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Incidence of unilateral consumption taxes on world carbon emissions*

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Abstract

This note investigates the suitability of unilateral consumption taxes for alleviating climate change in a two-period two-country general equilibrium model with a finite stock of fossil fuel. We analyze the incidence of a unilateral consumption tax in the first period on world carbon emissions. If countries are identical or if the taxing country imports both fossil fuel and consumption goods in the second period, increases in the tax rate lower first-period carbon emissions in both countries implying a negative rate of carbon leakage.

JEL classification: H22, Q38, Q58
Key words: unilateral consumption tax, world emissions, leakage

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1 Introduction

In the Kyoto Protocol a number of countries have committed to reduce unilaterally their emissions of greenhouse gases. The effort of the abating countries may be offset to some extent by *carbon leakage* (Felder and Rutherford 1993), that is, by increasing carbon emissions in non-abating countries. Even worse, demand-reducing measures of abating countries may increase rather than decrease aggregate world emissions, which has been labelled *green paradox* (Sinn 2008).

The literature so far has analyzed leakage rates resulting from unilateral emission taxes.[1] Eichner and Pethig (2011) find that positive leakage rates are unavoidable and identify the intertemporal substitution elasticity of consumption and the demand elasticity for fossil fuel as the main determinants of a green paradox. Empirical estimates of carbon leakage based on large-scale simulation models range from low levels (Burniaux and Martins 2010) to leakage rates implying a green paradox (Babiker 2005). Essentially, carbon leakage and the green paradox, a fortiori, compromize the use of unilateral emission taxes as climate policy instruments.

The present paper sets up a two-period two-country general equilibrium with a non-renewable fossil energy resource and examines the incidence of a unilateral first-period consumption tax on first-period world carbon emissions. That incidence appears to be straightforward, at first glance. Consumption and production is shifted from the first to the second period decreasing the demand for fossil fuel and the discharge of emissions in the first period. At second glance, we observe countervailing price effects in the general equilibrium model. Nonetheless, informative results can be obtained if either both countries are identical or if the taxing country imports fossil fuel and consumer goods in the second period. In those cases which appear to be relevant scenarios the consumption tax turns out to reduce total first-period emissions and not only avoids leakage rates exceeding 100% (green paradox) but, on the contrary, implies a negative leakage rate, because both countries reduce their production and with it their consumption of fossil fuel in the first period.[2]

[1] Cap and trade systems are equivalent to *emission taxes* in most abstract analytical models.

[2] Negative leakage rates may also result from unilateral emission taxes in one-period models as pointed out by Copeland and Taylor (2005), Karp (2010) or Fullerton et al. (2011).

2 The model

Using essentially the analytical framework of Eichner and Pethig (2011), we consider a two-period model with two countries A and B.[3] In each period $t = 1, 2$ each country i produces the output x_{it}^s of the consumption good X with the input of fossil fuel, e_{it}, according to the strictly increasing and concave production function

$$x_{it}^s = X^i(e_{it}) \qquad i = A, B \tag{1}$$

The representative consumer of country i derives utility from consumption x_{it} in period $t = 1, 2$ according to the intertemporal CES utility function[4]

$$u_i = U^i(x_{i1}, x_{i2}) = \left(\gamma_1 x_{i1}^{\frac{\sigma_i - 1}{\sigma_i}} + \gamma_2 x_{i2}^{\frac{\sigma_i - 1}{\sigma_i}}\right)^{\frac{h\sigma_i}{\sigma_i - 1}} \qquad i = A, B, \tag{2}$$

with $\gamma_1, \gamma_2, h > 0$ and $\sigma_A = \sigma_B \equiv \sigma > 0, \sigma \neq 1$. In each period, good X and fossil fuel are traded on perfectly competitive world markets at prices p_{xt} and p_{et}, respectively. For $t = 1, 2$ the market equilibrium conditions are

$$x_{At}^s + x_{Bt}^s = x_{At} + x_{Bt}, \tag{3}$$

$$e_t = e_{At} + e_{Bt}, \tag{4}$$

where e_t is the fossil fuel supply in period t. The intertemporal constraint for fossil fuel is then given by

$$\bar{e} = e_1 + e_2. \tag{5}$$

In (5), $\bar{e}$ is a finite stock of fossil fuel. Country i owns the share $\alpha_i \in [0, 1]$ of that stock, where $\alpha_A + \alpha_B = 1$. Carbon emissions are generated in strict proportion to the amount of fossil fuel consumed. Hence with suitable definitions of units, e_{it} denotes fuel consumption as well as carbon emissions. The government of country A levies a unit tax at rate τ on

