The Natural Interest Rate in OLG modelling -

A Rehabilitation

By

Ulrich van Suntum

Centrum für angewandte Wirtschaftsforschung CAWM
University of Münster
Am Stadtgraben 9
D-48143 Münster
ulivs@t-online.de
0176-62195932
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Abstract

A simple OLG model is used to show that the natural interest rate is superior to the golden rule. This remains valid with public goods, provided these are financed in an appropriate way. In order to preserve the natural interest rate, the so-called helicopter money appears to be more appropriate than the normal credit money. Dynamic inefficiency cannot occur, if either land or neutral (helicopter) money is available as an alternative store of private wealth. Thus, the frequently proposed failure of OLG-models to satisfy the first fundamental theorem of welfare economics does not exist. The paper both generalizes and summarizes some key results from my recent book (van Suntum 2017).

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1. Introduction

It is now 60 years ago that Paul Samuelson (1958) introduced the overlapping generations model (OLG) into economic theory, thereby undermining the first fundamental theorem of welfare economics. According to that theorem, in the absence of externalities or other distortions, markets should be generally Pareto-efficient, including the capital market. Until then this had been the common conviction of capital theoreticians including Wicksell (1898), who invented the notion of the natural interest rate as synonym for an efficient capital market equilibrium without any monetary disturbance. Samuelson however showed that, even in a simple stationary consumption loan model without any imperfections, this natural interest rate could easily get negative and thus cause what is nowadays known as "dynamic inefficiency". In particular, in his model with three generations, the middle aged save more in order to provide for retirement than the young would demand as loans with a zero interest rate. The stunning result is not only a negative interest rate but a Pareto-inefficient situation as well: As Samuelson argued a (both benevolent and omniscient) social planner could improve the welfare of all cohorts by forcing people to rearrange their lifecycle consumption path. (In particular, they would have to consume less in the first period of their life and more in the remaining periods, compared with the competitive solution). In other words, even under perfect conditions, a socialist dictator might do better than the free market.

In their first reactions, many economists have tried to brush aside this unpleasant result as a purely theoretical case, which was only based on the extreme simplicity of Samuelson's "Martian" model. However, further research by Phelps (1961), Diamond (1965) and others both confirmed and generalized his peculiar conclusions. In particular, it turned out that dynamic inefficiency generally occurs if the (steady state) natural interest rate is lower than the economy's (steady state) growth rate. An equivalent formulation of this condition is the requirement that total interest income must not fall below total investment in the steady state. This version is even more intuitive, because it appears indeed quite odd to have investors who save without getting any net consumption in return forever. On the other hand, this interpretation of the Samuelson verdict immediately raises the question if dynamic inefficiency could actually occur with rational savers.

For about 50 years these issues seemed to be of mainly theoretical interest, being a mere playing ground for mathematically oriented economists without much relevance for economic policy. Indeed, the violation of the first fundamental theorem turned out as a peculiarity of OLG modelling, which did not occur in the competing Ramsey/Koopmans/Cass general equilibrium world with undying individuals and infinite horizons. Thus OLG modelling has gradually become out of fashion, disregarding its merits particularly concerning intergenerational conflicts. This happened although empirical work by Abel et al (1989), Piketty (2014) and others suggested that dynamic inefficiency was observed rarely, if at all.

Meanwhile, however, the picture has substantially changed. Extremely low and even negative interest rates have become reality in many industrial countries in recent time, and so the discussion on over-accumulation of capital (now labelled as "savings glut") has celebrated a comeback (Bernanke 2005, Eugeni 2015). Interesting enough, it is now an alliance of Keynesian proponents of secular stagnation such as Summers 2014) and neoclassical growth-theorists like von Weizsäcker (2013) who argue in favor of more public debt as a cure. While the former see a lack of total demand, the latter diagnose over-investment (which is not necessarily consistent with each other). Nevertheless, their common recommendation is to boost consumption at the cost of savings in order to increase total income, welfare, and employment.

In the following, we use a simple model in order to rehabilitate both the concept of a natural interest rate and the OLG methodology. The model is a generalized version of that which is used in van Suntum (2017). In section 2, we use it to give the natural interest rate a clear definition, arguing that it is generally efficient and even superior to the golden rule. In particular, we show that the mere focus on steady state equilibria is generally misleading in OLG models and should be complemented by analyzing transition periods as well. In section 3, we show that the concept of the natural interest rate has also a clear meaning when both public goods are included. In section 4, we introduce money and argue that the so-called helicopter money is the best way to preserve the natural interest rate, while credit money tends to distort it even in the long term. Finally, in section 5, we argue that dynamic inefficiency is precluded if one allows for either land or neutral (and intrinsically valuable) money, at least with perfect capital markets.

