of its utility maximum. The x party's utility level for $F_y(\tau^*_x)$ is also shown.

However, the figure also shows that τ^*_x cannot be the x party's optimal choice. The $F_y(\tau)$ curve describes the feasible set for the period-1 x-party government, and as this curve crosses the indifference curve of the x party, there must be points on this curve

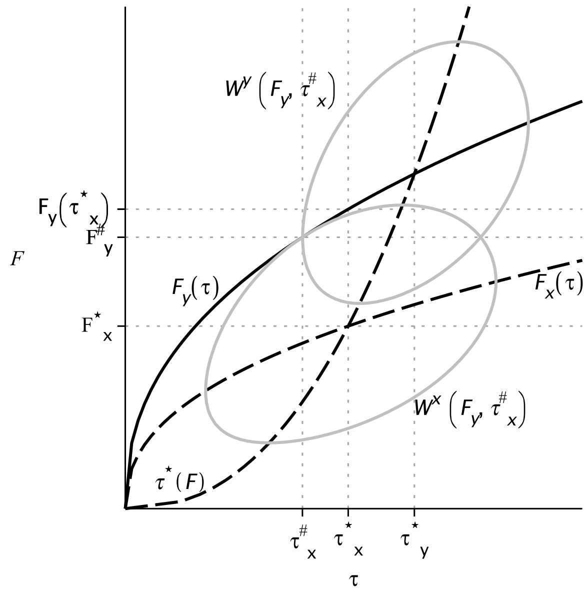

Figure 3: Investment determination in a democracy: Time-consistent investment

that give a higher utility. Put differently, τ_x^* is time-inconsistent in this situation; it would only be chosen by a myopic government.

Time-consistent investment In Figure 3, time-consistent investment of the first-period government is shown. We denote this investment level by $\tau_x^{\#}$. It is the τ level that maximizes the x party's utility given that $F_y(\tau)$ is its corresponding fuel consumption level. Obviously, in the figure this investment level is lower than τ_x^*.

The corresponding indifference curves for both parties are shown as well. For the x party, the F, τ combination of the time-inconsistent investment is outside of this indifference curve and so the time-consistent choice gives a higher utility. For the y party, the time-consistent equilibrium gives a lower utility level than the time-inconsistent $F_y(\tau_x^*)$, but higher utility than the x-party dictator's $F_x(\tau_x^*)$.

In the following analysis, we formally check under which circumstances these relations hold. To find the x-party government's investment, we again substitute the condition for the feasible set of F, τ combinations into the x party's utility function given by (3.4). This feasible set now is defined by $F = F_y$, so we get:

$$W^x\left(F_y(\tau), \tau\right) = \underbrace{-T(\tau)}_{w_1} + \underbrace{B\left(E_y\right) - \kappa_x \cdot Z\left(F_y\right)}_{w_2^x}. \tag{4.6}$$

First Order Condition The x-party government strives to equalize marginal costs and marginal welfare gains of investment. Marginal welfare gains, however, now depend on the y party's preferences. If an interior solution exists, the optimal productivity choice has to fulfill the first-order condition $\partial W^x/\partial\tau = 0$, which implies

$$-\frac{\partial T}{\partial \tau} + \frac{\partial B}{\partial E} \cdot \frac{\partial E_y}{\partial \tau} - \kappa_x \cdot \frac{\partial Z}{\partial F} \cdot \frac{\partial F_y}{\partial \tau} = 0. \tag{4.7}$$

Substituting $\partial E_y/\partial\tau = F_y + \tau \cdot (\partial F_y/\partial\tau)$, we have the following strategic optimality condition:

$$\frac{\partial T(\tau_x^{\#})}{\partial \tau} = \frac{\partial B\left(E_y^{\#}\right)}{\partial E} \cdot F_y^{\#} + \left[\frac{\partial B\left(E_y^{\#}\right)}{\partial E} \cdot \tau_x^{\#} - \kappa_x \cdot \frac{\partial Z\left(F_y^{\#}\right)}{\partial F}\right] \cdot \frac{\partial F_y(\tau_x^{\#})}{\partial \tau} \tag{4.8}$$

where $\tau_x^{\#}$ is the productivity level fulfilling this optimality condition, $F_y^{\#} \equiv F_y(\tau_x^{\#})$ and $E_y^{\#} \equiv \tau_x^{\#} \cdot F_y^{\#}$. Substituting the period-2 government's first-order condition (3.8) for $\lambda = y$ yields:

$$\frac{\partial T(\tau_x^{\#})}{\partial \tau} = \frac{\partial B\left(E_y^{\#}\right)}{\partial E} \cdot F_y^{\#} \cdot \left[1 + \eta_y \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right)\right]. \tag{4.9}$$

We assume that the second-order condition holds; it is discussed in Appendix A.3. It always holds for a conservative party expecting a greener successor and it always holds for energy-saving situations. For the remaing case in which a green party expects a more conservative successor and backfire, the following relation must hold in optimum:

$$\frac{\kappa_y}{\kappa_x} > \frac{1-\beta}{1+\varphi}. \tag{4.10}$$

This condition states that the expected expansion of fuel consumption must not be too strong. It also ensures that the square brackets term in (4.9) is positive. Given our assumptions $\beta > 0$, $\varphi \geq 1$, the fraction is always smaller than $1/2$; so if β and φ are both very small, the y party must value energy usage costs at least half as much the x party.

Discussion Even though (4.9) is more compact, we discuss the different considerations reflected by the optimality conditions for (4.8) because of its comprehensibility. The left-hand side of (4.8) represents marginal productivity costs. The first summand of the right-hand side reflects marginal benefits of productivity. So these two components together represent the investment considerations discussed in the preceding section. If the second summand of the right-hand side did not exist, the x-party government would invest as if the y party already was in power in the first period. So as the first summand represents the y party's preferences, we call it the "neutral expert" motive. It pulls the period-1 government's investment nearer to the y party's preferred level.

The second summand of (4.8) represents the strategic motive that we could call (subjective) "damage containment". The difference term in brackets represents the deviation from the x party's optimality that a y government will produce by choosing its optimal fuel use. Suppose $\kappa_x > \kappa_y$. Then the period-2 y-party government will choose too much fuel consumption from the x party's point of view: At that amount of F, marginal costs are higher than marginal benefits so that the term in brackets is negative. The multiplicand to the right of the brackets represents the reaction of the y government's optimal fuel usage to productivity investment. For an energy-saving benefit function, the elasticity of fuel consumption with respect to productivity, η_y, is negative so that for $\kappa_x > \kappa_y$ the second summand is positive: The x government in period 1 will invest more to reduce its successor's fuel consumption. By contrast, in a backfire case, η_y is positive so that the second summand is negative: The x government constrains its investment to make fuel consumption less attractive for its successor.

