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Thomas Eichner · Rüdiger Pethig

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Universität Siegen
Fachbereich 5
Wirtschaftswissenschaften, Wirtschaftsinformatik und Wirtschaftsrecht
Fachgebiet Volkswirtschaftslehre
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Taxing and trading cabon emissions in the EU: Distributional comparisons of mixed policies

Thomas Eichner[a] **and Rüdiger Pethig**[b]

a) Department of Economics, University of Bielefeld, Universitätsstr. 25, 33615 Bielefeld, Germany. E-mail: `teichner@wiwi.uni-bielefeld.de`

b) Department of Economics, University of Siegen, Hoelderlinstr. 3, 57068 Siegen, Germany. E-mail: `pethig@vwl.wiwi.uni-siegen.de`

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Abstract:

We model EU-type carbon emissions control in a group of countries to explore the distributional incidence of mixed policies that consist of an emissions trading scheme (ETS) and of emissions taxes overlapping with the ETS. Such policies impact on national welfares through both the overlapping taxes and the distribution of national emissions caps. Our main proposition is an equivalence result stating that for every mixed policy, there exists an ETS policy without overlapping taxes yielding the same levels of national welfare as the mixed policy. We also suggest two measures of the net distributional incidence of mixed policies.

JEL-classification: H23, Q52

Keywords: emissions cap, emissions tax, emissions trading

1 Introduction

In the Kyoto Protocol the EU committed to reduce its carbon emissions by 8% in 2012 from its baseline emissions in 1990 (EU 2000). Denoting by $\bar{e}$ the EU's baseline emissions in 1990, the EU commitment amounts to introducing an EU wide emissions cap $\bar{c} = 0.92 \cdot \bar{e}$ as an upper bound for total EU emissions in 2012. To share the burden of curbing emissions the EU member states agreed on national emissions limits (EU 2000)

$$\mathbf{c} := (c_1 \ldots, c_n) \in C := \left\{ \mathbf{c} \in \mathbb{R}^n_+ \;\middle|\; \sum_j c_j = \bar{c} \right\},$$

where c_i is the *national* emissions cap for each member state $i = 1, \ldots, n$. Complementing national emissions controls, the EU has established an emissions trading scheme (ETS) in 2005 (EU 2003a) allowing for EU wide trade in emissions permits. That scheme covers only part of each member state's economy (ETS sector), however. In the rest of their economies, called the non-ETS sectors, national governments are responsible to curb emissions by means of domestic regulation. Another distinctive feature of EU emissions control is the existence of national regulation, notably through energy taxes, in the ETS sector overlapping with the ETS (Johnstone 2003, Sorell and Sijm 2003).[1] Thus the EU emissions control is a hybrid policy characterized by

- a cap, $\bar{c}$, on overall EU emissions;
- a distribution of national emissions caps $\mathbf{c} \in C$ (burden sharing agreement);
- an ETS covering part of each member state's economy;
- additional national emissions controls in both sectors of the economy.

In the present paper we will capture these features of EU regulation in a stylized way to explore the *distributional incidence* of hybrid EU-type policies with special emphasis on emissions taxes overlapping with the ETS. In such mixed policies both policy components, the overlapping tax as well as the ETS (with its implied distribution of national emissions caps),

[1]For example, coal in industry is taxed in Austria and in Finland; heavy fuel oil for electricity generation is taxed in Austria, Germany, Ireland and in Poland (International Energy Agency, 2007). Moreover the EU Energy Tax Directive (EU 2003b) widens the scope of the EU's minimum energy tax rate system, previously limited to mineral oils, to all energy products including coal, natural gas and electricity.

impact on the distribution of national welfares. To begin with, variations in the distribution of national emissions caps, e.g. the replacement of $\mathbf{c} \in C$ by some $\mathbf{c}' \in C$, $\mathbf{c} \neq \mathbf{c}'$, clearly have distributional consequences.[2] Since the national cap c_i is a valuable asset for country i, the larger that asset the better off country i tends to be at the expense of the other countries. The observation that the choice of a vector of national emissions caps from the set C amounts to choosing the national burdens of emissions reductions is well understood although it is not easy to specify the precise impact (see below). However, one must not overlook that emissions taxes in the countries' ETS sectors also impact on national welfares in that they affect the equilibrium permit price and thus each country's export or import of permits. Thus, it is the overlapping tax as well as the distribution of national emissions caps that determine the distributional incidence of mixed policies. As an implication, the calculations of national net burdens of mixed policies are not correct unless the interaction of both instruments is accounted for. To address this issue we will consider variations in mixed policies and investigate their *net* effect on national welfares resulting from an integrated account of the *partial* welfare effects of both instruments that may point either in the same or in opposite directions. Policies using a tax without ETS or an ETS without a tax are included as limiting cases as illustrated in Table 1.

	Emissions control in the ETS sector via		
	ETS	ETS and sectoral tax	Sectoral tax
Emissions control in the non-ETS sector via sectoral tax	1	2	3

Table 1: EU-type emissions control in a two-sector economy

To keep the focus on distribution as clear as possible, we assume the overall emissions cap $\bar{c}$ constant throughout the paper and restrict most of our analysis to cost-effective policies.[3] For

[2]Recall that by definition of C the overall emissions cap $\bar{c}$ is fixed such that switching from one element of C to another describes a redistribution of national emissions caps.

[3]In practice the hybrid EU policy struggles, of course, with cost *in*effectiveness for various reasons.

the study of those policies we introduce the simplifying assumptions[4,5]

- that in their non-ETS sectors national governments effectively control emissions through a domestic sectoral emissions tax;
- that the rate of the emissions tax overlapping with the ETS sector is uniform across countries;[6]
- and that all countries choose their permit cap, as to equalize marginal abatement cost across the ETS sector and the non-ETS sector.

After having characterized the associated class of cost-effective hybrid policies we consider the limiting case of a tax-only policy (box 3 in Table 1) showing that the associated equilibrium allocation is unaffected by the introduction of an ETS and its partition into national caps (Proposition 1). This finding turns out to be a useful benchmark. Next we demonstrate the *distributional* consequences of mixed policies (box 2 in Table 1) by showing how a country's welfare varies in response either to changes in the emissions tax rate in the ETS sectors or to changes in its national emissions cap (with compensating changes in the caps of all other countries). Due to the interdependence of markets, the distributional effects of policy changes turn out to be not monotone, in general, and hence not easy to characterize. Our main proposition is an equivalence result (Proposition 2) stating that for every mixed policy (box 2 in Table 1) there exists an ETS-only policy (box 1 in Table 1) which provides all countries with the same level of welfare and the same allocation as the mixed policy. It is also possible to specify how the national caps in the equivalent ETS-only policy deviate from the caps assigned to the countries in the actually prevailing mixed policy (Proposition 3). In Proposition 4 we consider a

[4]Here we follow Eichner and Pethig (2009) who established conditions under which the policy mix is cost effective for the group of countries. They show, in particular, that the emissions tax can be fixed at different levels without compromising cost effectiveness if the overlapping tax is uniform across countries (and if some other qualifications are met). For more details see also Section 2.

[5]The relation between cost effectiveness and pareto efficiency has been clarified by Chichilnisky and Heal (1994) and Shiell (2003).

[6]This condition is not satisfied in the EU since in the 1990s the European Commission's proposal of introducing a uniform emissions tax had been turned down. More recently Nordhaus (2006) brought forward arguments in favor of an internationally harmonized emissions tax. In the present paper we deal in Proposition 4 with the case of overlapping taxes that differ across countries.

cost-ineffective policy mix with overlapping taxes whose rates differ across countries. We show that for every cost ineffective mixed policy there exists an ETS-only policy that provides all countries with a level of welfare that is higher than that in the mixed policy by some (uniform) percentage rate.

Making use of our analytical findings we proceed to propose two measures of the distributional incidence of EU-type emissions control. The first measure is non-monetary taking advantage of both the equivalence result and the benchmark property of the tax-only policy. With this measure one can identify winners and losers of a mixed policy relative to the tax-only policy. In the spirit of the welfare measure of equivalent variation the second measure consists of a monetary transfer payment a country needs to pay or receive in order to be indifferent between some given mixed policy and the tax-only benchmark policy.