2. Natural Rate of Interest vs. Golden Rule

While Samuelson used a three-period approach, in the subsequent literature the two-period version of the OLG-model has become more popular. In the following we use such a "Diamond"-version as well, because it is much simpler and not significantly less general than Samuelson`s original approach. Presumably, Samuelson would also have preferred the two-period-version in principle. However, this is not compatible with a pure consumption loan-model because, with only two cohorts, no intergenerational trade is possible. As Kotlikoff (2006) has vividly illustrated this, the young would only jape at the elderly by throwing candy-papers on them from top of the chocolate trees which the elderly are no longer able to climb.

In our model, we have labor N, capital K, and land B as production factors, where land and capital are variable but land is both non-extendable and scarce. The production function shall satisfy the usual neoclassical assumptions, but is less than linear homogeneous in labor and capital. This is because the fixed amount of land allows only for a disproportionate increase in total GDP (which we denote as Y). For simplicity, let the depreciation rate be unity, so gross investment in period t equals the total capital stock K in the next period t+1.

The two variable factors are paid their marginal product, while landowners receive the residual income. The income-share of the variable production factors is then given by their partial elasticities of production, which we call γ and β respectively.[i] Hence we have

(1) $$wN = \gamma Y$$

(2) $$(1+i)K = \beta Y$$

(3) $$\prod = (1 - \beta - \gamma)Y$$

where w is the real wage rate, i is the marginal product of capital (resp. the interest rate) and $\prod$ denotes residual profits, which are land rents in this model.

Individuals are identical in income and taste and live for two periods. We assume that the young are both workers and land owners. They save part of their income in order to provide for their retirement, thereby generating the capital stock of the economy. After retirement, they receive both redemption and interest on their savings, which is then their only income. Lifetime utility depends on consumption C_1 and C_2 in the two periods and shall satisfy the usual properties, in particular a diminishing (relative) marginal utility of consumption in each period. For the time being, we further assume that there are no durable goods, no monetary wealth stores like e.g. paper money, and no bequests. Hence, saving can only take place by lending out (resp. investing), which in turn implies that capital market equilibrium requires equality of total savings by the young S_1 in period t-1 and total capital available in period t. We also allow for a growing total income, either by (Harrod-neutral) technological progress or by population growth. For simplicity of notation, we generally omit the index t in our equations and only indicate variables which refer to the period before by the index t-1. With a growth rate n for total output, capital equilibrium requires:

(4) $$K = S_{1;t-1} = S_1 /(1 + n)$$

The individual`s maximization problem is

(5) $$Max\ U(C_1; C_2)$$

s.t.

(6) $$(1-\beta)Y = C_1 + S_1$$

(7) $$\beta Y = (1 + i)S_1 = C$$

By normalizing the initial number of individuals to unity and regarding (4) this leads to the general market equilibrium condition

(8) $$\left(\frac{\delta U / \delta C_2}{\delta U / \delta C_1} \right) \overset{!}{=} (1 + i^*) \overset{!}{=} \frac{\delta Y}{\delta K}$$

where i^* denotes the natural interest rate. In simple words, the intertemporal rate of consumption substitution must equal (gross) marginal product of capital. Mostly, the natural rate of interest is defined in terms of flows, namely by the equality of private savings and private investment. This is less general, because only the respective stocks can constitute a long-term equilibrium, as we know since Tobin (1961) at latest. In our model, where we have assumed a unity depreciation rate, the respective flows and stocks coincide, so we can use either definition.

Note that i^* increases in the growth rate n, because in a growing economy net investment is needed, as the capital market equilibrium reveals. This is an important issue which we will come back to in section 4 where we discuss the merits of neutral money above stable money.

Now we turn to the benevolent social planner`s approach, who also seeks to maximize individual utility. In contrast to the market approach, he is not restricted by the individual budget restrictions given above but must only regard the macroeconomic restriction that one cannot use more goods in total than are available in each period. Hence, he has to solve

(9) $$Max \ U(C_1; C_2)$$

s.t.

(10) $$Y(S_{1;t-1}) - C_1 - C_2 - S_1 = 0$$

Again we have $K = S_1/(1+n)$, which leads to the planner`s optimal solution

(11) $$\left(\frac{\delta U / \delta C_2}{\delta U / \delta C_1} \right) \overset{!}{=} (1+n) \overset{!}{=} \frac{\delta Y}{\delta K}$$

Obviously, the market condition (8) and the planner`s solution (11) do only coincide with $i = n$, i.e. if the natural interest rate happens to equal the economy`s growth rate. Since Phelps (1961) this is called the golden rule of accumulation. In any other case, the planner would realize a higher steady state utility than the market solution.