[3] The only difference to Eichner and Pethig (2011) is that their model features a third country which owns the resource fossil fuel whereas in the present paper the countries A and B are the owners of the fossil fuel stock. However, it can be shown that this modification leaves the results of Eichner and Pethig (2011) unaltered.

[4] The results derived in the sequel are also obtained for isoelastic utility functions of the form

$$U^i(x_{i1}, x_{i2}) = \begin{cases} \frac{\mu x_{i1}^{1-\eta}}{1-\eta} + \frac{1}{1+\rho} \cdot \frac{\mu x_{i2}^{1-\eta}}{1-\eta} & \text{for} \quad \mu > 0, \eta \neq 1, \\ \ln x_{i1} + \frac{1}{1+\rho} \ln x_{i2} & \text{for} \quad \eta = 1, \end{cases}$$

where $\rho \geq 0$. It is worth noting that both for CES and for isoelastic functions the intertemporal substitution elasticity is constant.

the domestic first-period consumption of good X.[5] In the (regulated) competitive market economy an aggregate resource firm extracts the entire stock of fossil fuel, $\bar{e}$, over both periods. In addition, in each country i an aggregate firm produces the consumption good X. With the discount rate being zero the profits are, respectively,

$$\begin{aligned} \Pi^i &:= \sum_t [p_{xt} X^i(e_{it}) - p_{et} e_{it}] \qquad i = A, B, \\ \Pi^F &:= \sum_t p_{et} e_t, \end{aligned}$$

where the extraction costs of the resource firm F are assumed to be zero. Maximizing profits yields the first-order conditions

$$p_{xt} X^i_{e_{it}} = p_{et} \qquad i = A, B \quad \text{and} \quad t = 1, 2, \tag{6}$$

$$p_{e1} = p_{e2}. \tag{7}$$

Equation (7) is a simplyfied Hotelling rule.

The representative consumer in country A and B, respectively, maximizes utility subject to her budget constraint

$$(p_{x1} + \tau) x_{A1} + p_{x2} x_{A2} = \Pi^{A*} + \alpha_A \Pi^{F*} + \tau x_{A1}, \tag{8}$$

$$p_{x1} x_{B1} + p_{x2} x_{B2} = \Pi^{B*} + \alpha_B \Pi^{F*}, \tag{9}$$

where Π^{i*} is the maximum profit of the firm in country $i = A, B$ and Π^{F*} is the maximum profit of the fossil fuel extracting firm. Utility maximization yields

$$\frac{U_{x_{A2}}}{U_{x_{A1}}} = \frac{p_{x2}}{p_{x1} + \tau} \quad \text{and} \quad \frac{U_{x_{B2}}}{U_{x_{B1}}} = \frac{p_{x2}}{p_{x1}}. \tag{10}$$

For convenience of exposition we take good X in period 1 as numéraire ($p_{x1} = 1$), and write $p_{x2} \equiv p_x$ and $p_{e1} = p_{e2} \equiv p_e$.