Most of the literature on hybrid EU-type carbon emissions control deals with allocative distortions of existing policies and/or with issues of policy design for allocative efficiency, e.g. Bovenberg and de Mooij (1994), Babiker et al. (2003), Bento and Jacobsen (2006), Böhringer et al. (2008) and Eichner and Pethig (2009). In contrast, only a few studies address issues of the international distribution of national welfares and burdens. The issue of *equitable* burden sharing has been addressed e.g. by Phylipsen et al. (1998) and Marklund and Samakovlis (2007). Yet our focus is not on equity or fairness but rather on the positive analysis of the distributional impact of mixed policies. There is an applied literature of numerical analysis in large-scale CGE models in which some distributional issues are investigated although not in a systematic *analytical* way. For example, Böhringer et al. (2008) consider a group of countries operating an ETS and they calculate how burdens change when an individual country successively raises the rate of the emissions tax in its ETS sector. Peterson and Klepper (2007) compare a harmonized international carbon tax to an ETS with different allocation rules for the emissions caps without considering the issue of overlapping instruments. Hence the distributional incidence of EU-type mixed policies appears to be under-researched.

The paper is organized as follows: Section 2 sets up the model, characterizes cost-effective equilibria, establishes the tax-only policy as a benchmark, and analyzes the welfare effects of changes in policy parameters. Section 3 presents and characterizes the equivalence result and extends it to mixed policies in which overlapping emissions taxes differ across countries. Section

4 suggests two measures of the distributional impact of mixed policies and Section 5 concludes.

2 The model

We consider an economy of n countries that are open to the rest of the world and that operate a joint ETS. Each country's economy consists of two sectors: One sector that is covered by the ETS, called the ETS sector, and the rest of the economy, called the non-ETS sector. The non-ETS sector of country $i = 1, \ldots, n$ uses the fossil fuel input e_{xi} to produce the output $x_{si} = X^i(e_{xi})$. Likewise, the ETS sector uses the fossil fuel input e_{yi} to produce the output $y_{si} = Y^i(e_{yi})$. All fossil fuel is assumed to be imported from the world market at the fixed price p_e. The energy costs of the firms in country i's ETS sector are $(p_e + t_{yi})e_{yi}$ if country i levies an energy tax at rate t_{yi}. We consider that tax as a tax on carbon emissions because the release of CO_2 is approximately proportional to the amount of fossil fuel burned. The imports of fossil fuel are (mainly) paid for by exporting good Y at the world market price p_y.[7] Good X^i is traded on a domestic market at price p_{xi} and the corresponding market clearance condition is

$$x_{si} = x_i \qquad \text{for } i = 1, \ldots, n, \tag{1}$$

where x_i is the domestic demand for good X^i. Given the overall emissions cap $\bar{c}$ for the group of countries and some partition $(c_1, \ldots, c_n)$ of $\bar{c}$ (as outlined in the Introduction) the government of each country i chooses the permit budget $c_{yi} \in [0, c_i]$. It issues and hands over to its ETS sector for free[8] the amount c_{yi} of emissions permits which can then be traded at price π_e among all firms in the ETS sectors of all countries. The condition for equilibrium on that permit market is

$$\sum_j c_{yj} = \sum_j e_{yj}. \tag{2}$$

[7]Part of the import bill may also be paid for by revenues from exporting permits. However, if permits are imported, the import of both fossil fuel and permits need to be paid for by revenues from exports of good Y. See trade balance equation (5) below.

[8]At the high level of abstraction in the present analysis, free allocation and auctioning of emissions permits are equivalent allocation procedures. For an analysis where the allocation rule matters we refer to Rosendahl (2008).

Each country also levies an emissions tax in its non-ETS sector whose rate t_{xi} is assumed to be chosen as to satisfy

$$c_i - c_{yi} = e_{xi} \qquad \text{for } i = 1, \dots, n. \tag{3}$$

Summing (3) over i and invoking (2) shows immediately that the EU-wide emissions cap $\bar{c}$ is met: $\sum_j c_j = \sum_j (e_{xj} + e_{yj}) = \bar{c}$.

The representative consumer of country i derives utility $U^i(x_i, y_i)$ from consuming the amounts x_i and y_i of the goods X^i and Y, respectively. His or her income is $z_i := g_{xi} + g_{yi} + t_{xi}e_{xi} + t_{yi}e_{yi}$. That income consists of transferred profits $g_{xi} := p_{xi}x_{si} - (p_e + t_{xi})e_{xi}$ and $g_{yi} := p_y y_{si} - \pi_e(e_{yi} - c_{yi}) - (p_e + t_{yi})e_{yi}$ and of recycled tax revenues $t_{xi}e_{xi}$ and $t_{yi}e_{yi}$. The consumer spends her income on the goods X^i and Y and hence observes the budget equation

$$z_i = p_{xi}x_i + p_y y_i. \tag{4}$$

The definitions of z_i, g_{xi} and g_{yi} combined with (1) and (4) yield country i's trade balance

$$p_y(y_{si} - y_i) + \pi_e(c_{yi} - e_{yi}) - p_e(e_{xi} + e_{yi}). \tag{5}$$

In the n-country economy described above a policy consists of a choice of instruments $\mathbf{t}_x := (t_{x1}, \dots, t_{xn}) \in \mathbb{R}^n_+, \mathbf{t}_y := (t_{y1}, \dots, t_{yn}) \in \mathbb{R}^n_+, \mathbf{c} := \{c_1, \dots, c_n\} \in C := \left\{\mathbf{c} \in \mathbb{R}^n_+ \;\middle|\; \sum_j c_j = \bar{c}\right\}$ and $\mathbf{c}_y(\mathbf{c}) := [c_{y1}(\mathbf{c}), \dots, c_{yn}(\mathbf{c})] \in C_y(\mathbf{c}) := [0, c_1] \times [0, c_2] \times \dots \times [0, c_n]$. With $\mathbf{c} \in C$ and $\mathbf{c}_y(\mathbf{c}) \in C_y(\mathbf{c})$ being fixed, the emissions ceiling (3) for country i's non-ETS sector is also determined. As noted above, this emissions cap $c_i - c_{yi}$ is implemented through an appropriate choice of t_{xi}. Hence if we take $\mathbf{c}, \mathbf{c}_y(\mathbf{c})$ and $\mathbf{t}_y$ as policy decision variables, as we will do, the tax rates $\mathbf{t}_x$ are endogenous variables rather than independent policy parameters.

Having introduced the necessary notation and described the structure of the model we now define the competitive equilibrium as follows:

Let the world market prices p_e and p_y, and some policy $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ be given. The prices[9] $\pi_e, \mathbf{p}_x$, *the tax rates $\mathbf{t}_x$, and the allocation $(\mathbf{x}_s, \mathbf{e}_x, \mathbf{y}_s, \mathbf{e}_y, \mathbf{x}, \mathbf{y})$ constitute a competitive equilibrium of the n-country economy, if (1), (2) and (3) hold and if for $i = 1, \dots, n$:*[10]

[9]Throughout the paper bold letters denote row vectors, e.g. $\mathbf{p}_x := (p_{x1}, \dots, p_{xn}) \in \mathbb{R}^n_+$.

[10]The variables below that are marked by a wiggle (like "$\tilde{e}_{xi}$") are meant to vary over $\mathbb{R}_+$. In contrast, when there is no wiggle we deal with an equilibrium value of the respective variable.

- (x_{si}, e_{xi}) *satisfies* $e_{xi} = \text{argmax}\ \left[p_{xi}X^i(\tilde{e}_{yi}) - (p_e + t_{xi})\tilde{e}_{xi}\right]$ *and* $x_{si} = X^i(e_{xi})$,

- (y_{si}, e_{yi}) *satisfies* $e_{yi} = \text{argmax}\ \left[p_y Y^i(\tilde{e}_{yi}) - \pi_e(\tilde{e}_{yi} - c_{yi}) - (p_e + t_{yi})\tilde{e}_{yi}\right]$ *and* $y_{si} = Y^i(e_{yi})$,

- (x_i, y_i) *satisfies* $(x_i, y_i) = \text{argmax}\ \left[U^i(\tilde{x}_i, \tilde{y}_i)\ s.t.\ (4)\right]$.