The net effect From (4.8) alone it is not yet clear whether the overall effect of a successor with different preferences will move actually chosen investment nearer to the level required by the opposition or away from it – does the neutral-expert motive dominate damage containment or vice versa? We differentiate (4.8) with respect to the period-2 government's fuel usage cost valuation parameter κ_y. Some rearrangements then yield the following result:

Proposition 3. *The elasticity of the x party's optimal investment in a democracy with respect to its successor's external effects valuation is:*

$$\mu_y \equiv \frac{\partial \tau_x^{\#}/\tau_x^{\#}}{\partial \kappa_y/\kappa_y} = \left(\xi_y\right)^2 \cdot (1+\varphi) \cdot \left[1 + \eta_y \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right)\right]^{-1} \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right) \cdot \left(\eta_y - \frac{1}{\omega}\right)^{-1}. \quad (4.11)$$

Proof: See Appendix A.4.

The first two multiplicands are positive. From Appendix A.3 we know that as long as condition (4.10) is fulfilled, the third multiplicand is positive as well. Therefore, the sign of the elasticity depends on the signs of the last two multiplicands.

First consider a backfire situation ($\eta_y > 0$). $\eta_y - \omega^{-1}$ is negative in this case, as discussed above. For $\kappa_y > \kappa_x$, the whole elasticity is negative and for $\kappa_y < \kappa_x$, it is positive. So for $\kappa_y = \kappa_x$, τ must be at its maximum. Similarly, τ is at its minimum value for $\kappa_y = \kappa_x$ in an energy-saving case. Put another way, given a period-1 government's preference parameter κ_x, expecting a successor with different preferences, no matter whether it is greener or more conservative, leads to reduced investment in a backfire case and more investment in an energy-saving case.

If the future government is less green than the current ($\kappa_x > \kappa_y$), the intuition for this behavior is as follows. In a backfire case, a given amount of investment induces additional perceived fuel consumption costs in period 2 due to the "too low" cost valuation, so it is better to invest less, save the investment costs and mitigate the damage. In an energy-saving case, the expectation of a government valuing energy usage costs less gives additional marginal benefits of productivity investment as it reduces energy usage and thus lowers usage costs.

If, by contrast, the future government is greener than the current ($\kappa_y > \kappa_x$), then in a backfire case the period 1 government considers that the amount of investment it principally would find reasonable would partly be wasted: It will not induce as much net benefit because for every degree of productivity the future government will allow less fuel consumption anyway. In an energy-saving case, today's government accepts even higher investment costs today: It can use raising productivity to directly raise benefits.

So the difference between the energy-saving and the backfire case is that in the former the period-1 government invests in a substitute for the period-2 governement's choice variable, while in the latter it invests in a complement. With rising marginal costs of investment, investing in a complement pays off less if your successor uses too much or too little of the other complementary part, fuel. However, exactly this expectation makes it more worthwhile to invest in a substitute: It makes less dependent on your successor.

Escalation and compromise The derived relations between τ_x^* and $\tau_x^\#$ are enough to show an inefficiency implied by the model. To see this, first consider the period-1 x-party government anticipating that its conservative opposition ($\kappa_y < \kappa_x$) will take over, in a backfire situation. By (4.4) it is implied that this opposition will demand a higher level of investment. Proposition 3, however, reveals that the expectation of a successor with different preferences causes the period-1 government to reduce investment. So judging by the difference between preferred and realized investment, the inefficiency is given by the fact that chosen investment is less than what both parties want. In an energy-saving situation, the argument is mirrored: Investment is chosen higher than the desired level of each of the parties. In other words, a green party expecting a conservative successor "escalates" the political differences; the damage containment motive always dominates the neutral-expert motive for the Greens.

The situation is different if a conservative party is in power in period 1, again discussed for a backfire situation. (4.4) shows that the green opposition will demand lower investment than the conservative party would choose if it was sure to stay in power. However, as (4.4) states that the $\tau_x^\#$ reaches its maximum for $\kappa_x = \kappa_y$, expecting a greener opposition ($\kappa_y > \kappa_x$) changes investment only marginally. Therefore, τ_y^* must be lower than $\tau_x^\#$ which again is lower than τ_x^* (while in an energy-saving situation we

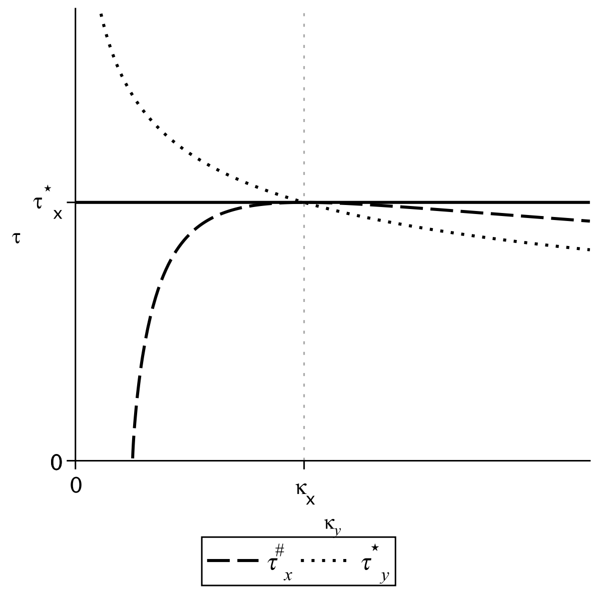

Figure 4: Investment of the period-1 government and opposition's desired investment: Backfire case

have $\tau^*_y > \tau^\#_x > \tau^*_x$). Therefore, expecting a greener successor causes the government to choose a compromise level of investment: For the conservative government, the neutral-expert motive dominates damage containment.

So far we can conclude that if in period 1 a conservative government was in power and the green opposition was sure to win the next elections, then the opposition would want the government to share this expectation because this induces a compromise investment level. By contrast, if the green party was in power, the Conservatives would prefer to take power by surprise, because the knowledge to be replaced influences the green government's investment in an undesirable way.

Figure 4 illustrates the relation for a backfire case. Because the investment level desired by the x party if it is sure to stay in power, τ^*_x, is independent of κ_y, it serves as reference, while $\tau^\#_x$ and τ^*_y depend on κ_y. On the κ_y axis, κ_x is the reference point: Points to the right of it imply that the period 2 government is greener than the period-1 government (and vice versa).