Hence, at first sight at least, the golden rule looks like an optimum optimorum, as it is actually sometimes called (von Weizsäcker 2015, p. 196; Weil 2008, p 125): The intertemporal rate of consumption substitution equals marginal product of capital like in the competitive case, and steady state utility is even higher (and actually at its absolute maximum). So who could ask for more?

However, as already Starret (1972) has pointed out, for a more general assessment we must not stick to steady state considerations but have to regard transition periods as well. In particular, as is well known meanwhile, with $i > n$ we cannot unambiguously call the natural rate of interest inefficient. The reason is that a switch towards the (higher) golden rule level of capital stock would then require at least a one off extra investment in the transition

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period and, hence, a one-off decline in consumption as well (because total output increases only from the next period on). Thus, the transition cohort has to suffer a sacrifice and the switch cannot be assessed as a clear Pareto improvement. Only with $i < n$ in the beginning the switch to $i = n$ would be beneficial for everyone, because in that case increasing future consumption would require a decline in the capital stock and, hence, even lead to an extra consumption of the transition cohort.

However, we can even say more because, with $i > n$, immediately the compensation idea by Kaldor (1939) and Hicks (1939) springs to mind: Could the transition cohort not be compensated for their loss from the gains of future generations? Alternatively, looking at it from the other side, could it even be beneficial for all to switch back from the golden rule to a natural interest rate $i > n$ by compensating the future cohorts for their permanent utility loss from the one-off gain of the transition cohort? Indeed, the answer to both questions is yes.

In order to show this, we first examine the two equilibrium equations (8) and (11) more closely. They both imply that the (intertemporal) marginal rate of consumption transformation (MRS), which is defined as $\left| dC_2 / dC_1 \right|_{\overline{U}}$, equals the marginal product of capital dY / dK. With respect to the compensation idea, the latter can be interpreted as the (intertemporal) marginal rate of transformation (MRT), which is defined as $\left| dC_2 / dC_1 \right|_{\overline{Y}}$. This is the amount of C_2 in the next period which the transition cohort can c.p. earn by reallocating one unit of C_1 in the transition period to investment (with given total income in the transition period and with given consumption of every future generation). Equivalently, $\left| dC_2 / dC_1 \right|_{\overline{U}}$ denotes the amount of C_2 in the next period which the transition cohort would at least *need* as a compensation for a respective reallocation of their resources. Hence, a sufficient compensation is only possible if $\left| dC_2 / dC_1 \right|_{\overline{Y}} \geq \left| dC_2 / dC_1 \right|_{\overline{U}}$, i.e. if the marginal rate of transformation exceeds the marginal rate of substitution.

With $i^* > n$, this requirement is only satisfied by the natural rate of interest, but not by the golden rule, as can be illustrated with the help of figure 1. As the figure reveals, between the two equilibria we have MRT < MRS, which must always be true with $i^* > n$ because otherwise i^* could not be the equilibrium interest rate. This is because, with the capital stock being below its equilibrium level, marginal productivity of capital (which is equal to MRT here) obviously exceeds MRS, while with a capital stock above its equilibrium level the opposite is true. [ii]
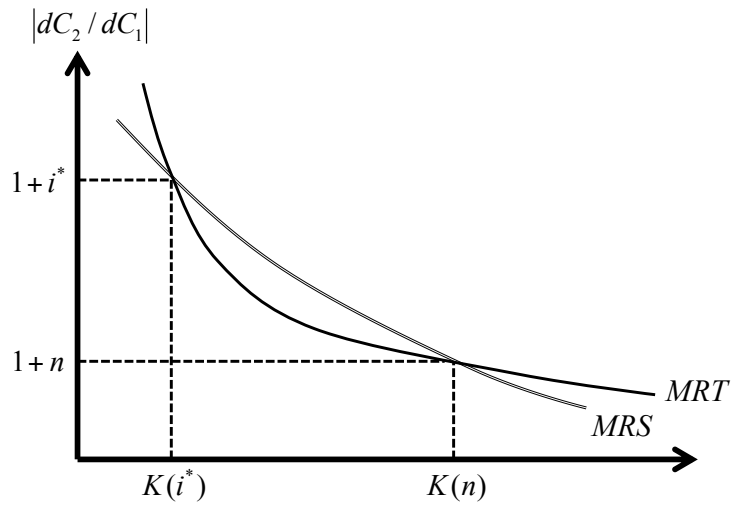
Figure 1

Now suppose that the economy is initially in its market equilibrium, but the social planner forces the capital stock above its natural level $K(i^*)$ towards the golden rule. The respective consumption sacrifice in the transition period must be borne by those who are young in that period, because the elderly cannot be compensated in the next period any more (when they are already dead). The maximum compensation which can be granted is given by the respective area below MRT, while the minimum amount of C_2 which is needed for a sufficient compensation of the young (in their second period of life) is given by the respective area below MRS. Because the latter exceeds the former, a sufficient compensation is obviously impossible in this case.