Assuming that production functions are concave and utility functions are quasi-concave, it can be shown that for given p_e, p_y and policy[11] $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ (with appropriate upper bounds on the tax rates $\mathbf{t}_y$) a competitive equilibrium exists and is unique. However, such equilibria are not cost effective, in general, and the equilibrium distribution of utilities will crucially depend on the policy $[\mathbf{c}, \mathbf{c}_y, (\mathbf{c}), \mathbf{t}_y]$ chosen. To obtain a clear focus on distribution, we disentangle distribution from allocative inefficiency (for the major part of the paper) by restricting our analysis to cost-effective policies. The next Section 3 serves to characterize such policies and to explore the distributional impact of variations in cost-effective policies that implement the predetermined overall cap $\bar{c}$.

3 Distributional impacts of variations in cost-effective carbon control

Eichner and Pethig (2009) show that cost effectiveness is attained if and only if

$$t_{xi} = t_x \quad \text{and} \quad t_{yi} = t_y \quad \text{for all} \quad i = 1, \ldots, n \quad \text{and} \quad t_x = \pi_e + t_y. \tag{6}$$

In policies satisfying (6) the national permit caps $\mathbf{c}_y(\mathbf{c})$ are endogenously determined as to equalize the marginal abatement costs across sectors and across countries so that the caps $\mathbf{c}_y(\mathbf{c})$ are no independent policy parameters anymore. Therefore, *cost-effective* policies are completely described by $(\mathbf{c}, t_y)$. For convenience of notation, we will write $\pi_e(\mathbf{c}, t_y), p_{xi}(\mathbf{c}, t_y), x_i(\mathbf{c}, t_y)$ etc. when referring to the levels of variables belonging to the *cost-effective* competitive equilibrium associated to the policy $(\mathbf{c}, t_y)$. Moreover, we will denote the entire equilibrium as $E(\mathbf{c}, t_y) := [P(\mathbf{c}, t_y), A(\mathbf{c}, t_y)]$, where $P(\mathbf{c}, t_y) := [\pi_e(\mathbf{c}, t_y), \mathbf{p}_x(\mathbf{c}, t_y)]$ are the equilibrium prices and where $A(\mathbf{c}, t_y) := [\mathbf{x}_s(\mathbf{c}, t_y), \mathbf{e}_x(\mathbf{c}, t_y), \mathbf{y}_s(\mathbf{c}, t_y), \mathbf{e}_y(\mathbf{c}, t_y), \mathbf{y}(\mathbf{c}, t_y)]$ is the equilibrium allocation for the group of countries.

[11] We postpone the specification of the domain of tax rates $\mathbf{t}_y$ that are consistent with equilibrium. See the equations (7).

We have yet to specify the domain of all $(\mathbf{c}, t_y)$ for which a cost-effective equilibrium exists. Since we keep the overall cap $\bar{c}$ for the group of countries constant throughout the paper, we have $\mathbf{c} \in C$. To determine the domain of t_y for which cost-effective equilibria exist, observe that raising t_y reduces aggregate emissions, ceteris paribus. Increasing t_y tends to reduce the equilibrium permit price π_e and perhaps also increases the price p_{xi}. However, for sufficiently large t_y the permit price π_e is eventually driven down to zero, and with further increases in t_y total emissions would fall short of the EU cap $\bar{c}$. To capture the borderline case let $\mathbf{c} \in C$ be given and consider $\bar{t}_y > 0$ defined by

$$\begin{aligned} &\sum_j \left[e_{xj}(\mathbf{c}, \bar{t}_y) + e_{yj}(\mathbf{c}, \bar{t}_y)\right] = \bar{c} \quad \text{and} \quad \pi_e(\mathbf{c}, \bar{t}_y) = 0 \\ &\text{and} \quad \sum_j \left[e_{xj}(\mathbf{c}, t_y) + e_{yj}(\mathbf{c}, t_y)\right] < \bar{c} \quad \text{and} \quad \pi_e(\mathbf{c}, t_y) = 0 \quad \text{for all} \quad t_y > \bar{t}_y. \end{aligned} \tag{7}$$

According to (7) the value of $\bar{t}_y$ is contingent on the choice of the distribution $\mathbf{c} \in C$, which we indicate by writing $\bar{t}_y(\mathbf{c})$. Yet closer inspection shows

Proposition 1.

(i) $\bar{t}_y(\mathbf{c}) = \bar{t}_y(\mathbf{c}') = \bar{t}_y$ *and hence* $E\left[\mathbf{c}, \bar{t}_y(\mathbf{c})\right] = E\left[\mathbf{c}', \bar{t}_y(\mathbf{c}')\right] = E(\bar{t}_y)$ *for all* $\mathbf{c}, \mathbf{c}' \in C$.

(ii) Suppose an emissions target $\bar{c}$ *for the group of countries is to be met exclusively through an emissions tax at uniform rate* $t_{xi} = t_{yi} = t$*, all* i*, and denote by* $E(\bar{t})$ *the tax-only equilibrium, in which the emissions tax rate* $\bar{t}$ *implements the cap* $\bar{c}$*. Comparing* $E(\bar{t})$ *and* $E(\bar{t}_y)$ *from Proposition 1(i) yields* $E(\bar{t}) = E(\bar{t}_y)$ *and* $\bar{t} = \bar{t}_y$*.*

Proof.

Ad (i). Observe that depending on the choice of $\mathbf{c} \in C$, exports and imports of permits may still be carried out in the equilibrium $E(\mathbf{c}, \bar{t}_y)$ but at zero price, because those transactions leave the consumer's income unchanged. Therefore the equilibria $E(\mathbf{c}, \bar{t}_y) = E(\mathbf{c}', \bar{t}_y)$ for all $\mathbf{c}, \mathbf{c}' \in C$ are identical, if one keeps the price $p_{xi}(\mathbf{c}, \bar{t}_y)$ also unchanged under the policies $(\mathbf{c}, \bar{t}_y), (\mathbf{c}', \bar{t}_y)$, for all $\mathbf{c}, \mathbf{c}' \in C$.

Ad (ii). Suppose an equilibrium $E(\mathbf{c}, \bar{t}_y)$ is given for some $\mathbf{c} \in C$. With $t_y = \bar{t}_y$ being presupposed, the equations (6) and $\pi_e(\mathbf{c}, \bar{t}_y) = 0$ imply $t_x = \bar{t}_y$. Hence there is an emissions tax that is uniform across all sectors and countries. In that scenario the ETS can be abolished without any displacement effects because owing to $\pi_e(\mathbf{c}, \bar{t}_y) = 0$ the ETS neither creates nor transfers any wealth. Thus we can switch from a limiting case of mixed policy instruments (formally belonging to box 2 in Table 1) to a tax-only policy (box 3 in Table 1). ■

We now address the question as to how equilibria differ between alternative policies $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$. Primarily we are interested in the distribution of national welfares associated with different policies. For convenience of notation let us refer to that distribution as $\mathbf{u}(\mathbf{c}, t_y) := [u_1(\mathbf{c}, t_y), \dots, u_n(\mathbf{c}, t_y)]$, where $u_i(\mathbf{c}, t_y)$ is country i's welfare associated to the equilibrium $E(\mathbf{c}, t_y)$. We seek to answer the question as to what the impact of country i's welfare is of variations

- in the distribution of permit endowments when the overlapping tax rate remains constant ($\mathrm{d}c_i = -\sum_{j \neq i} \mathrm{d}c_j \neq 0$ and $\mathrm{d}t_y = 0$) and
- in the overlapping tax rate when the distribution of permit endowments remains constant ($\mathrm{d}c_j = 0$ for all j and $\mathrm{d}t_y \neq 0$).

Consider first policies $(\mathbf{c}, t_y)$ with t_y being fixed. If we start from an equilibrium $E(\mathbf{c}, t_y)$ and consider small changes $\mathrm{d}c_i$ in country i's cap ($i = 1, \dots, n$) under the constraint $\sum_j \mathrm{d}c_j = 0$, the comparative static effects of $\mathrm{d}c_i$ (Appendix) are[12]

$$\frac{\mathrm{d}u_i}{\lambda_i \mathrm{d}c_i} = \left[\frac{t_y(\alpha_i\delta_i - \beta_i\gamma_i + \alpha_i D_z^i \Delta e_{yi}) + \gamma_i \Delta e_{yi}}{\gamma_i}\right] \frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} + \frac{\alpha_i t_y D_z^i \pi_e}{\gamma_i} + \pi_e, \tag{8a}$$

$$\frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} = \frac{\frac{\alpha_i D_z^i \pi_e}{\gamma_i}}{\sum_j \frac{\beta_j\gamma_j - \alpha_j\delta_j - D_z^j \Delta e_{yj}}{\gamma_j}}, \tag{8b}$$

$$\frac{\mathrm{d}p_{xi}}{\mathrm{d}c_i} = \frac{(\delta_i + D_z^i \Delta e_{yi})}{\gamma_i} \frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} + \frac{D_z^i \pi_e}{\gamma_i}. \tag{8c}$$

Although the sign of the effects of increasing c_i are ambiguous in general, the terms (8a) - (8c) simplify considerably if we restrict our attention to quasi-linear utility functions $U^i(x_i, y_i) = V^i(x_i) + y_i$ with V^i being increasing and strictly concave.[13] For this special functional form the income effect of the demand for good X^i is zero ($D_z^i = 0$) which turns the equations (8a) - (8c)

[12] $\alpha_i, \beta_i, \delta_i, \gamma_i$ are defined in the Appendix and $\Delta e_{yi} := c_i - e_{xi} - e_{yi}$ is the amount of permits exported or imported by country i.