The dotted curve demonstrates the dependence of τ^*_y on κ_y: If $\kappa_y < \kappa_x$ – so that the

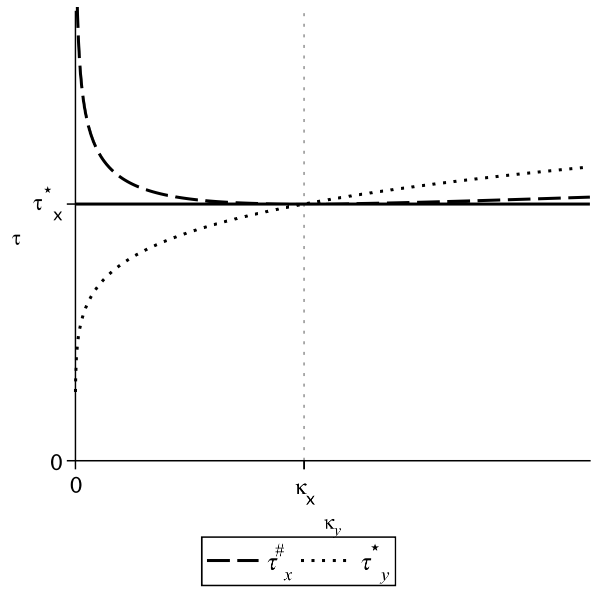

Figure 5: Investment of the period-1 government and opposition's desired investment: Energy-saving case

opposition is more conservative than the government – then the y party would demand a level of productivity investment that is higher than τ_x^*. If, contrarily, $\kappa_y > \kappa_x$, the opposition is greener than the government and so the opposition would want a lower level of investment than the one preferred by an x-party dictator. The dashed curve shows the investment choice of the government in a democracy. As analyzed above, this choice reflects a compromise level if the government expects a greener successor, but it deviates from both parties' originally preferred investment level if the expected successor is more conservative. Figure 5 illustrates the relations for an energy-saving case, where – as described above – expecting a conservative successor ($\kappa_y < \kappa_x$) makes the government raise its investment level while the conservative successor would want lower investment instead.

4.3 The opposition's utility in dictatorship and democracy

We have shown that the investment level which the x-party government chooses in period 1 if it expects a more conservative successor ($\kappa_x > \kappa_y$) deviates from what both parties want. Now consider that the y party has to decide before period 1 whether to run for office after period 1, being able to win the election for sure. Just deciding on the basis of investment levels, a conservative y party could be expected to simply relinquish government to the green x party. However, this also precludes the conservatives from regulating the level of fuel consumption given the chosen level of productivity $\tau_x^{\#}$. Which of these effects dominates?

For this decision, the difference between utility levels in both of the possible situations is crucial:

$$\begin{aligned}\Delta W^y &\equiv W^y\left(F_y^{\#}, \tau_x^{\#}\right) - W^y\left(F_x^*, \tau_x^*\right) \\ &= -\left[T(\tau_x^{\#}) - T(\tau_x^*)\right] + \left[B\left(E_y^{\#}\right) - B\left(E_x^*\right)\right] - \kappa_y \cdot \left[Z\left(F_y^{\#}\right) - Z\left(F_x^*\right)\right]. \qquad (4.12)\end{aligned}$$

This is zero at $\kappa_x = \kappa_y$ and it cannot be negative for $\kappa_y > \kappa_x$ because the knowledge of voting out induces a compromise investment of a conservative government. But what about $\kappa_y < \kappa_x$?

Analytically, we cannot say much here, but trying numerical values with example functions shows that reasonable parameters exist for which the y party is better off in a dictatorship than in a democracy. Figure 6 shows an example with $\kappa_x = 1$, $\kappa_y = 1/2$, and the elasticities are constant as $\varphi = 1$, $\theta = 1$, and $\beta = 3/10$. In a democracy, the x government in period 1 chooses its investment so that productivity is given by $\tau_x^{\#}$. In period 2, the y government chooses $F_y^{\#}$. The respective indifference curves are indicated. The important feature of this constellation is the fact that the F, τ combination that an x-party dictator would choose – τ_x^*, F_x^* – yields higher utility for the y party as it lies between the y party's democracy indifference curve and its utility maximum at $\tau_y^*, F_y(\tau_y^*)$.

For these example functions, Figure 7 shows ΔW^y for different values of β as functions of κ_y. Backfire cases are shown as solid curves, energy-saving cases are shown as dashed curves; the darker the curve, the further β lies above or below 1. The important feature demonstrated is the fact that all of the differences turn negative for low values of κ_y. The point where the curves cut the κ_y axis is further to the left for β near unity

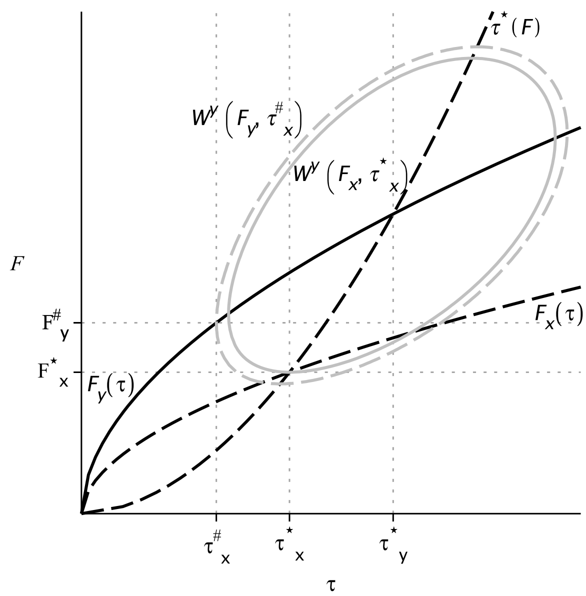

Figure 6: The conservative opposition put in a worse position by the prospect of taking over

(that is, for low reactions of fuel consumption to energy productivity) and this critical κ_y value always is very low for energy-saving functions. The presumable reason for the latter point is that in case of energy-saving functions, the importance of strategic investment is lower: Both parties principally appreciate the productivity progress when it brings benefits and saves costs. We sum up the point as follows:

Proposition 4. *It is possible that a period-1 y-party opposition with $\kappa_y << \kappa_x$ would be better off with a x-party dictator. This is most likely for large backfire ($\beta << 1$) and least likely for $\beta \approx 1$ as in the latter situation strategic considerations are relatively unimportant.*