Vice versa, suppose that the social planner has already forced the capital stock to its golden rule level $K(n)$, but the economy returns to the natural level $K(i^*)$. This would mean a gain for the transition cohort (as K is decreased and C_1 can be increased respectively in that period), while all future generations would face a lower total income and, thus, suffer a loss. However, there is also a possible compensation-scheme which works as follows: The winning transition cohort gives part of their consumption C_2 to those who are young in the very next period, thereby increasing C_1 of that cohort. In reverse, the latter gives part of their consumption C_2 to those who are young in the following period and so on. By this means all cohorts could be compensated although the capital stock is now permanently lower than at its golden rule level, provided that the initial increase of C_1 in the transition period is sufficiently high. Obviously, the latter is the case if we have MRS > MRT for $K < K(n)$, because this would mean that the amount of C_2 which the transition cohort is *prepared* to sacrifice for an additional unit of C_1 exceeds the amount of C_2 which is *actually lost* by a respective reallocation of the resources. Apparently, this requirement is met according to figure 1, i.e. all future cohorts can be compensated for a re-switch to the natural level of the capital stock, while the transition cohort still has left a gain. Concerning the normal case $i^* > n$, we thus conclude that the natural rate of interest is superior to the golden rule.

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According to the normal interpretation of the Kaldor-Hicks-criterion, the compensation need not be actually executed in practice. Generally, it is held to be sufficient that a respective compensation was possible in principle, e.g. concerning the gains from free trade (or from competition in general), where we usually have some losers as well. Hence, accepting this argument, there are good reasons to make sure that the interest rate is normally at its natural level. In particular, manipulating it down in order to increase total income appears dubious from the Kaldor-Hicks point of view. Even if the economy would benefit from that in the long term, one would at least have to account for potential losses of the transition cohort. Currently, such losses affect e.g. the savers who are forced more or less to finance the respective (over-)investment at a nearly zero interest rate. According to our model, their respective sacrifice is bigger than the future gains from that investment presumably are.

3. Allowing for Public Goods

As has frequently be shown with the help of OLG-modelling, in the absence of Ricardian equivalence, public debt has an impact on the equilibrium interest rate (Barro 1974). In principle, the same is true for income taxes, because they tend to change individual savings. Hence, the equilibrium interest rate is obviously affected by the way how public goods are financed. On the other hand, the natural rate of interest is normally defined with respect to an economy where no public sector exists. This immediately raises the question, what the whole concept is good for in the real world. However, as will be shown in this section, it is well possible to provide public goods to the economy without thereby distorting the natural rate of interest. For simplicity, we confine ourselves to a stationary economy in this section.

Public consumption goods

Public goods are normally provided by the state because non-payers cannot be excluded. Moreover, most of these goods are also non-rival. However, this does not at all prevent them from being provided competitively, as can be seen from numerous examples (like e.g. software, television programs, or digitally stored music). Thus, non-excludability is the really decisive issue here.

Concerning the optimal supply and funding of public goods, one could be tempted to simply include them into an OLG-model and then optimize the utility function in the normal way. However, by doing that one would easily fall to a trap, because the conventional steady-state-approach implicitly leads to a golden-rule-solution again, which we have already proved to be generally inferior to the market solution. On the other hand, explicitly allowing for adequate compensation schemes as above is not at all a trivial project.

Fortunately, there is a way out: Because we already know that a perfect market generates the natural rate of interest, we can simply ask what the equilibrium supply of public goods

would be if they were excludable and, thus, could be provided by the market. Assume e.g. that G is the amount of a non-rival good which is consumed by both the young and the elderly in our model from above. If there are V (identical) individuals in the economy, each of them has to bear only G/V of the costs of the non-rival good, while they have still to pay the full costs of each unit of the normal consumption good C. For simplicity, we assume that the price of both goods is unity and that the economy is stationary. Then the individual maximization problem with a competitive supply of both goods reads

(12) $$Max\ U(C_j; G_j) \quad j = 1;2$$

s.t.