3 Incidence of the consumption tax

In this section we consider the competitive equilibrium in the two-country model (1) - (10) and explore the comparative static effects of increasing the tax rate τ in country A. We wish to determine, in particular, the sign of $\frac{\mathrm{d}e_1}{\mathrm{d}\tau} = \frac{\mathrm{d}e_{A1}}{\mathrm{d}\tau} + \frac{\mathrm{d}e_{B1}}{\mathrm{d}\tau}$, where $\mathrm{d}e_{A1}/\mathrm{d}\tau$ and $\mathrm{d}e_{B1}/\mathrm{d}\tau$ may take on any sign. If $\mathrm{d}e_1/\mathrm{d}\tau > 0$ the tax steepens rather than flattens the carbon

[5] For the benefit of a clear focus on the consumption tax incidence we refrain from adding emissions taxes or cap and trade systems to the model.

emission path and thus exacerbates climate change.[6] The consumption tax qualifies as an instrument of unilateral climate policy under the following constellations of tax incidence:

$$\frac{\mathrm{d}e_1}{\mathrm{d}\tau} < 0 \quad \text{and} \quad \frac{\mathrm{d}e_{B1}}{\mathrm{d}\tau} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \quad \Longrightarrow \quad \left\{ \begin{array}{l} \text{leakage rate positive, but} < 100\%, \\ \text{no leakage,} \\ \text{negative leakage.} \end{array} \right\}$$

The comparative statics (in the Appendix) reveal that

$$\operatorname{sign}\frac{\mathrm{d}e_{A1}}{\mathrm{d}\tau} = \operatorname{sign}\frac{\mathrm{d}e_{B1}}{\mathrm{d}\tau} = -\operatorname{sign}\frac{\mathrm{d}p_e}{\mathrm{d}\tau} = -\operatorname{sign}\frac{\mathrm{d}p_x}{\mathrm{d}\tau}. \tag{11}$$

The first equality sign in (11) means that the consumption tax either exacerbates climate change ($\mathrm{d}e_{A1}/\mathrm{d}\tau > 0$) or is a promising climate policy instrument that even reduces country B's first-period emissions (negative leakage) although country B is politically inactive.

It is interesting to compare that observation with the scenario studied by Eichner and Pethig (2011) where country A increases the rate π of a tax on its first-period emissions. They show that $\mathrm{d}e_{A1}/\mathrm{d}\pi < 0$ holds 'unconditionally'[7] but that the leakage rate is always positive, possibly larger than 100% (green paradox).

Now the crucial question is how restrictive the conditions are for $\mathrm{d}e_{A1}/\mathrm{d}\tau < 0$ or, equivalently, for $\mathrm{d}e_1/\mathrm{d}\tau < 0$. We derive in the Appendix that

$$\frac{\mathrm{d}e_1}{\mathrm{d}\tau} = -\frac{1}{\rho(\tau) - \psi(\tau)\Delta e_A - \theta(\tau)\Delta x_{A2}} \tag{12}$$

where $\Delta e_A := \alpha_A \bar{e} - e_{A1} - e_{A2}$, $\Delta x_{A2} := x^s_{A2} - x_{A2}$ and where $\rho(\tau)$, $\xi(\tau)$ and $\theta(\tau)$ are parameters defined in the Appendix that satisfy $\rho(\tau) > 0$ for $\tau \geq 0$, $\xi(\tau) = \theta(\tau) = 0$ for $\tau = 0$, and $\xi(\tau) > 0$, $\theta(\tau) > 0$ for $\tau > 0$.

Proposition 1. *An increase in the consumption tax rate flattens the carbon extraction path and implies negative leakage, if the following conditions (i) or (ii) hold:*

(i) The countries differ with respect to production functions and energy resource endowments $\left(\alpha_A \neq \frac{1}{2}\right)$*, and the initial competitive equilibrium is characterized*

(a) either by $\tau = 0$

(b) or by $\tau > 0$ *and by imports of fossil fuel* $(\Delta e_A \leq 0)$ *and second-period consumption good* $(\Delta x_{A2} \leq 0)$ *on the part of country A.*

[6] The case of $\mathrm{d}e_1/\mathrm{d}\tau > 0$ where the direct effect $\mathrm{d}e_{A1}/\mathrm{d}\tau$ is negative could be called a 'green paradox' because it implies a carbon leakage rate exceeding 100%.

[7] What is needed is the textbook assumption that the firm's demand for fossil fuel is strictly decreasing in the fuel price.