4.4 Welfare gains by binding agreements

The strategic investment of the government in period 1 arises from the fact that both parties know that the y-party government of period 2 will choose a point on the $F_y(\tau)$ curve – this is the only time-consistent behavior. We now show that the resulting equilibrium is inefficient, no matter whether the x party (governing in period 1) is greener

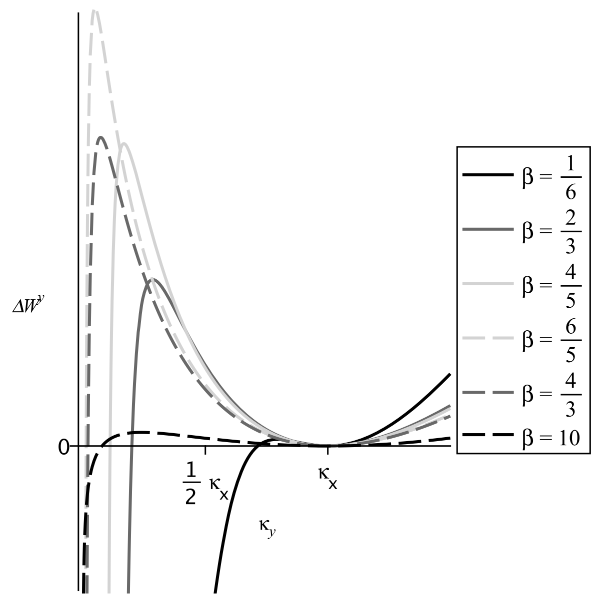

Figure 7: The y party's utility differences between dictatorship and democracy

or more conservative than the y party (governing in period 2). The approach is to show that a Pareto-superior allocation can be found if binding agreements between the two parties are possible. We state:

Proposition 5. *Binding agreements between the parties allow τ, F combinations that are Pareto-superior to the strategic equilibrium.*

Proof: In the strategic democracy equilibrium, $F_y^\# \equiv F_y(\tau_x^\#)$ is chosen. However, for any F value, the $\tau^*(F)$ curve gives the productivity level τ that both parties consider optimal. Therefore, if the parties could agree to invest $\tau^*(F_y^\#)$ in period 1 and consume $F_y^\#$ in period 2, both parties' utility would be higher.

However, $\tau^*(F_y^\#)$ is not the only Pareto-superior investment level. We illustrate this claim graphically. Figure 8 shows the same situation as Figure 3 did. The utility levels associated with the time-consistent democracy behavior, $W^x(F_y^\#, \tau_x^\#)$ and $W^y(F_y^\#, \tau_x^\#)$, are indicated. For both parties, higher utility levels are attained for any F, τ combination that lies inside of their indifference curve. We can see that there is a range that lies between both curves – therefore, all points inside of this range must be Pareto-

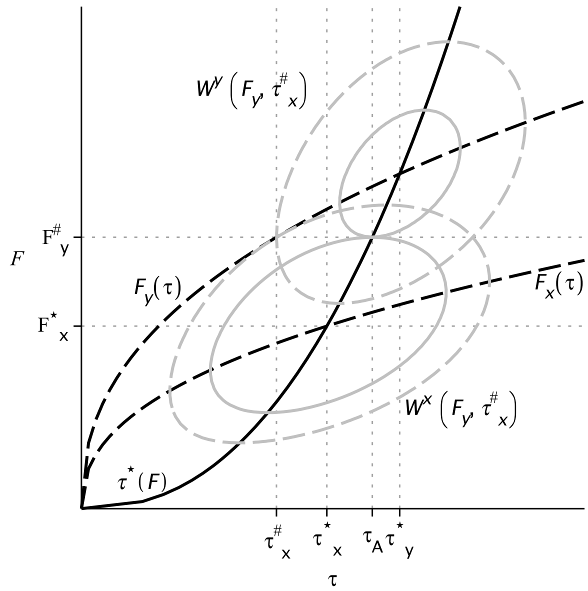

Figure 8: Binding agreements: The case of a green period-1 government

superior to $F_y^{\#}$. In this range, Pareto-efficient points can be found on the $\tau^*(F)$ curve by definition: Given any amount of fuel consumption F, both parties' utility would be maximized on this curve. Indifference curves of $\tau_A \equiv \tau^*(F_y^{\#})$ are shown in the figure.

The exact point on $\tau^*(F)$ that is agreed upon can be chosen by bargaining. Any agreement must make both parties better off than on their common threat point, $F_y^{\#}, \tau_x^{\#}$. We can see in the figure that the $\tau^*(F)$ points that yield higher utility than the threat point are not on the preferred-fuel consumption curves. Therefore, an agreement implies choosing an amount of fuel consumption that represents a compromise level for the chosen τ. Given such a compromise F value, the chosen investment level is a compromise between τ_x^* and τ_y^* as well.

An alternative situation where $\kappa_x < \kappa_y$ is shown in Figure 9. Even though in this constellation the conservative period-1 government chooses a compromise investment level even if there are no binding agreements, there is again a range of F, τ combinations that make both parties better off: The government reduces investment even further, both agree to a compromise level of fuel consumption and a Pareto-optimal point on $\tau^*(F)$ can be chosen.

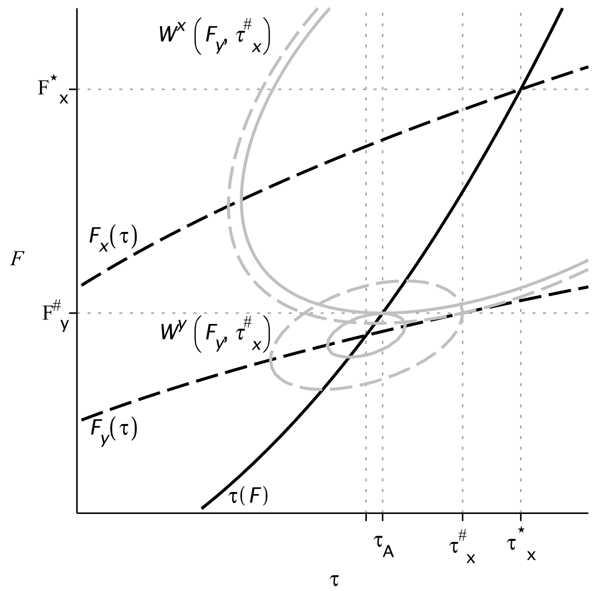

Figure 9: Binding agreements: The case of a conservative period-1 government

5 Conclusions

In our model, different parties assess the social costs of fuel usage differently, and so they disagree about optimal energy usage. This leads to different preferred investment levels: If higher productivity raises energy usage (so-called "backfire"), then a "Greens" will want to invest less than "Conservatives"; if, by contrast, higher energy productivity implies "energy-saving", then Greens want to invest more.