[13] It is sufficient for the purpose of the present paper to demonstrate the distributional consequences for quasi-linear utility functions only, because our focus is on distributional *equivalence* of policies' rather than on a full characterization of those policies distributional *impacts*.

into

$$\frac{\mathrm{d}u_i}{\lambda_i \mathrm{d}c_i} = \pi_e > 0, \qquad \frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} = \frac{\mathrm{d}p_{xi}}{\mathrm{d}c_i} = 0. \tag{8d}$$

The results in (8d) are as expected. Increasing country i's cap increases private income in country i with the straightforward consequence that its residents are better off. In addition, (8d) reveals that quasi-linear utility functions eliminate spillover effects on the market of good X^i.

Suppose next that $\mathbf{c} \in C$ is fixed and that starting from $t_y = 0$, the tax rate t_y is successively raised. Eichner and Pethig (2008) determine the comparative static effects of $\mathrm{d}t_y$ as

$$\frac{\mathrm{d}u_i}{\lambda_i \mathrm{d}t_y} = t_y \left(\frac{\alpha_i\delta_i - \beta_i\gamma_i}{\gamma_i}\right)\left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1\right) + \left(\frac{\alpha_i t_y D_z^i + \gamma_i}{\gamma_i}\right)\Delta e_{yi}\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}, \tag{9a}$$

$$\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -\frac{1}{1 + \frac{\sum_j \frac{\alpha_j D_z^i}{\gamma_j}\Delta e_{yj}}{\sum_j \frac{\alpha_j\delta_j - \beta_j\gamma_j}{\gamma_j}}}, \tag{9b}$$

$$\frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} = \frac{\delta_i + \Delta e_{yi} D_z^i}{\gamma_i}\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + \frac{\delta_i}{\gamma_i}. \tag{9c}$$

Again, for quasi-linear utility functions there are no interdependence effects on the market of good X^i such that the equations (9a) - (9c) simplify to[14]

$$\frac{\mathrm{d}u_i}{\mathrm{d}t_y} \begin{cases} > 0, & \text{if country } i \text{ imports permits,} \\ < 0, & \text{if country } i \text{ exports permits,} \end{cases} \tag{9d}$$

$$\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -1 \qquad \text{and} \qquad \frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} = 0. \tag{9e}$$

To sum up, the policy space under review is $C \times [0, \bar{t}_y]$ and every policy $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$ has an impact on the welfare distribution $\mathbf{u}(\mathbf{c}, t_y)$ through the policy parameter $\mathbf{c} \in C$ as well as through the policy parameter $t_y \in [0, \bar{t}_y]$. Since in general the distributional effects of both policy parameters are interdependent, we need to consider both of them for assessing correctly the net impact of variations in either parameter. In the next section we show that for all mixed policies (box 2 in Table 1) as well as for the tax-only policy (box 3 in Table 1) there is a policy $(\mathbf{c}, 0) \in C \times \{0\}$ (box 1 in Table 1) that leaves the welfare distribution unchanged.

[14]Eichner and Pethig (2008) show that these results do not hold, in general, when utility functions are not quasi-linear.

4 Equivalence of policies with and without overlapping instruments

We have demonstrated that changes in the utility profile $\mathbf{u}(\mathbf{c}, t_y)$ can be brought about either by varying t_y while keeping $\mathbf{c}$ constant or by varying $\mathbf{c}$ while keeping t_y constant (setting perhaps $t_y = 0$). This observation suggests to examine the possibility of neutralizing the welfare effects of an exogenous change in t_y by an appropriate change in $\mathbf{c}$. In other words, we want to answer the question whether for some given policy $(\mathbf{c}, t_y) \in C \times]0, \bar{t}_y[$ one can find $\tilde{\mathbf{c}} \in C, \tilde{\mathbf{c}} \neq \mathbf{c}$, such that $\mathbf{u}(\tilde{\mathbf{c}}, 0) = \mathbf{u}(\mathbf{c}, t_y)$. Such an equivalence exists, indeed, and is established in

Proposition 2.

Define $\tilde{\mathbf{c}}(\mathbf{c}, t_y) := [\tilde{c}_1(\mathbf{c}, t_y), \ldots, \tilde{c}_n(\mathbf{c}, t_y)]$ *by*

$$\tilde{c}_i(\mathbf{c}, t_y) := \frac{\pi_e(\mathbf{c}, t_y)c_i + t_y[e_{xi}(\mathbf{c}, t_y) + e_{yi}(\mathbf{c}, t_y)]}{\pi_e(\mathbf{c}, t_y) + t_y}, \quad i = 1, \ldots, n. \tag{10}$$

(i) For every $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$ *it is true that* $\tilde{\mathbf{c}}(\mathbf{c}, t_y) \in C$.

(ii) For every $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$ *the competitive equilibrium* $E[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ *associated to policy* $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ *satisfies*

$$A[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0] = A(\mathbf{c}, t_y) \tag{11}$$

and

$$\pi_e[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0] = \pi_e(\mathbf{c}, t_y) + t_y \quad \text{and} \quad p_{xi}[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0] = p_{xi}(\mathbf{c}, t_y) \qquad i = 1, \ldots, n. \tag{12}$$

Proof. To avoid clumsy notation we use the short form $(\cdot)$ for $(\mathbf{c}, t_y)$.

Ad (i). Summation of (10) and invoking (2) immediately yields

$$\sum_j \tilde{c}_j(\cdot) = \frac{\pi_e(\cdot) \sum_j c_j + t_y \sum_j [e_{xj}(\cdot) + e_{yj}(\cdot)]}{\pi_e(\cdot) + t_y} = \frac{[\pi_e(\cdot) + t_y] \sum_j c_j}{\pi_e(\cdot) + t_y} = \bar{c}.$$

Ad (ii). Let $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$, consider $\tilde{\mathbf{c}}(\mathbf{c}, t_y)$ as defined in (11) and suppose the prices

$$\tilde{\pi}_e = \pi_e(\cdot) + t_y \quad \text{and} \quad \tilde{p}_{xi} = p_{xi}(\cdot) \qquad i = 1, \ldots, n \tag{13}$$

prevail. First we determine the allocation $\tilde{A} := (\tilde{\mathbf{x}}_s, \tilde{\mathbf{e}}_x, \tilde{\mathbf{y}}_s, \tilde{\mathbf{e}}_y, \tilde{\mathbf{x}}, \tilde{\mathbf{y}})$ that results from optimizing behavior of all agents when they face the prices $(\tilde{\pi}_e, \tilde{\mathbf{p}}_x)$. In equilibrium the first-order conditions of profit maximization are

$$p_y Y^i\left[e_{yi}(\cdot)\right] = p_e + \pi_e(\cdot) + t_y \quad \text{and} \quad p_{xi} X^i\left[e_{xi}(\cdot)\right] = p_e + t_x = p_e + \pi_e(\cdot) + t_y. \tag{14}$$

On the other hand, if policy $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ and prices $(\tilde{\pi}_e, \tilde{\mathbf{p}}_x)$ are given, the firms in country i choose $\tilde{e}_{xi}$ and $\tilde{e}_{yi}$, respectively, which are implicitly defined by

$$p_y Y^i(\tilde{e}_{yi}) = p_e + \tilde{\pi}_e \quad \text{and} \quad \tilde{p}_{xi}\tilde{X}^i(\tilde{e}_{xi}) = p_e + \tilde{\pi}_e. \tag{15}$$