(13) $$Y_1 - C_1 - G_1/V - \frac{C_2}{1+i} - \frac{G_2/V}{1+i} = 0$$

where G_1 and G_2 are the units of the non-rival good which are consumed by each individual in the two periods of their life (e.g. the number of visits in a zoo), i.e. we have $G_1 + G_2 = G$. Then, with the production assumptions from above, the relevant optimality condition is

(14) $$\left|\frac{dC_2}{dC_1}\right|_{\bar{U}} \overset{!}{=} \left|\frac{dG_2}{dG_1}\right|_{\bar{U}} \overset{!}{=} (1+i^*) \overset{!}{=} \frac{\delta Y}{\delta K}$$

(15) $$\left|\frac{dG_1}{dC_1}\right|_{\bar{U}} \overset{!}{=} \left|\frac{dG_2}{dC_2}\right|_{\bar{U}} \overset{!}{=} V$$

Now suppose that the non-rivalry good G is also non-excludable and, thus, must be provided by the state. In principal, the publicly provided G can be financed by a tax on the income of the young, by a tax on the income of the elderly, or by public debt D (which is equal to the public deficit in our two-period model). Hence, with τ_1 and τ_2 denoting the respective tax rates, the individual budget restriction now reads

(16) $$Y_1(1-\tau_1) - C_1 - \frac{C_2}{1+i(1-\tau_2)} = 0$$

where we have $Y_1 = (1-\beta)Y$ like before. The new individual optimality condition is

(17) $$\left|\frac{dC_2}{dC_1}\right|_{\bar{U}} \overset{!}{=} 1 + i^*(1-\tau_2) \overset{!}{=} \frac{\delta Y}{\delta K}$$

From the comparison of (15) and (17) it immediately follows that τ_2 must be zero in order to generate the same allocation as the (hypothetical) market solution. Hence, we are left with the public budget restriction

(18) $$\tau_1 Y_1 - G_1 - G_2 - iD = 0$$

Because the public solution shall mimicry the market, the equilibrium values of all consumption quantities C_j^* and G_j^*, of total income Y^*, and of the (natural) interest rate i^* are required to be equal in both cases.

Principally, the optimal amount G_1^* could be funded by an appropriately chosen tax rate τ_1^* such that $\tau_1^* Y_1^* = G_1^*$, thereby generating the same revenue which would be spent by the young individuals if they had to purchase G_1^* at the market. However, with the public provision of G , private saving S_1 will be lower than S_1^* , because there is no more need for individual providing for the future consumption G_2^*. Hence, in order to balance the capital market with the competitive capital demand K^*, we need an additional capital supply by the government, which is

(19) $$-D^* = \frac{G_2^*}{1+i^*}$$

Thus the hypothetical market result can be imitated by using a combination of taxes and a *negative* public debt for financing the public consumption good G ! The public saving $-D^*$ just substitutes that part of private savings S_1^* which would be devoted by the individuals for their future consumption of the public good if the latter was provided by the market. Regarding the public budget restriction (18), for the optimal tax on Y_1^* we then have

(20) $$\tau_1^* Y_1^* = G_1^* + G_2^* - i^* D^*$$

Hence, our model in principle confirms the classical rule (of thumb) that public consumption goods are best financed by taxes only. Only to the extent at which the good is consumed in future (i.e. by the elderly), a perfect imitation of the market requires that the respective costs are covered by additional (public) savings, as it would be necessary in a competitive equilibrium as well. As a result, in this case tax revenues need not fully cover the out of pocket costs of the public good because of the public interest revenues.

Public investment goods

Now suppose that the government provides a public investment good I instead of a public consumption good, a case which is e.g. dealt with by (Yakita 2018). Analogously to the private capital good K , we assume that I also lives for only one period. Formally, it can be

included into the production function as a second capital good. Still assuming a normal (though less than linear homogeneous) neoclassical production function, we now have $Y = Y(N; K; I)$. Setting the price of both capital goods to unity and labelling the partial elasticity of production of the public investment good as ϕ, the optimal relation of the two capital goods is $(K/I)^* = \beta/\phi$.

Again, suppose first that the public investment good I can be provided by the market. Then the competitive capital equilibrium requires that

(21)
$$S_1^* = K^* + I^* = \frac{(\beta + \phi)Y^*}{1 + i^*}$$

Analogously to (5), equation (21) determines the natural rate of interest i^*. The individual budget constraints for the young and the elderly in equilibrium are given by

(22)
$$Y_1^* = (1 - \beta - \phi)Y^* = C_1^* + S_1^*$$

(23)
$$Y_2^* = S_1^*(1 + i^*) = (\beta + \phi)Y^* = C_2^*$$

Now suppose that I is non-excludable and thus must be provided by the government. Again, we look for a public funding scheme that perfectly imitates the competitive solution. The respective individual budget restrictions must satisfy

(24)
$$Y_1 = (1 - \beta)Y^*(1 - \tau_1) = C_1^* + S_1$$

(25)
$$Y_2 = S_1(1 + i^*(1 - \tau_2)) = (\beta + \phi)Y^* = C_2^*$$

For the same reason as above, τ_2^* is zero, because otherwise we would immediately have a distortion of the interest rate which also distorts the equilibrium marginal rate of substitution between C_1 and C_2. Thus, the public budget restriction is