If the different parties succeed each other in government, strategic considerations distort the first government's investment. In particular, the prospect of a successor who is an environmental bully makes a green government reduce its investment even further in case of backfire, and it expands even further in case of energy-saving effects. If the party preferences diverge very much, this reduction of investment – which reduces benefits of energy consumption in the second period – can be so severe that the conservative party would prefer not to come to power at all.

To derive our results, we used some simplifications. One is the benefit function: energy-saving and backfire situations were modeled as different benefit functions. Another example is the energy-use cost function: One party's cost perception of a given amount of fuel usage is proportional to the other's. Also, our results depend on the assumption that the elasticity of fuel usage with respect to productivity is approximately constant. Finally, we model the election outcome as deterministic. However, these assumptions seem uncritical: They help demonstrating the more basic principle that the combination of disagreements about social costs of energy use and investment-dependent marginal benefits of fuel consumption should influence rational politicians' investment behaviour.

We have also shown that, in principle, binding agreements could make both parties better off. However, such agreements are not credible in a democracy. For reasons that are outside the scope of our model, they might not even be desirable. Not only may binding future governments be seen as illegitimate, but commitments also reduce flexibility in the face of unexpected events or improved understanding of optimal choices (cf. Brunner et al., 2012). The most prominent external effects of fuel consumption are the costs of climate change (and the costs of its abatement). Uncertainty in this field is very significant (cf. McKibbin and Wilcoxen, 2002).

It would be interesting to explicitly model elections, particularly because one result of our model was that the Conservatives would like to take power by suprise, while the Greens would like a conservative government to know its successor's preferences. However, the diversity of political issues that influence elections make it plausible to treat the election outcome as exogenous in a model focussing on energy productivity and energy's social costs. List and Sturm (2006) mention environmental issues as a typical example for a secondary policy issue which may be important to only few voters. This consideration also qualifies the result that a conservative party might prefer not to come to power at all: In reality, other political fields have to be considered as well.

One obvious shortcoming of the model is the lack of easily testable empirical implications. If governments in the real world behave rationally, we should expect to see, for example, green governments overinvesting in productivity for "energy-saving" sectors when they expect an electoral defeat. But in a changing environment, overinvestment is hard to distinguish from normal investment that should already be higher under a green government. Also, governmental actions concerning productivity investment might often take place as administrative orders and standards that are hard to quantify.

However, a short note about empirical magnitudes might help to put the discussion in perspective. Rebound estimates for single kinds of energy services range between $R = 0$ and $R = 0.5$ (cf. Greening et al., 2000), which implies $-1 < \eta < -0.5$. This seems to indicate relatively little importance of strategic factors (which mostly appear with backfire). However, for economy-wide productivity gains, the situation is much less clear. It is hard to quantify macroeconomic rebound effects empirically, but theoretical (growth) models imply that energy productivity improvements are likely to raise fuel demand (cf. Saunders, 1992).

Finally, we would like to point out that for simplicity we have identified "green" parties with higher cost valuation and "conservative" parties with lower. This is appropriate for the case of fuel consumption discussed, but the situation is likely reversed for regenerative energy sources and becomes totally unclear if we discuss a fuel like natural gas that is more climate-friendly than coal, but more damaging than wind power. Heterogenous energy sources, with different benefit functions and different cost functions, would expand the area for strategic conflict: Not only the amount, but also the kind of investment would be subject to strategic distortions. As this adds realism, it offers an interesting direction for future research.

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A Appendix

A.1 Second-order condition for a dictator's optimal investment

To guarantee that we have a maximum at τ_x^*, F_x^*, it is required that the following condition holds at this point:

$$\begin{aligned}\frac{\partial^2 W^x(F_x(\tau),\tau)}{\partial\tau^2} = & -\frac{\partial^2 T}{\partial\tau^2} + \frac{\partial^2 B}{\partial E^2}\cdot\left(\frac{\partial E_x}{\partial\tau}\right)^2 - \kappa_x\cdot\frac{\partial^2 Z}{\partial F^2}\cdot\left(\frac{\partial F_x}{\partial\tau}\right)^2 \\ & + \frac{\partial B}{\partial E}\cdot\frac{\partial^2 E_x}{\partial\tau^2} - \kappa_x\cdot\frac{\partial Z}{\partial F}\cdot\frac{\partial^2 F_x}{\partial\tau^2} \qquad < 0.\end{aligned}$$

Substituting $\partial^2 E_x/\partial\tau^2 = 2 \cdot \partial F_x/\partial\tau + \tau \cdot \partial^2 F_x/\partial\tau^2$ and the optimal fuel use condition (3.8) yields:

$$\begin{aligned}\frac{\partial^2 W^x(F_x(\tau),\tau)}{\partial\tau^2} &= -\frac{\partial^2 T}{\partial\tau^2} + \frac{\partial^2 B}{\partial E^2} \cdot \left(\frac{\partial E_x}{\partial\tau}\right)^2 \\ &\quad - \kappa_x \cdot \frac{\partial^2 Z}{\partial F^2} \cdot \left(\frac{\partial F_x}{\partial\tau}\right)^2 + 2 \cdot \frac{\partial B}{\partial E} \cdot \frac{\partial F_x}{\partial\tau}.\end{aligned} \tag{A.1}$$

In an energy-saving situation, $\partial F_x/\partial\tau$ is negative so that all summands are negative; then the second-order condition is always fulfilled. So we have to derive under which circumstances the second-order condition is fulfilled in a backfire situation. Factoring out $\partial B/\partial E$ and F_x/τ and substituting the first order conditions (3.8) (for $\lambda = x$) and (4.3), we get:

$$\frac{\partial^2 W^x(F_x^*,\tau_x^*)}{\partial\tau^2} = \frac{F_x^*}{\tau_x^*} \cdot \frac{\partial B}{\partial E} \cdot \left[-\theta - \beta \cdot \left(\frac{\partial E_x}{\partial\tau} \cdot \frac{1}{F_x^*}\right)^2 - \varphi \cdot \left(\frac{\partial F_x/F_x^*}{\partial\tau/\tau_x^*}\right)^2 + 2 \cdot \frac{\partial F_x/F_x^*}{\partial\tau/\tau_x^*}\right].$$