Combining (13), (14) and (15) immediately yields

$$\tilde{e}_{hi} = e_{hi}(\cdot) \quad \text{and} \quad \tilde{h}_{si} = h_{si}(\cdot) \quad \text{for} \quad h = x, y. \tag{16}$$

Next, we wish to show that $\tilde{h}_i = h_i(\cdot)$ for $h = x, y$. To that end invoke (13) to transform (10) as follows:

$$\begin{aligned}
\tilde{\pi}_e \tilde{c}_i(\cdot) &= \pi_e(\cdot)c_i + t_y[e_{xi}(\cdot) + e_{yi}(\cdot)], \\
\tilde{\pi}_e \tilde{c}_i(\cdot) - \tilde{\pi}_e(\cdot)[e_{xi}(\cdot) + e_{yi}(\cdot)] &= \pi_e(\cdot)c_i + t_y[e_{xi}(\cdot) + e_{yi}(\cdot)] - \tilde{\pi}_e[e_{xi}(\cdot) + e_{yi}(\cdot)], \\
\tilde{\pi}_e\left[\tilde{c}_i(\cdot) - e_{xi}(\cdot) - e_{yi}(\cdot)\right] &= \pi_e(\cdot)[c_i - e_{xi}(\cdot) + e_{yi}(\cdot)] - \tilde{\pi}_e[e_{xi}(\cdot) + e_{yi}(\cdot)]. \quad (17)
\end{aligned}$$

In equilibrium $E(\cdot)$, the consumer's income is

$$z_i(\cdot) = p_{xi}(\cdot)x_{si}(\cdot) + p_y y_{si}(\cdot) - p_e[e_{xi}(\cdot) + e_{yi}(\cdot)] - \pi_e(\cdot)[c_i - e_{xi}(\cdot) - e_{yi}(\cdot)]. \tag{18}$$

If policy $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ and prices $(\tilde{\pi}_e, \mathbf{p}_x)$ are given, the income is

$$\tilde{z}_i = \tilde{p}_{xi}\tilde{x}_{si} + p_y\tilde{y}_{si} - p_e(\tilde{e}_{xi} + \tilde{e}_{yi}) - \tilde{\pi}_e[\tilde{c}_i(\cdot) - \tilde{e}_{xi} - \tilde{e}_{yi}].$$

From (13), (16), (17) and (18) follows $\tilde{z}_i = z_i(\cdot)$. Consequently the consumer's budget constraint in $E(\mathbf{c}, t_y)$ is the same as under policy $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ when prices satisfy (13). The straightforward conclusion is $\tilde{x}_i = x_i(\cdot)$ and $\tilde{y}_i = y_i(\cdot)$. Thus we have shown that $\tilde{A} = A(\mathbf{c}, t_y)$. Since the allocation $A(\mathbf{c}, t_y)$ is an equilibrium allocation, so is $\tilde{A}$ when prices are given by (13). This observation proves (11) and (12). ■

According to Proposition 2, for each policy $(\mathbf{c}, t_y)$ there exists a unique *equivalent* policy $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ that does not make use of any overlapping emissions tax in the countries' ETS sectors. The governments of all countries are indifferent with respect to these policies because each policy produces the same resource allocation: $A[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0] = A(\mathbf{c}, t_y)$ and hence switching policies leaves all countries' welfare positions unchanged.

In view of $e_{hi}(\mathbf{c}, t_y) = e_{hi}[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ for $h = x, y$ and $i = 1, \ldots, n$ equation (10) implies, in fact, that the values of permits imported or exported are the same under both policy schemes $(\mathbf{c}, t_y)$ and $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$. As a consequence, country i's income remains unchanged which leaves the representative consumer's demand for consumption goods unaffected when $p_{xi}[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0] = p_{xi}(\mathbf{c}, t_y)$.

Proposition 2 also covers the special case $t = \bar{t}_y$, since $\tilde{c}_i(\bar{t}_y)$ follows immediately from (10) for $\pi_e = 0$. We have thus supplemented Proposition 1 by showing that there exists a unique vector of national caps, $\tilde{\mathbf{c}}(\bar{t}_y)$, defined in (10) such that the policy $[\tilde{\mathbf{c}}(\bar{t}_y), 0]$ belonging to box 1 in Table 1 is equivalent to the tax-only policy (box 3 in Table 1). In other words, the tax-only policy is equivalent to a particular vector of property-right shares of the group's total endowment of tradable permits, $\bar{c}$, that the individual countries hold. If the vector $\tilde{\mathbf{c}}(\bar{t}_y)$ of national emissions caps is chosen, net exports and imports of permits are zero in the equilibrium $E\left[\tilde{\mathbf{c}}(\bar{t}_y), 0\right]$. In other words, the tax-only policy $\bar{t}_y$ is equivalent to the ETS-only policy $[\tilde{\mathbf{c}}(\bar{t}_y), 0]$ characterized by zero permit exports and imports.

Given the equivalence between $(\mathbf{c}, t_y)$ and $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ it is natural to ask what the sign and the magnitude are of the differences $c_i - \tilde{c}_i(\mathbf{c}, t_y)$ and $c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)$ and how these differences vary with c_i and t_y, respectively. The answers are provided in

Proposition 3.

Suppose the policy $(\mathbf{c}, t_y) \in C \times]0, \bar{t}_y]$ is applied with $t_y > 0$.

(i) $c_i < \tilde{c}_i(\mathbf{c}, t_y)$ [$c_i > \tilde{c}_i(\mathbf{c}_i, t_y)$], if country i imports [exports] permits.

(ii) Consider an economy with quasi-linear utility functions. The impact of changes in c_i and t_y on the differences $c_i - \tilde{c}_i(\mathbf{c}, t_y)$ and $c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)$ are, respectively,[15]

$$\frac{\mathrm{d}[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{\mathrm{d}c_i} = \frac{t_y}{(\pi_e + t_y)^2} \quad \textit{and} \quad \frac{\mathrm{d}[c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)]}{\mathrm{d}c_i} = \frac{t_y}{(\pi_e + t_y)^2}, \tag{19a}$$

$$\frac{\mathrm{d}[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{\mathrm{d}t_y} = \frac{\Delta e_{yi}}{\pi_e + t_y} \quad \textit{and} \quad \frac{\mathrm{d}[c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)]}{\mathrm{d}t_y} = \frac{\Delta e_{yi}}{\pi_e + t_y}, \tag{19b}$$

where $\Delta e_{yi} := c_i - e_{xi}(\mathbf{c}, t_y) - e_{yi}(\mathbf{c}, t_y)$.

[15] The variation $\mathrm{d}c_i$ is carried out under the constraint $\sum_j \mathrm{d}c_j = 0$.

Proof.

Ad (i). Proposition 3(i) is straightforward from rewriting (10) as

$$\tilde{c}_i(\mathbf{c}, t_y) = c_i - \frac{t_y \Delta e_{yi}}{\pi_e + t_y}.$$

Ad (ii). Differentiation of $\tilde{c}_i(\mathbf{c}, t_y)$ with respect to c_i and t_y yields, after some rearrangement of terms,

$$\frac{\mathrm{d}\tilde{c}_i}{\mathrm{d}c_i} = 1 - \frac{t_y \frac{\mathrm{d}\Delta e_{yi}}{\mathrm{d}c_i} - t_y \frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} \Delta e_{yi}}{(\pi_e + t_y)^2}, \tag{20a}$$

$$\frac{\mathrm{d}\tilde{c}_i}{\mathrm{d}t_y} = -\frac{\pi_e \left(1 - \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} \cdot \frac{t_y}{\pi_e}\right)}{(\pi_e + t_y)^2} \Delta e_{yi} - \frac{t_y}{\pi_e + t_y} \cdot \frac{\mathrm{d}\Delta e_{yi}}{\mathrm{d}t_y}. \tag{20b}$$

Differentiate $c_i - \tilde{c}_i(\mathbf{c}, t_y)$ with respect to c_i and t_y, respectively, and make use of (20a) and (20b) to get

$$\frac{\mathrm{d}[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{\mathrm{d}c_i} = \frac{t_y \frac{\mathrm{d}\Delta e_{yi}}{\mathrm{d}c_i} - t_y \frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} \Delta e_{yi}}{(\pi_e + t_y)^2}, \tag{21a}$$

$$\frac{\mathrm{d}[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{\mathrm{d}t_y} = \frac{\pi_e \left(1 - \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} \cdot \frac{t_y}{\pi_e}\right)}{(\pi_e + t_y)^2} \Delta e_{yi} + \frac{t_y}{\pi_e + t_y} \frac{\mathrm{d}\Delta e_{yi}}{\mathrm{d}t_y}. \tag{21b}$$