(ii) The countries are identical (same production functions and $\alpha_A = \frac{1}{2}$) and the initial competitive equilibrium is characterized by $\tau \geq 0$.[8]

Proposition 1(i) is straightforward from (12). In order to assess the conditions (b) in Proposition 1(i) observe that $\Delta e_A < 0$ is the more likely the smaller is the energy resource endowment of country A (α_A small). Moreover, if production capacities do not differ too much in both countries, $\Delta x_{A2} < 0$ is also plausible because the reduction in the relative consumer price of the second-period consumption good is lower in country A than in country B.[9] Thus the conditions (b) appear to approximate the characteristics of the group of countries that have committed to emission reduction in the Kyoto Protocol. It is also worth mentioning that the conditions (b) are sufficient but not necessary.

Unfortunately, reversals in the sign of Δe_A and Δx_{A2} cannot be excluded when the tax rate is successively raised or lowered. Such reversals may, in turn, switch the sign of $\mathrm{d}e_1/\mathrm{d}\tau$ in (12). It is therefore not quite clear how restrictive the conditions (b) in Proposition 1(i) are or, to put it differently, how attractive the consumption tax is as an instrument for fighting climate change. Proposition 1(ii) provides additional support for a role of the consumption tax in climate policy by establishing for the case of symmetry that e_1 is monotone decreasing in τ. The proof in the Appendix shows that under the condition of identical countries $\Delta e_A = 0$ and $\Delta x_{A2} < 0$ holds for all $\tau \geq 0$ such that in (12) $\mathrm{d}e_1/\mathrm{d}\tau < 0$ for all $\tau \geq 0$.

It is interesting to compare the price effects of an increase in the consumption tax τ with those of an increase in the unilateral first-period emission tax π under the condition that in both cases total first-period emissions are curbed ($\mathrm{d}e_1/\mathrm{d}\tau < 0$ and $\mathrm{d}e_1/\mathrm{d}\pi < 0$). We infer $\mathrm{d}p_e/\mathrm{d}\tau > 0$ and $\mathrm{d}p_x/\mathrm{d}\tau > 0$ from (11), whereas Eichner and Pethig (2011) find $\mathrm{d}p_e/\mathrm{d}\pi < 0$ and $\mathrm{d}p_x/\mathrm{d}\pi < 0$. Obviously, the emission tax reduces the world demand for fossil fuel at any given price and thus lowers the equilibrium fuel price. Consumption in period 1 becomes relatively more expensive and thus induces the necessary shift of consumption from the first into the second period. In sharp contrast, p_e needs to rise in case of an increase of the consumption tax rate because otherwise producers would not reduce their first-period demand for fossil fuel. $\mathrm{d}\tau > 0$ discourages first-period consumption in country A such that the increase in country B's first-period consumption (due to $\mathrm{d}p_x > 0$) is overcompensated.

[8] The proof of Proposition 1(ii) is provided in the Appendix.

[9] Observe that the budget constraints (8) and (9) can be rearranged to the intertemporal trade balances $\Delta x_{i1} + p_x \Delta x_{i2} + p_e \Delta e_i = 0$ for $i = A, B$. Then $\Delta e_A < 0$ and $\Delta x_{A2} < 0$ implies that country A exports consumption goods in the first period ($\Delta x_{A1} > 0$).

4 Concluding remarks

In general, those tax instruments are best suited to reduce pollution-generating emissions whose tax base are emissions rather than (non-polluting) consumer goods. Accordingly, in the climate-change context the perfect tax for reducing total first-period carbon emissions is a *uniform* emission tax on *all* first-period emissions. However, unilateral emission taxes inevitably create distortions and excess costs as do unilateral consumption taxes. This note suggests that the unilateral consumption tax has the potential of an instrument for fighting climate change, if full cooperation cannot be attained. It might thus be a promising alternative or supplement to unilateral emission taxes or other unilateral conventional emission-reducing environmental policy instruments.