Noting that $(\partial E_x/\partial\tau) \cdot 1/F_x = 1 + \eta_x$, we can substitute the elasticities and simplify:

$$\frac{\partial^2 W^x(F_x^*,\tau_x^*)}{\partial\tau^2} = \frac{F_x^*}{\tau_x^*} \cdot \frac{\partial B}{\partial E} \cdot (1-\beta) \cdot \left(\eta_x - \frac{1}{\omega}\right). \tag{A.2}$$

Thus, in a backfire situation where $1-\beta > 0$, the second-order condition is fulfilled if

$$\eta_x < \omega^{-1}, \tag{A.3}$$

which means that a F, τ combination can be a utility maximum if for these values the $F_x(\tau)$ function is less steep than the inverse of the $\tau^*(F)$ function. In an energy-saving case where $1-\beta < 0$,

$$\eta_x > \omega^{-1}, \tag{A.4}$$

is implied so that the $F_x(\tau)$ function is steeper than the $\tau^*(F)$ function. Substituting η_x

and ω in (A.2), we see that the general requirement for the inequality to be fulfilled is

$$\frac{1 - 2 \cdot \beta - \theta \cdot \varphi - \theta \cdot \beta - \varphi \cdot \beta}{\beta + \varphi} < 0$$

in optimum. After further rearrangements, we can state this condition as

$$1 - \beta < \frac{(1 + \varphi) \cdot (1 + \theta)}{(1 + \varphi) + (1 + \theta)}. \tag{A.5}$$

In (3.12), we assumed $\varphi \geq 1, \theta \geq 1$; as $\beta > 0$, this is sufficient for (A.5) to be fulfilled and then any extremum of the welfare function is a maximum.

A.2 The impact of fuel usage cost valuation on a dictator's investment

We differentiate (4.2) with respect to κ_x and τ. This yields:

$$\begin{aligned}
0 = & \left(\frac{\partial B}{\partial E} \cdot \tau - \kappa_x \cdot \frac{\partial Z}{\partial F}\right) \cdot d\left(\frac{\partial F_x}{\partial \tau}\right) \\
& + \left\{\frac{\partial^2 B}{\partial E^2} \cdot \left[{F_x}^2 + 2 \cdot E_x \cdot \frac{\partial F_x}{\partial \tau} + \tau^2 \cdot \left(\frac{\partial F_x}{\partial \tau}\right)^2\right]\right. \\
& \quad \left. - \kappa_x \cdot \frac{\partial^2 Z}{\partial F^2} \cdot \left(\frac{\partial F_x}{\partial \tau}\right)^2 + 2 \cdot \frac{\partial B}{\partial E} \cdot \frac{\partial F_x}{\partial \tau} - \frac{\partial^2 T}{\partial \tau^2}\right\} \cdot d\tau \\
& + \left[\left(\frac{\partial B}{\partial E} + \frac{\partial^2 B}{\partial E^2} \cdot E_x\right) \cdot \frac{\partial F_x}{\partial \kappa_x} - \frac{\partial Z}{\partial F} \cdot \frac{\partial F_x}{\partial \tau}\right. \\
& \quad \left. + \left(\frac{\partial^2 B}{\partial E^2} \cdot \tau^2 - \kappa_x \cdot \frac{\partial^2 Z}{\partial F^2}\right) \cdot \frac{\partial F_x}{\partial \tau} \cdot \frac{\partial F_x}{\partial \kappa_x}\right] \cdot d\kappa_x.
\end{aligned}$$

The first line of the right-hand side equals zero because of the period-2 government's optimality condition. The square brackets term in the second line can be replaced as

$${F_x}^2 + 2 \cdot E_x \cdot \frac{\partial F_x}{\partial \tau} + \tau^2 \cdot \left(\frac{\partial F_x}{\partial \tau}\right)^2 = \left(\frac{\partial E_x}{\partial \tau}\right)^2.$$

Doing so and rearranging gives:

$$0=\left[-\frac{\partial^2 T}{\partial\tau^2}+\frac{\partial^2 B}{\partial E^2}\cdot\left(\frac{\partial E_x}{\partial\tau}\right)^2-\kappa_x\cdot\frac{\partial^2 Z}{\partial F^2}\cdot\left(\frac{\partial F_x}{\partial\tau}\right)^2+2\cdot\frac{\partial B}{\partial E}\cdot\frac{\partial F_x}{\partial\tau}\right]\cdot\frac{d\tau}{d\kappa_x}$$
$$+\left[\frac{\partial B}{\partial E}\cdot(1-\beta)\cdot\frac{\partial F_x}{\partial\kappa_x}-\frac{\partial Z}{\partial F}\cdot\frac{\partial F_x}{\partial\tau}+\left(\frac{\partial^2 B}{\partial E^2}\cdot\tau^2-\kappa_x\cdot\frac{\partial^2 Z}{\partial F^2}\right)\cdot\frac{\partial F_x}{\partial\kappa_x}\cdot\frac{\partial F_x}{\partial\tau}\right].\quad\text{(A.6)}$$

The square brackets term in the first line is equivalent to $\partial^2 W^x/\partial\tau^2$, as given in (A.1). We can also substitute the elasticities, factor out $\partial B/\partial E$ and F_x/κ_x, substitute the optimality condition (3.8) and rearrange to get:

$$\frac{\partial\tau_x^*}{\partial\kappa_x}=\left[\frac{\partial^2 W^x(\kappa_x,\tau)}{\tau^2}\right]^{-1}\cdot\frac{F_x^*}{\kappa_x}\cdot\frac{\partial B}{\partial E}\cdot\eta_x.$$

Now we substitute $\partial^2 W^x/\partial\tau^2$ from (A.2):

$$\frac{\partial\tau_x^*}{\partial\kappa_x}=\left[\frac{F_x^*}{\tau_x^*}\cdot\frac{\partial B}{\partial E}\cdot(1-\beta)\cdot\left(\eta_x-\frac{1}{\omega}\right)\right]^{-1}\cdot\frac{F_x^*}{\kappa_x}\cdot\frac{\partial B}{\partial E}\cdot\eta_x$$

Simplifying finally yields:

$$\psi_x\equiv\frac{\partial\tau_x^*/\tau_x^*}{\partial\kappa_x/\kappa_x}=-\xi_x\cdot\left(\eta_x-\frac{1}{\omega}\right)^{-1}.$$