Consider first (21a). From (8a) and (8c) we infer that $\frac{\mathrm{d}p_{xi}}{\mathrm{d}c_i} = \frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} = 0$ if utility functions are quasi-linear $\left(D_z^i = 0\right)$. In addition, (A23) in the Appendix implies $\frac{\mathrm{d}e_{xi} + \mathrm{d}e_{yi}}{\mathrm{d}c_i} = 0$ for $\frac{\mathrm{d}p_{xi}}{\mathrm{d}c_i} = \frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} = 0$ and hence $\frac{\mathrm{d}\Delta e_{yi}}{\mathrm{d}c_i} = 1$ follows. Making use of this information in (21a) we get $\frac{\mathrm{d}[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{\mathrm{d}c_i} = \frac{t_y}{(\pi_e + t_y)^2}$. Next, we differentiate $c_{yi}(\mathbf{c}, t_y) - \tilde{c}_{yi}(\mathbf{c}, t_y)$ with respect to c_i to obtain $\frac{\mathrm{d}[c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)]}{\mathrm{d}c_i} = \frac{\mathrm{d}c_{yi}}{\mathrm{d}c_i} - \frac{\mathrm{d}\tilde{c}_{yi}}{\mathrm{d}c_i}$. Accounting for $c_{yi} = c_i - e_{xi}$, $\tilde{c}_{yi} = \tilde{c}_i - \tilde{e}_{xi}$ and $\frac{\mathrm{d}e_{xi}}{\mathrm{d}c_i} = \frac{\mathrm{d}\tilde{e}_{xi}}{\mathrm{d}c_i} = 0$ from (A22) yields $\frac{\mathrm{d}[c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)]}{\mathrm{d}c_i} = \frac{\mathrm{d}[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{\mathrm{d}c_i}$ which in turn establishes (19a).

Now we turn to (21b). Implicit in (14) the demand for fossil fuel (and for emissions permits) of country i is given by the functions $E^{xi}(p_{xi}, \pi_e + t_y)$ and $E^{yi}(\pi_e + t_y)$. Totally differentiating these functions E^{xi} and E^{yi} with respect to t_y gives us

$$\frac{\mathrm{d}E^{xi}(p_{xi}, \pi_e + t_y)}{\mathrm{d}t_y} = \frac{\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1}{p_{xi} X_{ee}^i} - \frac{x_{si}}{p_{xi} X_{ee}^i} \cdot \frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} \quad \text{and} \quad \frac{\mathrm{d}E^{yi}(\pi_e + t_y)}{\mathrm{d}t_y} = \frac{\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1}{Y_{ee}^i}. \tag{22}$$

Obviously, in view of (9e), i.e. $\frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} = 0$ and $\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -1$ for $i = 1, \ldots, n$, the equations (22) imply

$$\frac{\mathrm{d}E^{xi}(p_{xi}, \pi_e + t_y)}{\mathrm{d}t_y} = \frac{\mathrm{d}E^{yi}(\pi_e + t_y)}{\mathrm{d}t_y} = \frac{\mathrm{d}(e_{xi} + e_{yi})}{\mathrm{d}t_y} = 0. \tag{23}$$

Next, we differentiate $\Delta e_{yi} = c_i - e_{xi}(\mathbf{c}, t_y) - e_{yi}(\mathbf{c}, t_y)$ with respect to t_y to obtain $\frac{\Delta e_{yi}}{\mathrm{d}t_y} = 0$. Using this information and $\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -1$ in (21b) establishes $\frac{\mathrm{d}[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{\mathrm{d}t_y} = \frac{\Delta e_{yi}}{\pi_e + t_y}$. Invoking the same arguments as above straightforwardly leads to $\frac{\mathrm{d}[c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)]}{\mathrm{d}t_y} = \frac{\mathrm{d}[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{\mathrm{d}t_y}$. ■

According to Proposition 3(i), replacing policy $(\mathbf{c}, t_y)$ through the equivalent policy $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ implies that country i's emissions cap under the new policy $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ is greater [smaller] than under the policy $(\mathbf{c}, t_y)$, if country i imports [exports] permits when the policy $(\mathbf{c}, t_y)$ is applied. Moreover, the gap $|c_i - \tilde{c}_{yi}(\mathbf{c}, t_y)|$ is greater for a permit-exporting country i and smaller for a permit-importing country i, the greater is country i's initial emissions cap, c_i. However, raising the tax rate t_y in the initial policy $(\mathbf{c}, t_y)$ widens the gap $|c_i - \tilde{c}_{yi}(\mathbf{c}, t_y)|$ for both permit-exporting and permit-importing countries.

It should be noted, however, that if the assumption of quasi-linear utility functions is relaxed, the distributional impact of changes in c_i and t_y will be less clear-cut. Since markets are interdependent, an exogenous change in t_y must be expected to trigger repercussions in other markets so that the crucial presupposition of Proposition 3(ii), $(\mathrm{d}p_{xi}/\mathrm{d}c_i) = (\mathrm{d}p_{xi}/\mathrm{d}t_y) = 0$ *for all* i, is not satisfied, in general. However, if interdependence effects are present, general information cannot be gained from (20a) and (20b) neither on the sign nor on the magnitude of the differential quotients. In particular, the results (19a) and (19b) that changes in $\mathrm{d}c_i$ and $\mathrm{d}t_{yi}$ fully translate into a change in the permit cap $\tilde{c}_{yi}$ must be considered special cases.

To highlight the relevance of Proposition 3 regarding the distributional impact of fixing $\mathbf{c}$ and t_y in policies $(\mathbf{c}, t_y)$, suppose the group of countries has agreed on some distribution of emissions caps, $\mathbf{c} \in C$, satisfying certain equity criteria as in case of the EU burden sharing agreement. If the countries should have determined their "fair" distribution $\mathbf{c}$ without accounting for the preexisting overlapping tax(es), the true distributional impact of the policy $(\mathbf{c}, t_y)$ is unfair according to the equity criteria chosen.

The equivalence result established above is valid for cost-effective policies. However, the policy $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ applied in the EU is not cost effective because member states levy taxes in their ETS sectors at different rates $\mathbf{t}_y = (t_{y1}, \ldots, t_{yn})$ and may have failed to fix cost effective permit caps.[16] For these reasons there does not exist a cost-effective policy $(\tilde{\mathbf{c}}, t_y = 0), \tilde{\mathbf{c}} \in C$,

[16] Since the national tax rates $\mathbf{t}_y$ are not uniform in the EU, it is not second best, in general, to choose the allocation of national permit caps $\mathbf{c}_y(\mathbf{c})$, such that marginal abatement costs are the same across sectors and

that is equivalent to the EU policy $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$. Addressing this scenario, in the spirit of the equivalence result of Proposition 2 we are able to state

Proposition 4.

Let $\mathbf{c} \in C$ *and consider a policy* $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ *satisfying* $t_{yi} \neq t_{yj}$ *for some* $i, j = 1, \ldots, n$ *for which an equilibrium* $E[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ *exists. There is* $\hat{\mathbf{c}}[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] \in C$ *and a real number* $\alpha > 1$ *such that* $\mathbf{u}[\hat{\mathbf{c}}[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y], 0] = \alpha\, \mathbf{u}[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$.