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Appendix

Derivation of (11): When first-period consumption is taxed in country A the competitive equilibrium is characterized by the equations

$$
\begin{aligned}
X^A_{e_{A1}} - p_e &= 0, && \text{(A1)}\\
X^B_{e_{B1}} - p_e &= 0, && \text{(A2)}\\
p_x X^A_{e_{A2}} - p_e &= 0, && \text{(A3)}\\
p_x X^B_{e_{B2}} - p_e &= 0, && \text{(A4)}\\
e_1 - e_{A1} - e_{B1} &= 0, && \text{(A5)}\\
\bar{e} - e_1 - e_{A2} - e_{B2} &= 0, && \text{(A6)}\\
X^A(e_{A1}) - x_{A1} + p_x[X^A(e_{A2}) - x_{A2}] + p_e \Delta e_A &= 0, && \text{(A7)}\\
X^B(e_{B1}) - x_{B1} + p_x[X^B(e_{B2}) - x_{B2}] + p_e \Delta e_B &= 0, && \text{(A8)}\\
X^A(e_{A2}) + X^B(e_{B2}) - x_{A2} - x_{B2} &= 0, && \text{(A9)}\\
\frac{U_{x_{A2}}}{U_{x_{A1}}} - \frac{p_x}{1+\tau} &= 0, && \text{(A10)}\\
\frac{U_{x_{B2}}}{U_{x_{B1}}} - p_x &= 0, && \text{(A11)}
\end{aligned}
$$

where $\Delta e_i := \alpha_i \bar{e} - e_{i1} - e_{i2}$ for $i = A, B$. The endogenous variables determinded by the 11 equations (A1) - (A11) are $e_{A1}, e_{A2}, e_{B1}, e_{B2}, e_1, x_{A1}, x_{A2}, x_{B1}, x_{B2}, p_e$ and p_x. The tax rate τ is treated here as exogenous parameter. Total differentation of (A1) - (A11) yields, after some rearrangement of terms,

$$
\begin{aligned}
\frac{1}{\eta_{A1}} \hat{e}_{A1} - \hat{p}_e &= 0, && \text{(A12)}\\
\frac{1}{\eta_{B1}} \hat{e}_{B1} - \hat{p}_e &= 0, && \text{(A13)}\\
\hat{p}_x + \frac{1}{\eta_{A2}} \hat{e}_{A2} - \hat{p}_e &= 0, && \text{(A14)}\\
\hat{p}_x + \frac{1}{\eta_{B2}} \hat{e}_{B2} - \hat{p}_e &= 0, && \text{(A15)}\\
e_1 \hat{e}_1 - e_{A1} \hat{e}_{A1} - e_{B1} \hat{e}_{B1} &= 0, && \text{(A16)}\\
-e_1 \hat{e}_1 - e_{A2} \hat{e}_{A2} - e_{B2} \hat{e}_{B2} &= 0, && \text{(A17)}\\
-x_{A1} \hat{x}_{A1} - p_x x_{A2} \hat{x}_{A2} + \Delta e_A p_e \hat{p}_e + \Delta x_{A2} p_x \hat{p}_x &= 0, && \text{(A18)}\\
-x_{B1} \hat{x}_{B1} - p_x x_{B2} \hat{x}_{B2} + \Delta e_B p_e \hat{p}_e + \Delta x_{B2} p_x \hat{p}_x &= 0, && \text{(A19)}\\
p_e e_{A2} \hat{e}_{A2} + p_e e_{B2} \hat{e}_{B2} - p_x x_{A2} \hat{x}_{A2} - p_x x_{B2} \hat{x}_{B2} &= 0, && \text{(A20)}\\
\hat{x}_{A2} - \hat{x}_{A1} + \sigma \hat{p}_x - \frac{\sigma \tau}{1+\tau} \hat{\tau} &= 0, && \text{(A21)}\\
\hat{x}_{B2} - \hat{x}_{B1} + \sigma \hat{p}_x &= 0, && \text{(A22)}
\end{aligned}
$$

where $\Delta x_{it} := x_{it}^s - x_{it}$ and $\eta_{it} := X^i_{e_{it}}/(e_{it}X^i_{e_{it}e_{it}}) < 0$ for $t = 1, 2$ and $i = A, B$. We insert (A21) in (A18) and (A22) in (A20) to obtain

$$\hat{x}_{A2} = \frac{\Delta e_A p_e}{y_A}\hat{p}_e + \frac{p_x \Delta x_{A2} - \sigma x_{A1}}{y_A}\hat{p}_x + \frac{\sigma\tau x_{A1}}{(1+\tau)y_A}\hat{\tau}, \tag{A23}$$

$$\hat{x}_{B2} = \frac{\Delta e_B p_e}{y_B}\hat{p}_e + \frac{p_x \Delta x_{B2} - \sigma x_{B1}}{y_B}\hat{p}_x. \tag{A24}$$