(26)
$$\tau_1(1 - \beta)Y^* - I - iD = 0$$

where public debt D may principally be positive or negative. Capital market equilibrium requires that

(27)
$$S_1 = K^* + D.$$

With these equations, it is easily shown that a perfect imitation of the market requires [iii]

(28)
$$\tau_1^*(1 - \beta)Y^* = (1 + i)I^*$$

and

(29)
$$D^* = I^*$$

In other words: The appropriate funding of a public investment good in order to mimicry a respective (hypothetical) market result is public debt, supplemented by a tax in order to meet the annual debt service.

Again, this result nearly perfectly meets the respective classical rule of thumb (the so-called golden rule of public funding). It is also quite intuitive: Because I is financed by (private) debt in the market case, it appears natural to finance its public provision by (public) debt as well. Moreover, in order to pay the interest bill (including depreciation), we need a respective tax in addition. Again, it is appropriate to impose that tax on the non-interest-income alone, because otherwise we would distort the intertemporal rate of consumption substitution according to (15).

4. Allowing for money

After we have shown that the natural interest rate can be preserved with public goods in principal, what about money? Money has been included in OLG models by many authors, see a.g. McCallum (1983), Abel (1987), Maeda (1991), and Zhu (2008). As is well known since Metzler (1951), it may have not only a temporary, but also a lasting effect on the interest rate depending on the way how it is brought into circulation (see also Niehans 1987, p. 78). Moreover, it is also important if there is Ricardian equivalence or not. With perfect Ricardian equivalence, individuals would recognize that both the government and the central bank are lastly part of their own wealth, because all expenditures and revenues of these institutions are mirrored in a respective change in their future tax burden. As a result, an increase in public debt might be compensated by an increase in private savings, while an increase in central bank assets might have the opposite effect. However, a full compensation by such counter effects is unlikely to occur, if it occurs at all. Hence, we will neglect Ricardian equivalence in the following.

Helicopter money

By money we generally mean some sort of asset which is both generally acknowledged as currency and limited in quantity, such that it can serve as a means of payment and as a store of wealth as well. Today it is standard to give money an intrinsic value, namely by incorporating it either into the production function or into the utility function (Walsh 2010 pp 33). In the so-called cash-in- advance (CIA) framework money is seen as a factor of production which reduces transaction costs. In contrast, in the money-in-utility (MIU) approach, liquidity holding directly appears in the individual utility function because of its convenience and other advantages above pure barter trade. In the following, we use the latter approach.

We also distinguish between two different means of bringing money into circulation, namely helicopter money vs. credit money. Unlike normal credit money, helicopter money is not lend out but donated by the central bank, either to the government or directly to the private sector. It is also conceivable that helicopter money is not provided by any monetary

authority at all, but simply exists, e.g. in the form of any precious metals, rare shells or anything else like this.

In order to introduce money it into our model, we include real liquidity L/p into the individual utility function, where L denotes the number of money units at the disposal of the individual, and p is the overall price level in the economy. In equilibrium, the resulting demand for real liquidity must equal real money supply, which is the quantity of helicopter money $\overline{M}$ divided by p. Thus the equilibrium price level is generally given by

(30)
$$p^* = \frac{\overline{M}}{(L/p)^*}$$

where real liquidity demand $(L/p)^*$ normally depends on both total income and the interest rate. However, because the creation of helicopter money does not tackle the capital market, the interest rate is independent of its quantity. Thus changes in the numerator of the price formula do not affect its denominator, permanently at least. This in turn suggests that with pure helicopter money the quantity theory of money is valid.

Formally, with a constant amount of helicopter money $\overline{M}$, the individual optimization problem in an economy growing by n reads

(31)
$$Max\ U(C_1;C_2;L/p)$$

s.t.

(32)
$$(1\text{-}\beta)Y = C_1 + S_1 + (L/p)$$

(33)
$$(1+i)S_1 + (1+n)(L/p) = C_2$$

All variables including the interest rate are defined in real terms, so there is no need to explicitly regard inflation or deflation here. Real liquidity (L/p) is built by the individuals when they are young and spent for consumption in total when they are old. By this way, it permanently circulates in the economy. Note that, with a constant quantity of helicopter money $\overline{M}$, the real value of liquidity increases by the economy`s growth rate, as it is indicated in the budget restriction for the elderly.