Proof. Define

$$\rho_i[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] := \frac{u_i[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]}{u_1[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]} \qquad \text{for} \quad i = 1, \ldots, n$$

and determine the policy $(\hat{\mathbf{c}}, 0) \in C \times \{0\}$ by the equations

$$u_i(\hat{\mathbf{c}}, 0) = \rho_i[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] \cdot u_1(\hat{\mathbf{c}}, 0) \qquad \text{for} \quad i = 1, \ldots, n. \tag{24}$$

A distribution of national caps $\hat{\mathbf{c}} \in C$ satisfying (24) clearly exists and is located on the welfare possibility frontier generated by the welfare distributions $\mathbf{u}(\mathbf{c}, 0)$ prevailing in the equilibria associated to all policies $(\mathbf{c}, 0) \in C \times \{0\}$. By definition of $\hat{\mathbf{c}}$ it is true that

$$\frac{u_i(\hat{\mathbf{c}}, 0)}{u_i[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]} = \frac{u_j(\hat{\mathbf{c}}, 0)}{u_j[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]} \quad \text{for} \quad i, j = 1, \ldots, n.$$

Denote this ratio by α. Since $E[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ is not cost effective (because the tax rates t_{y_i} are presupposed to differ across countries) the welfare distribution of the equilibrium $E[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ is clearly located below the welfare distribution frontier. As a consequence, $\alpha > 1$. ■

Proposition 4 provides an interesting policy proposal for emissions control in the EU. It suggests to switch from the present cost-ineffective policy with overlapping emissions taxes to a cost-effective ETS without overlapping emissions taxes making all member states better off at a rate that is uniform across countries.

countries, as is optimal in case of cost-effective policies (Eichner and Pethig 2008). It is unlikely that the permit caps $\mathbf{c}_y(\mathbf{c})$ laid down in the national allocation plans of all member states are the second-best permit caps because there are no indications that the governments of the EU member states have (appropriately) accounted for the preexisting tax rates $\mathbf{t}_y$ in calculating those caps.

5 Methods of measuring the (re)distributional impact of carbon emissions control

This section focuses on cost-effective policies again and aims at *measuring* the distributional impact of those policies $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$. Our finding that the equilibrium associated to all policies $(\mathbf{c}, \bar{t}_y)$ is independent of $\mathbf{c}$ suggests taking the distributional impact of the tax-only policy as a benchmark for assigning national emissions caps. Recall that, according to Proposition 2, a given policy $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$ is equivalent to a pure ETS with $\tilde{\mathbf{c}}(\mathbf{c}, t_y)$, and that the tax-only policy $\bar{t}_y$ is equivalent to a pure ETS with $\tilde{\mathbf{c}}(t_y)$. From these observations the following measure of distribution is straightforward:

Measure I of Distribution.
Relative to the policy of implementing $\bar{c}$ with an emissions tax only that is uniform across all sectors and all countries the redistributional implication of policy $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$ is measured by

$$[\tilde{\mathbf{c}}(\mathbf{c}, t_y) - \tilde{\mathbf{c}}(\bar{t}_y)] \in \mathbb{R}^n. \tag{25}$$

Under conditions specified in Section 3 we know that switching from the tax-only policy to the policy $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$ makes country i better [worse] off if and only if $\tilde{c}_i(\mathbf{c}, t_y) > \tilde{c}_i(\bar{t}_y)$ $[\tilde{c}_i(\mathbf{c}, t_y) < \tilde{c}_i(\bar{t}_y)]$. The advantage of Measure I is to translate tax policies and policies mixing taxes and emissions trading into shares of permit endowments. Its downside is, however, that its link to the utility distribution is not unambiguous under general forms of utility functions and that it is not a monetary measure.

These limitations are overcome, however, by another straightforward measure that takes as benchmark the welfare associated to the tax-only policy. To construct that measure we first introduce a vector of transfer payments $\boldsymbol{\theta} := (\theta_{y1}, \ldots, \theta_{yn}) \in \mathbb{R}^n$ in an equilibrium with policy $(\mathbf{c}, t_y)$. As a result, the welfare of country i becomes equal to

$$u_i(\mathbf{c}, t_y; \theta_i) := U^i \left[D^i(\cdot), z_i(\mathbf{c}, t_y) + \theta_i - p_{xi}(\mathbf{c}, t_y) D^i(\cdot) \right], \tag{26}$$

when it receives the positive or negative transfer θ_i. In (26) $D^i(\cdot) := D^i \left[p_{xi}(\mathbf{c}, t_y), z_i(\mathbf{c}, t_y) + \theta_i \right]$ is the demand for good X^i.

Measure II of Distribution.

Relative to the policy of implementing $\bar{c}$ with an emissions tax only that is uniform across all sectors and all countries the redistributional implication of policy $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$ is measured by the monetary transfer $\boldsymbol{\theta}(\mathbf{c}, t_y) := [\theta_1(\mathbf{c}, t_y), \ldots, \theta_n(\mathbf{c}, t_y)] \in \mathbb{R}^n$, where for all i the monetary transfer $\theta_i = \theta_i(\mathbf{c}, t_y)$ is defined by[17]

$$u_i[\tilde{\mathbf{c}}(\bar{t}_y), 0; \theta_i] = u_i(\mathbf{c}, t_y; 0). \tag{27}$$

According to (27) $\theta_i(\mathbf{c}, t_y)$ is the amount of money country i needs to receive or to pay in order to shift its utility from the level $u_i[\tilde{\mathbf{c}}(\bar{t}_y), 0; \theta_i]$ to the level $u_i(\mathbf{c}, t_y; 0) = u_i(\mathbf{c}, t_y)$ which it actually enjoys in the equilibrium attained under the policy $(\mathbf{c}, t_y)$. Switching from $[\tilde{\mathbf{c}}(\bar{t}_y), 0]$ to $(\mathbf{c}, t_y)$ creates winners and losers. If $\theta_i(\mathbf{c}, t_y) > 0$, country i loses through that policy switch because it needs the compensation $\theta_i(\mathbf{c}, t_y) > 0$ in order to be indifferent between both policy schemes. Conversely, if $\theta_i(\mathbf{c}, t_y) < 0$ it gains through that policy switch because its income under policy $(\mathbf{c}, t_y)$ needs to be reduced by $\theta_i(\mathbf{c}, t_y)$ to make its utility level under policy $(\mathbf{c}, t_y)$ equal to the level it enjoys under the tax-only policy.

6 Concluding Remarks

Burden sharing is well known to be a crucial precondition for successful international carbon emissions control within the EU as well as world wide. In the present paper we do not address fairness in burden sharing but focus, instead, on the questions preceding the fairness issue, namely what the true national burdens are in hybrid EU-type emissions control policies and how to measure them. We show that when an ETS covering only part of all participating countries' economies is combined with an overlapping emissions tax the net impact on national welfares results from an integrated account of the partial welfare effects of both instruments. Our equivalence result allows expressing each country's net burden carried in a mixed policy as the net burden it carries in a hypothetical but equivalent ETS-only policy. In other words, the distributional impact of a uniform overlapping tax is thus 'translated' into changes in national emissions caps. The national net burdens are shown to be measurable as deviations from the burdens implied by the tax-only policy.

[17] θ_i is in spirit analogous to the Hicksean equivalent variation.

Our paper provides a message for parties involved in negotiations about an agreement on the distribution of national emissions caps in the context of a joint ETS. When major emissions taxes overlapping with the ETS exist, the negotiated national emissions caps are distorted indicators of national burdens, when the burdens implicit in the overlapping taxes are not taken into account. Rational burden sharing negotiations need to consider the 'burden impact' of both instruments. There are reasons to doubt whether the parties in the EU burden sharing agreement had at their disposal all the information about the incidence of their agreed-upon national caps that is needed to share the burden according to their own fairness criteria. The parties are advised to calculate their 'true' net burdens invoking the equivalence result established above and the associated measures.

In the major part of the paper we assume cost-effective mixed policies to avoid blurring distributional and efficiency effects. Yet the hybrid EU policy is not cost effective because, among other things, the extant national overlapping taxes are not uniform across countries. We were able to show that our procedure of specifying burdens for cost-effective mixed policies can be extended to the empirically relevant scenario of non-uniform taxes. In this case distributional equivalence is combined with an overall efficiency gain that may be distributed by increasing the welfare of all countries at a uniform rate. The economist's recommendation would be, of course, to eliminate the inefficiency through tax harmonization in the first place.