A.3 Second-order condition for optimal investment in a democracy

To guarantee an x-party utility maximum for $\tau_x^\#, F_y^\#$, it is required that the following condition holds at this point:

$$\frac{\partial^2 W^x(F_y(\tau),\tau)}{\partial\tau^2}=-\frac{\partial^2 T}{\partial\tau^2}+\frac{\partial^2 B}{\partial E^2}\cdot\left(\frac{\partial E_y}{\partial\tau}\right)^2-\kappa_x\cdot\frac{\partial^2 Z}{\partial F^2}\cdot\left(\frac{\partial F_y}{\partial\tau}\right)^2$$
$$+\frac{\partial B}{\partial E}\cdot\frac{\partial^2 E_y}{\partial\tau^2}-\kappa_x\cdot\frac{\partial Z}{\partial F}\cdot\frac{\partial^2 F_y}{\partial\tau^2}<0.$$

Substituting $\partial^2 E_y/\partial\tau^2 = 2 \cdot \partial F_y/\partial\tau + \tau \cdot \partial^2 F_y/\partial\tau^2$ yields:

$$\begin{aligned}\frac{\partial^2 W^x(F_y(\tau),\tau)}{\partial\tau^2} &= -\frac{\partial^2 T}{\partial\tau^2} + \frac{\partial^2 B}{\partial E^2} \cdot \left(\frac{\partial E_y}{\partial\tau}\right)^2 - \kappa_x \cdot \frac{\partial^2 Z}{\partial F^2} \cdot \left(\frac{\partial F_y}{\partial\tau}\right)^2 + 2 \cdot \frac{\partial B}{\partial E} \cdot \frac{\partial F_y}{\partial\tau} \\ &\quad + \left(\frac{\partial B}{\partial E} \cdot \tau - \kappa_x \cdot \frac{\partial Z}{\partial F}\right) \cdot \frac{\partial^2 F_y}{\partial\tau^2}. \end{aligned} \tag{A.7}$$

Substituting the first order conditions (3.8) (for $\lambda = y$) and (4.9) and factoring out $\partial B/\partial E$ and F_y/τ, we get:

$$\begin{aligned}\frac{\partial^2 W^x(F_y(\tau_x^\#),\tau_x^\#)}{\partial\tau^2} &= \frac{F_y^\#}{\tau_x^\#} \cdot \frac{\partial B}{\partial E} \cdot \Bigg\{-\theta \cdot \left[1 + \left(1 - \frac{\kappa_x}{\kappa_y}\right) \cdot \eta_y\right] \\ &\quad - \beta \cdot \left(\frac{\partial E_y}{\partial\tau} \cdot \frac{1}{F_y^\#}\right)^2 - \frac{\kappa_x}{\kappa_y} \cdot \varphi \cdot \left(\frac{\partial F_y/F_y^\#}{\partial\tau/\tau_x^\#}\right)^2 \\ &\quad + 2 \cdot \frac{\partial F_y/F_y^\#}{\partial\tau/\tau_x^\#} + \tau_x^\# \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right) \cdot \frac{\partial^2 F_y}{\partial\tau^2} \cdot \frac{\tau_x^\#}{F_y}\Bigg\}. \end{aligned} \tag{A.8}$$

For the second derivative of the period 2 government's fuel consumption choice with respect to energy productivity, rearranging and differentiating (3.11) yields:

$$\frac{\partial^2 F_y}{\partial\tau^2} = \frac{F_y}{\tau} \cdot \left[\frac{\partial\eta_y}{\partial\tau} + \frac{1}{\tau} \cdot \eta_y \cdot \left(\eta_y - 1\right)\right]. \tag{A.9}$$

Substituting (A.9) and $(\partial E_y/\partial\tau) \cdot 1/F_y = 1 + \eta_y$ into (A.8) and simplifying yields:

$$\begin{aligned}\frac{\partial^2 W^x(F_y(\tau_x^\#),\tau_x^\#)}{\partial\tau^2} &= \frac{F_y^\#}{\tau_x^\#} \cdot \frac{\partial B}{\partial E} \cdot (1-\beta) \cdot \Bigg\{\eta_y - \frac{1}{\omega} \\ &\quad + \frac{1}{\beta+\varphi} \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right) \cdot \left[\eta_y \cdot (1+\varphi) - 1 - \theta\right] \\ &\quad + \frac{1}{\beta+\varphi} \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right) \cdot \left(\frac{\partial\eta_y/\eta_y}{\partial\tau/\tau_x^\#}\right)\Bigg\}. \end{aligned}$$

With some rearrangements, we get:

$$\begin{aligned}\frac{\partial^2 W^x(F_y(\tau_x^\#),\tau_x^\#)}{\partial\tau^2} &= \frac{F_y^\#}{\tau_x^\#} \cdot \frac{\partial B}{\partial E} \cdot (1-\beta) \cdot \Bigg\{\left(\eta_y - \frac{1}{\omega}\right) \cdot \left[1 + \eta_y \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right)\right] \\ &\quad + \frac{1}{\beta+\varphi} \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right) \cdot \left(\frac{\partial\eta_y/\eta_y}{\partial\tau/\tau_x^\#}\right)\Bigg\}. \end{aligned} \tag{A.10}$$

We assume that the elasticity of η_y with respect to τ is neglectable.[9] Then, finding out under which conditions this second derivative is smaller than zero is equivalent to finding out whether the following inequality holds:

$$(1-\beta)\cdot\left(\eta_y-\frac{1}{\omega}\right)\cdot\left[1+\eta_y\cdot\left(1-\frac{\kappa_x}{\kappa_y}\right)\right]<0.$$

From Appendix A.1, we know that the product of the first two multiplicands is negative, so we now have to analyze whether the third multiplicand is positive. Note that $1-\kappa_x/\kappa_y$ is bounded between $-\infty$ (for $\kappa_x\to\infty$ and $\kappa_y\to 0$) and 1 (for $\kappa_x\to 0$ and $\kappa_y\to\infty$) and that $1>\eta_y>0$ in the backfire case and $0>\eta_y>-1$ in the energy-saving case, given our assumptions in subsection 3.4.

First consider the energy-saving case in which $\eta_y<0$. Then for $\kappa_x>\kappa_y$, no part in the square brackets is negative. For $\kappa_x<\kappa_y$, the square brackets term must be larger than $1+(-1)\cdot 1=0$, so the energy-saving case is never a problem.