The solution of the individual maximization problem must satisfy (8) as above and requires in addition

(34)
$$\left|\frac{\delta(L/p)}{\delta C_1}\right|_{\overline{U}} = \frac{1+i^*}{n-i^*}$$

(35)
$$S_1 = \frac{C_2}{1+i^*} - \frac{1+n}{1+i^*}\left(\frac{L}{p}\right)$$

From (35) it is easily seen that, unlike real savings S_1, liquidity does not contribute to the capital stock, although it absorbs part of private asset holdings. For this reason, real savings S_1 are c.p. lower and, hence, the natural interest rate is higher in an economy with money than in a pure barter economy, as is well known. However, this effect is independent of the nominal quantity of helicopter money $\overline{M}$. This is because real savings are (partly) replaced by *real* liquidity in the individual`s asset planning, and $(L/p)^*$ in turn does not depend on the nominal quantity of helicopter money in circulation because the quantity theory is valid here, as we have seen above. Hence a (one off) increase in $\overline{M}$ tends to increase the price level, but does not permanently affect the (natural) interest rate.

This is different with a *permanent* change in $\overline{M}$ which creates either inflation or deflation, because a permanent change in the price level changes the demand for real liquidity. In particular, inflation reduces liquidity demand and hence tends to decrease the (real) interest rate, which can be interpreted as a kind of Phillips curve effect. Conversely, deflation tends to increase the interest rate, because it equals a return on cash and thus tends to reduce real savings. Thus, with a given amount of helicopter money, the interest rate increases in the economy`s growth rate – just as it would in a pure barter economy, as we have seen above. In other words, helicopter money is neutral with respect to the natural rate of interest if its quantity is either constant by nature (as it is the case with unreproducible goods), or if it is voluntarily held constant by the monetary authorities.

Credit money

Now we consider credit money, i.e. money which the central bank creates by lending out to the economy. In this case, unlike with helicopter money, an additional capital supply emerges. Denoting the relation of real value of credit money and total product as m, the new equilibrium condition at the capital market reads

(36) $$S_1 = K + mY$$

Apparently, the "artificial" capital supply generated by credit money tends to reduce the interest rate, if one rules out Ricardian counter effects (as we do here). Thus credit money enables the central bank to manipulate the natural rate of interest in whatever direction, even in the long run. In principle, they can even enforce a negative interest rate, provided both cash and land are not available (or prohibited) as an escape store of wealth. (Otherwise, the increase in credit money supply would be absorbed by a respective increase in credits by the private sector, which in turn would be invested in both land and other stable assets.)

Note that we always need some helicopter money in addition, because with credit money alone the price level was undetermined. Hence, monetary equilibrium with credit money requires

(37)
$$\overline{M} / p + mY = \left(L/p\right)^*$$

from which the price level can be easily derived as

(38)
$$p^* = \frac{\overline{M}}{\left(L/p\right)^* - mY}$$

In contrast to a widespread believe, even a huge increase in credit money does not necessarily lead to inflation. This is because a decreasing interest rate at the same time tends to increase the demand for real liquidity, as the opportunity costs of holding liquidity decline as well. Hence, in contrast to the helicopter case, we have two conflicting effects in our price equation.
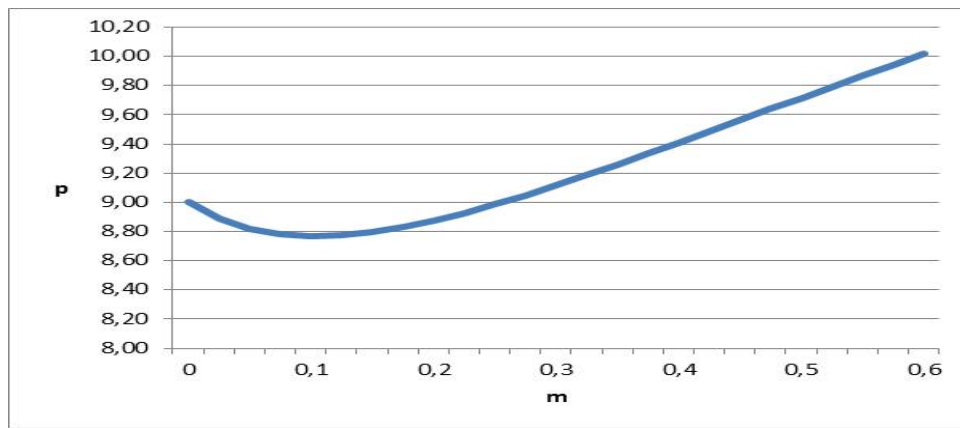

Figure 2

For example, an increase in m (i.e. a rising share of credit money) may lead to a U-shaped reaction of the price level according to figure 2,[iv] i.e. while with a moderate rise in m at first *deflation* occurs, an even more expansionary monetary policy leads to an increase in the price level sooner or later.