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Appendix: The comparative statics of changing the permit cap c_i

The cost-effective competitive equilibrium is determined by the equations

$$\begin{aligned}
\sum_j c_j &= \sum_j (e_{xj} + e_{yj}), && \text{(A1)}\\
x_{si} &= x_i \qquad i = 1, \ldots, n, && \text{(A2)}\\
x_{si} &= X^i(e_{xi}), \qquad i = 1, \ldots, n, && \text{(A3)}\\
x_i &= D^i(p_{xi}, z_i), \qquad i = 1, \ldots, n, && \text{(A4)}\\
z_i &= p_{xi} + y_{si} - p_e(e_{xi} + e_{yi}) + \pi_e(c_i - e_{xi} - e_{yi}), \quad i = 1, \ldots, n, && \text{(A5)}\\
y_{si} &= Y^i(e_{yi}), \qquad i = 1, \ldots, n, && \text{(A6)}\\
z_i &= p_{xi}x_i + y_i, \qquad i = 1, \ldots, n, && \text{(A7)}\\
p_{xi}X_e^i(e_{xi}) &= p_e + t_x, \qquad i = 1, \ldots, n, && \text{(A8)}\\
Y_e^i &= p_e + \pi_e + t_y, \qquad i = 1, \ldots, n, && \text{(A9)}\\
t_x &= \pi_e + t_y. && \text{(A10)}
\end{aligned}$$

In (A1) - (A10) good Y is chosen as numeraire. The demand function (A4) follows from the first-order condition for utility maximization. It is convenient to compress the system of equations

(A1) - (A10) as follows

$$\begin{aligned}
\sum_j c_j &= \sum_j (e_{xj} + e_{yj}), && \text{(A11)}\\
X^i(e_{xi}) &= D^i(p_{xi}, z_i), && \text{(A12)}\\
z_i &= p_{xi}X^i(e_{xi}) + Y^i(e_{yi}) - p_e(e_{xi} + e_{yi}) + \pi_e \Delta e_{yi}, && \text{(A13)}\\
p_{xi}X^i_e(e_{xi}) &= Y^i_e(e_{yi}), && \text{(A14)}\\
Y^i_e(e_{yi}) &= p_e + \pi_e + t_y, && \text{(A15)}\\
y_i &= Y^i(e_{yi}) - p_e(e_{xi} + e_{yi}) + \pi_e \Delta e_{yi}, && \text{(A16)}
\end{aligned}$$

where $\Delta e_{yi} := c_i - e_{xi} - e_{yi}$. Our aim is to determine through a comparative static analysis the impact of exogenous variations in the caps c_i **subject to the constraint** $\sum_j \mathrm{d}c_j = 0$. To that end (A11) - (A15) are totally differentiated.

$$\sum_j (\mathrm{d}e_{xj} + \mathrm{d}e_{yj}) = 0, \tag{A17}$$

$$X^i_e \mathrm{d}e_{xi} - D^i_p \mathrm{d}p_{xi} - D^i_z \mathrm{d}z_i = 0, \tag{A18}$$

$$\mathrm{d}z_i - x_i \mathrm{d}p_{xi} - t_y(\mathrm{d}e_{xi} + \mathrm{d}e_{yi}) - \Delta e_{yi}\mathrm{d}\pi_e - \pi_e \mathrm{d}c_i = 0, \tag{A19}$$

$$X^i_e \mathrm{d}p_{xi} + p_{xi}X^i_{ee}\mathrm{d}e_{xi} - Y^i_{ee}\mathrm{d}e_{yi} = 0, \tag{A20}$$

$$Y^i_{ee}\mathrm{d}e_{yi} - \mathrm{d}\pi_e = 0. \tag{A21}$$

Inserting $\mathrm{d}e_{yi} = \frac{\mathrm{d}\pi_e}{Y^i_{ee}}$ from (A21) in (A20) yields

$$\mathrm{d}e_{xi} = \frac{\mathrm{d}\pi_e}{p_{xi}X^i_{ee}} - \frac{X^i_e \mathrm{d}p_{xi}}{p_{xi}X^i_{ee}}. \tag{A22}$$

Summation of $\mathrm{d}e_{xi}$ from (A22) and $\mathrm{d}e_{yi}$ from (A21) gives

$$\mathrm{d}e_{xi} + \mathrm{d}e_{yi} = \alpha_i \mathrm{d}p_{xi} - \beta_i \mathrm{d}\pi_e, \tag{A23}$$

where $\alpha_i := -\frac{X^i_e}{p_{xi}X^i_{ee}} > 0$ and $\beta_i := -\left(\frac{1}{Y^i_{ee}} + \frac{1}{p_{xi}X^i_{ee}}\right) > 0$. Inserting (A23) in (A17) we obtain

$$\frac{\sum_j \alpha_j \mathrm{d}p_{xj}}{\sum_j \beta_j} = \mathrm{d}\pi_e. \tag{A24}$$

Next, we take advantage of (A23) to turn (A19) into

$$\mathrm{d}z_i = (x_i + \alpha_i t_y)\mathrm{d}p_{xi} + (\Delta e_{yi} - \beta_i t_y)\mathrm{d}\pi_e + \pi_e \mathrm{d}c_i. \tag{A25}$$

We make use of (A22) and (A25) to transform (A18) into

$$\mathrm{d}p_{xi} = \frac{(\delta_i + D_z^i \Delta e_{yi})}{\gamma_i}\mathrm{d}\pi_e + \frac{D_z^i \pi_e}{\gamma_i}\mathrm{d}c_i, \tag{A26}$$

where $\delta_i := \alpha_i - \beta_i t_y D_z^i$ and $\gamma_i := \alpha_i X_e^i - D_p^i - (x_i + \alpha_i t_y) D_z^i$. We insert (A26) into (A24) to obtain, after some rearrangement of terms,

$$\mathrm{d}\pi_e \left[\sum_j \left(\frac{\beta_j \gamma_j - \alpha_j \delta_j - D_z^j \Delta e_{yj}}{\gamma_j}\right)\right] = \sum_j \frac{\alpha_j D_z^j \pi_e}{\gamma_j}\mathrm{d}c_j. \tag{A27}$$

Next, we differentiate the utility function $u_i = U^i(x_i, y_i)$ and use the first-order condition of the consumer's utility maximization problem to get

$$\frac{\mathrm{d}u_i}{\lambda_i} = p_{xi}\mathrm{d}c_i + \mathrm{d}y_i, \tag{A28}$$

where λ_i is the marginal utility of income. From (A3), (A8) and (A10) we infer

$$\mathrm{d}x_i = X_{ee}^i \mathrm{d}e_{xi} = \frac{p_e + \pi_e + t_y}{p_{xi}}\mathrm{d}e_{xi}. \tag{A29}$$

From (A16) we obtain with the help of (A15)

$$\mathrm{d}y_i = t_y \mathrm{d}e_{yi} - (p_e + \pi_e)\mathrm{d}e_{xi} + \Delta e_{yi}\mathrm{d}\pi_e + \pi_e \mathrm{d}c_i. \tag{A30}$$

Inserting (A30) and (A29) in (A28) yields after some rearrangement of terms

$$\frac{\mathrm{d}u_i}{\lambda_i \mathrm{d}c_i} = t_y \frac{\mathrm{d}e_{xi} + \mathrm{d}e_{yi}}{\mathrm{d}c_i} + \Delta e_{yi}\frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} + \pi_e. \tag{A31}$$

From (A23) it follows that

$$\frac{\mathrm{d}e_{xi} + \mathrm{d}e_{yi}}{\mathrm{d}c_i} = \alpha_i \frac{\mathrm{d}p_{xi}}{\mathrm{d}c_i} - \beta_i \frac{\mathrm{d}\pi_e}{\mathrm{d}c_i}. \tag{A32}$$

(A26) yields

$$\frac{\mathrm{d}p_{xi}}{\mathrm{d}c_i} = \frac{(\delta_i + D_z^i \Delta e_{yi})}{\gamma_i}\frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} + \frac{D_z^i \pi_e}{\gamma_i}. \tag{A33}$$

Making use of (A33) in (A32) yields

$$\frac{\mathrm{d}e_{xi} + \mathrm{d}e_{yi}}{\mathrm{d}c_i} = \left(\frac{\alpha_i \delta_i - \beta_i \gamma_i + \alpha_i D_z^i \Delta e_{yi}}{\gamma_i}\right)\frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} + \frac{\alpha_i D_z^i \pi_e}{\gamma_i}. \tag{A34}$$

Finally, taking advantage of (A34) in (A31) establishes

$$\frac{\mathrm{d}u_i}{\lambda_i \mathrm{d}c_i} = \left[\frac{t_y(\alpha_i \delta_i - \beta_i \gamma_i + \alpha_i D_z^i \Delta e_{yi}) + \gamma_i \Delta e_{yi}}{\gamma_i}\right]\frac{\mathrm{d}\pi_e}{\mathrm{d}c_i} + \frac{\alpha_i t_y D_z^i \pi_e}{\gamma_i} + \pi_e. \tag{A35}$$