Using (A12), (A13) in (A16) we find

$$\hat{p}_e = \frac{1}{\sum_{i=A,B} e_{i1}\eta_{i1}} e_1 \hat{e}_1, \tag{A25}$$

Inserting (A13), (A14) in (A17) and taking advantage of (A24) we get after rearrangement of terms

$$\hat{p}_x = \left(\frac{1}{\sum_{i=A,B} e_{i1}\eta_{i1}} + \frac{1}{\sum_{i=A,B} e_{i2}\eta_{i2}} \right) e_1 \hat{e}_1. \tag{A26}$$

(A12), (A13) (A25) and (A26) establish (11).

Derivation of (12): Inserting (A17) in (A20) yields

$$-p_e e_1 \hat{e}_1 = p_x \sum_{i=A,B} x_{i2}\hat{x}_{i2}. \tag{A27}$$

Making use of (A23) and (A24) in (A27) we obtain

$$\begin{aligned} \frac{p_x x_{A2} x_{A1} \sigma \mathrm{d}\tau}{(1+\tau)y_A} &= -p_e e_1 \hat{e}_1 - \left(\frac{p_x x_{A2}}{y_A} - \frac{p_x x_{B2}}{y_B} \right) \Delta e_A p_e \hat{p}_e \\ - &\left(\frac{p_x x_{A2}}{y_A} - \frac{p_x x_{B2}}{y_B} \right) p_x \Delta x_{A2} \hat{p}_x + \sigma \left(\frac{p_x x_{A1} x_{A2}}{y_A} + \frac{p_x x_{B1} x_{B2}}{y_B} \right) \hat{p}_x. \end{aligned} \tag{A28}$$

With the help of (A25) and (A26) equation (A28) can be rearranged to

$$\begin{aligned} \frac{p_x x_{A2} x_{A1} \sigma \mathrm{d}\tau}{(1+\tau)y_A \mathrm{d}e_1} &= -p_e - \left(\frac{p_x x_{A2}}{y_A} - \frac{p_x x_{B2}}{y_B} \right) \frac{p_e \Delta e_A}{\sum_{i=A,B} e_{i1}\eta_{i1}} \\ - \left(\frac{p_x x_{A2}}{y_A} - \frac{p_x x_{B2}}{y_B} \right) & p_x \Delta x_{A2} \left(\frac{1}{\sum_{i=A,B} e_{i1}\eta_{i1}} + \frac{1}{\sum_{i=A,B} e_{i2}\eta_{i2}} \right) \\ +\sigma \left(\frac{p_x x_{A1} x_{A2}}{y_A} + \frac{p_x x_{B1} x_{B2}}{y_B} \right) & \left(\frac{1}{\sum_{i=A,B} e_{i1}\eta_{i1}} + \frac{1}{\sum_{i=A,B} e_{i2}\eta_{i2}} \right). \end{aligned} \tag{A29}$$