Now consider the backfire case with $\eta_y>0$. For $\kappa_x<\kappa_y$, $1-\kappa_x/\kappa_y$ is positive so that the whole term is positive again. However, for $\kappa_x>\kappa_y$, the square brackets term can take negative values. The condition for this not to happen is:

$$\frac{\kappa_y}{\kappa_x}>\frac{1-\beta}{1+\varphi}.$$

To sum up, the second-order condition is always fulfilled with energy-saving benefit functions. For a backfire case, the second-order condition is fulfilled if the investing government is more conservative than its successor ($\kappa_x<\kappa_y$), or if the future government will not expand fuel consumption too much in optimum.

[9]If it is not, little can be said about optimality. Our model constitutes a second-best problem for the first-period government, which requires some assumptions about cost and benefit functions (cf. Lipsey and Lancaster, 1956-1957).

A.4 The impact of the period-2 government's fuel usage cost valuation on investment in a democracy

To analyze how a change in the period-2 government's valuation parameter κ_y changes the x-party government's investment, we proceed along the lines of Appendix A.2, but now it is not the period-1 government's own cost valuation parameter that varies, but the period-2 government's. First, we differentiate the strategic first-order condition (4.8) with respect to τ and κ_y. Rearranging yields:

$$\begin{aligned} 0 = &\left[-\frac{\partial^2 T}{\partial \tau^2} + \frac{\partial^2 B}{\partial E^2} \cdot \left(\frac{\partial E_y}{\partial \tau} \right)^2 - \kappa_x \cdot \frac{\partial^2 Z}{\partial F^2} \cdot \left(\frac{\partial F_y}{\partial \tau} \right)^2 \right. \\ &\left. \quad + 2 \cdot \frac{\partial B}{\partial E} \cdot \frac{\partial F_y}{\partial \tau} + \left(\frac{\partial B}{\partial E} \cdot \tau - \kappa_x \cdot \frac{\partial Z}{\partial F} \right) \cdot \frac{\partial^2 F_y}{\partial \tau^2} \right] \cdot \frac{d\tau}{d\kappa_y} \\ &+ \left[\frac{\partial B}{\partial E} \cdot (1-\beta) \cdot \frac{\partial F_y}{\partial \kappa_y} + \left(\frac{\partial^2 B}{\partial E^2} \cdot \tau^2 - \kappa_x \cdot \frac{\partial^2 Z}{\partial F^2} \right) \cdot \frac{\partial F_y}{\partial \tau} \cdot \frac{\partial F_y}{\partial \kappa_y} \right. \\ &\left. \quad + \left(\frac{\partial B}{\partial E} \cdot \tau - \kappa_x \cdot \frac{\partial Z}{\partial F} \right) \cdot \frac{\partial^2 F_y}{\partial \tau \partial \kappa_y} \right]. \end{aligned} \tag{A.11}$$

The square-brackets term in the first two lines is equivalent to $\partial^2 W^x(F_y(\tau), \tau)/\partial \tau^2$ as given in (A.7). From the definitions of ξ_λ and η_λ in (3.10) and (3.11), we can derive the following equivalence for $\partial^2 F_y/\partial \tau \partial \kappa_y$:

$$\frac{\partial^2 F_y}{\partial \tau \partial \kappa_y} = \eta_\lambda \cdot \frac{F_y}{\tau \cdot \kappa_y} \cdot \left(\frac{\partial \eta_y / \eta_y}{\partial \kappa_y / \kappa_y} + \xi_y \right). \tag{A.12}$$

Substituting both findings and rearranging yields:

$$\begin{aligned} \frac{d\tau}{d\kappa_y} = -&\left[\frac{\partial^2 W^x(F_y(\tau), \tau)}{\partial \tau^2} \right]^{-1} \\ &\cdot \left[\frac{\partial B}{\partial E} \cdot (1-\beta) \cdot \frac{\partial F_y}{\partial \kappa_y} + \left(\frac{\partial^2 B}{\partial E^2} \cdot \tau^2 - \kappa_x \cdot \frac{\partial^2 Z}{\partial F^2} \right) \cdot \frac{\partial F_y}{\partial \tau} \cdot \frac{\partial F_y}{\partial \kappa_y} \right. \\ &\left. \quad + \left(\frac{\partial B}{\partial E} \cdot \tau - \kappa_x \cdot \frac{\partial Z}{\partial F} \right) \cdot \eta_\lambda \cdot \frac{F_y}{\tau \cdot \kappa_y} \cdot \left(\frac{\partial \eta_y / \eta_y}{\partial \kappa_y / \kappa_y} + \xi_y \right) \right]. \end{aligned}$$

We factor out $\partial B(E_y)/\partial E$ and F_y/κ_y and substitute the period-2 government's optimality condition, (3.8) for $\lambda = y$:

$$\frac{\partial \tau_x^\#}{\partial \kappa_y} = -\left[\frac{\partial^2 W^x(F_y(\tau),\tau)}{\partial \tau^2}\right]^{-1} \cdot \left[\varphi \cdot \xi_y + \left(\frac{\partial \eta_y/\eta_y}{\partial \kappa_y/\kappa_y} + \xi_y\right)\right] \cdot \frac{F_y}{\kappa_y} \cdot \frac{\partial B}{\partial E} \cdot \eta_y \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right).$$

Now substitute $\partial^2 W^x(F_y(\tau),\tau)/\partial \tau^2$ from (A.10) and assume that the elasticity of η_y with respect to κ_y (and, as we assumed before, with respect to τ) is negligible:

$$\begin{aligned}\frac{\partial \tau_x^\#}{\partial \kappa_y} = &-\left\{\frac{F_y}{\tau_x^\#} \cdot \frac{\partial B}{\partial E} \cdot (1-\beta) \cdot \left(\eta_y - \frac{1}{\omega}\right) \cdot \left[1 + \eta_y \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right)\right]\right\}^{-1} \\ &\cdot \left[\varphi \cdot \xi_y + \xi_y\right] \cdot \frac{\partial B}{\partial E} \cdot \frac{F_y}{\kappa_y} \cdot \eta_y \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right).\end{aligned}$$

Simplifying yields:

$$\mu \equiv \frac{\partial \tau_x^\#/\tau_x^\#}{\partial \kappa_y/\kappa_y} = \left(\xi_y\right)^2 \cdot \left(\eta_y - \frac{1}{\omega}\right)^{-1} \cdot (1+\varphi) \cdot \left[1 + \eta_y \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right)\right]^{-1} \cdot \left(1 - \frac{\kappa_x}{\kappa_y}\right).$$