(A29) can be rewritten in the form

$$\frac{\mathrm{d}e_1}{\mathrm{d}\tau} = -\frac{1}{\rho(\tau) - \xi(\tau)\Delta e_A - \theta(\tau)\Delta x_{A2}}, \tag{A30}$$

where

$$\rho(\tau) := \frac{(1+\tau)y_A}{p_x x_{A2} x_{A1}} \left[p_e - \sigma \left(\frac{1}{\sum_{i=A,B} e_{i1}\eta_{i1}} + \frac{1}{\sum_{i=A,B} e_{i2}\eta_{i2}} \right) \cdot \left(\frac{p_x x_{A1} x_{A2}}{y_A} + \frac{p_x x_{B1} x_{B2}}{y_B} \right) \right], \quad \text{(A31)}$$

$$\xi(\tau) := -\frac{(1+\tau)y_A}{p_x x_{A2} x_{A1}} \left[\frac{p_e}{\sum_{i=A,B} e_{i1}\eta_{i1}} \left(\frac{p_x x_{A2}}{y_A} - \frac{p_x x_{B2}}{y_B} \right) \right], \quad \text{(A32)}$$

$$\theta(\tau) := -\frac{(1+\tau)y_A}{x_{A2} x_{A1}} \left[\left(\frac{1}{\sum_{i=A,B} e_{i1}\eta_{i1}} + \frac{1}{\sum_{i=A,B} e_{i2}\eta_{i2}} \right) \left(\frac{p_x x_{A2}}{y_A} - \frac{p_x x_{B2}}{y_B} \right) \right]. \quad \text{(A33)}$$

The properties of $\rho(\tau)$, $\xi(\tau)$ and $\theta(\tau)$ are specified in

Lemma 1.

(i) $\dfrac{p_x x_{A2}}{x_{A1} + p_x x_{A2}} \left\{ \begin{matrix} > \\ = \end{matrix} \right\} \dfrac{p_x x_{B2}}{x_{B1} + p_x x_{B2}} \iff \tau \left\{ \begin{matrix} > \\ = \end{matrix} \right\} 0.$

(ii) $\rho(\tau) > 0$, $\xi(\tau) = \theta(\tau) = 0$ *for* $\tau = 0$.

(iii) $\rho(\tau) > 0$, $\xi(\tau) > 0$, $\theta(\tau) > 0$ *for* $\tau > 0$.

Proof: (i) Observe that CES utility functions imply $x_{A1} = x_{A2} \cdot \left(\frac{\gamma_1 p_x}{\gamma_2 (1+\tau)} \right)^{\sigma}$, $x_{B1} = x_{B2} \cdot \left(\frac{\gamma_1 p_x}{\gamma_2} \right)^{\sigma}$ and hence

$$\frac{p_x x_{A2}}{x_{A1} + p_x x_{A2}} = \frac{p_x}{p_x + \left(\frac{\gamma_1 p_x}{\gamma_2 (1+\tau)} \right)^{\sigma}} \geqq \frac{p_x}{p_x + \left(\frac{\gamma_1 p_x}{\gamma_2} \right)^{\sigma}} = \frac{p_x x_{B2}}{x_{B1} + p_x x_{B2}}, \quad \text{(A34)}$$

where the equality in the $\geqq$ sign in (A34) holds if and only if $\tau = 0$.
The Lemmas 1(ii) and 1(iii) follow from Lemma 1(i), $\eta_{i1} < 0$ for $i = 1, 2$ and $(1+\tau) > 0$.■

Proof of Proposition 1(ii): An equilibrium with identical production functions is characterized by $e_{A1} = e_{B1}$ and $e_{A2} = e_{B2}$. Differentiation of these equalities yields

$$\frac{\mathrm{d}e_{A1}}{\mathrm{d}\tau} = \frac{\mathrm{d}e_{B1}}{\mathrm{d}\tau} \quad \text{and} \quad \frac{\mathrm{d}e_{A2}}{\mathrm{d}\tau} = \frac{\mathrm{d}e_{B2}}{\mathrm{d}\tau}. \quad \text{(A35)}$$

Next we differentiate $\Delta e_A = \alpha_A \bar{e}_1 - e_{A1} - e_{A2}$ and $\Delta e_B = (1 - \alpha_A) - e_{B1} - e_{B2}$ to obtain

$$\frac{\mathrm{d}(\Delta e_A)}{\mathrm{d}\tau} = \alpha_A \frac{\mathrm{d}e_1}{\mathrm{d}\tau} - \frac{\mathrm{d}e_{A1}}{\mathrm{d}\tau} - \frac{\mathrm{d}e_{A2}}{\mathrm{d}\tau}, \quad \text{(A36)}$$

$$\frac{\mathrm{d}(\Delta e_B)}{\mathrm{d}\tau} = (1 - \alpha_A) \frac{\mathrm{d}e_1}{\mathrm{d}\tau} - \frac{\mathrm{d}e_{B1}}{\mathrm{d}\tau} - \frac{\mathrm{d}e_{B2}}{\mathrm{d}\tau}. \quad \text{(A37)}$$

Substracting (A37) from (A36) and accounting for (A35) yields

$$\frac{\mathrm{d}(\Delta e_A)}{\mathrm{d}\tau} - \frac{\mathrm{d}(\Delta e_B)}{\mathrm{d}\tau} = (2\alpha_A - 1) \frac{\mathrm{d}e_1}{\mathrm{d}\tau}. \quad \text{(A38)}$$

Since $\Delta e_A = -\Delta e_B$ and hence $\mathrm{d}(\Delta e_A) = -\mathrm{d}(\Delta e_B)$ we infer from (A38)

$$\alpha_A = \frac{1}{2} \quad \Longrightarrow \quad \frac{\mathrm{d}(\Delta e_A)}{\mathrm{d}\tau} = \frac{\mathrm{d}(\Delta e_B)}{\mathrm{d}\tau} = 0. \tag{A39}$$

Next, we use (A39), (A22) in (A19) to get

$$\frac{\mathrm{d}x_{B1}}{\mathrm{d}\tau} = \frac{(x_{B2}\sigma + \Delta x_{B2})x_{B1}}{y_B}\frac{\mathrm{d}p_x}{\mathrm{d}\tau}. \tag{A40}$$

In addition, from (A13) and differentiating $x^s_{B1} = X(e_{B1})$ we get

$$\frac{\mathrm{d}x^s_{B1}}{\mathrm{d}\tau} = e_{B1}\eta_{B1}\frac{\mathrm{d}p_e}{\mathrm{d}\tau}, \tag{A41}$$

$$\frac{\mathrm{d}(\Delta x_{B1})}{\mathrm{d}\tau} = \frac{\mathrm{d}x^s_{B1}}{\mathrm{d}\tau} - \frac{\mathrm{d}x_{B1}}{\mathrm{d}\tau} = e_{B1}\eta_{B1}\frac{\mathrm{d}p_e}{\mathrm{d}\tau} - \frac{(x_{B2}\sigma + \Delta x_{B2})x_{B1}}{y_B}\frac{\mathrm{d}p_x}{\mathrm{d}\tau}. \tag{A42}$$

Consider an unregulated equilibrium ($\tau = 0$) which is characterized by $\Delta e_A = \Delta x_{A1} = \Delta x_{A2} = 0$ for identical production functions and $\alpha_A = \frac{1}{2}$. Suppose next that this initial equilibrium is disturbed by a tax increase $\mathrm{d}\tau > 0$. Applying Proposition 1(i)(a) we know that

$$\left.\frac{\mathrm{d}e_1}{\mathrm{d}\tau}\right|_{\tau=0} < 0, \quad \left.\frac{\mathrm{d}p_x}{\mathrm{d}\tau}\right|_{\tau=0} > 0 \quad \text{and} \quad \left.\frac{\mathrm{d}p_e}{\mathrm{d}\tau}\right|_{\tau=0} > 0. \tag{A43}$$

Accounting for (A43) in (A42) we get $\left.\frac{\mathrm{d}(\Delta x_{B1})}{\mathrm{d}\tau}\right|_{\tau=0} < 0$. Due to (A39) the new regulated equilibrium for ($\tau = \mathrm{d}\tau > 0$) has the properties

$$\Delta x_{A1} = -\Delta x_{B1} > 0, \quad \Delta x_{A2} = -\Delta x_{B2} < 0, \quad \Delta e_A = \Delta e_B = 0. \tag{A44}$$

Hence the presuppositions of Proposition 1(i)(b) are satisfied. Using the same arguments as before it is straightforward to show that further increases of τ result in equilibria at which (A44) holds. ■