Anyway, if one wants to preserve the natural interest rate, our analysis supports the idea of helicopter money instead of credit money. This does not preclude that the central bank temporarily intervenes at the capital market in order to mitigate business fluctuations (or for the sake of financial stability). However, with helicopter money, the long-term interest rate in the Taylor equation it is much more likely to meet its natural level than with credit money.

5. Can dynamic inefficiency really occur?

Turning towards the $i^* < n$ case now, why does it only occur in OLG-models, but not in the Ramsey/Koopmans/Cass framework? Even more interesting: Is it really relevant for the real world or just a theoretical chimaera? These questions have been intensively discussed in the past, with major contributions by Shell (1971), Okuno and Zilcha (1981), Blanchard (1985), Weil (1987, 1989, 2008) and others. In general, there are two competing explanations for

the possibility of $i^* < n$ in OLG models: One of these builds on the limited trading options in these models, due to the limited lifetime of agents. The other answer rejects this view and argues instead that and endless chain of overlapping generations implies a troublesome double infinity (Shell, 1971), i.e. "an infinite number of households and dated goods" (Weil 2008, p. 123). This in turn would allow for Pareto-improving Ponzi-games which are, however, only available to the (permanently existing) government but not to the early mortal market agents.

In fact, the two answers are not that different as it might seem at first glance. In the end, both refer to mutually advantageous arrangements which are possible in principal, but not feasible under market conditions with the normal assumptions made in OLG models. As a consequence, public interventions such as establishing a pay-go-pension system or creating public debt as an alternative drain channel for excess saving are frequent proposals for a cure, not only in the model world but also in reality (von Weizsäcker 2013; Weil 2008, pp 125). Indeed, as is well known, these measures tend to increase the interest rate, because in both cases part of private saving is actually diverted to consumption, while it creates net wealth only from the individual´s point of view.

However, a similar effect can also be achieved by allowing for private assets which do not add to the real capital stock. Obvious candidates are land, precious metals, or paper money, which all have in common that they can store wealth without depreciation. Moreover, if these assets are both scarce and non-extendable, their value tends to increase by the economy`s growth rate (as it is true for the claims in a pay-go system as well). With such alternative saving options at hand, individuals are no longer forced to lend out at an interest rate below the growth rate and, thus, dynamic inefficiency can no longer occur. Formally, this can be seen from equation (34) which implies that, with i approaching n, the demand for real liquidity would tend to infinity, while real savings get smaller and smaller

Already Samuelson in his original 1958 contribution pointed to money as a way out of the dilemma. However, he only dealt with the special case of stable money rather than the more general requirement that money should be neutral in order to prevent $i^* < n$. Moreover, Samuelson`s money was just "printed paper" without any intrinsic value. Hence, as Weil (2008, pp 126) rightly remarks, it would have no value in an economy with $i > n$ and, moreover, even with $i < n$ its value would only rest on the (unfounded) confidence that future generations will accept it in exchange for real goods. In other words, Samuelson`s money was little more than an object of speculation (like bitcoins in todays world), which could rarely be trusted to solve the problem. Indeed, by just introducing any intrinsically worthless asset into the budget restrictions as we have done in (32) and (33), but without having liquidity in the utility function (31), the latter would simply drop out of the solution and, hence, we would still be left with the possibility of $i < n$.

In the section above we have already mentioned that with neutral money the real value of liquidity increases by the growth rate n. Neutral money, which is generally achieved by

holding its nominal quantity constant irrespective of economic growth, was already preferred above stable money by Milton Friedman (1969), as is well known. Obviously, with neutral money, the interest rate cannot fall below the growth rate because, unlike liquidity, real savings do not even have an intrinsic value. Hence, as we have already argued above, with i approaching n nearly all private assets will be held in the form of liquidity in the end, while the real capital stock dramatically declines (with a low marginal, but a high average productivity). Conversely, in contrast to Samuelson`s intrinsically useless money, even with an interest rate far above the growth rate at least some liquidity will be held because of its non-pecuniary advantages.

The same is principally true for every other durable goods such as artwork, classic cars, or precious metals, provided that they have any – however small – non-pecuniary value, such that they can be included in the utility function. Under this assumption, a move of the interest rate towards the growth rate will boost the demand for these goods as well. The sharp increase in the price of these goods (as well as in land prices) in recent time supports this theoretical result.

Admittedly, in a growing economy, neutral money would cause deflation, which is generally seen as a knock out argument for its invention. However, it is well conceivable that a neutral parallel currency exists which is mainly used as a store of wealth, while the main currency is non-neutral and masters the bulk of daily transactions. The bitcoin might be an example, because it´s nominal quantity is indeed absolutely fixed (at approximately 21 billion units), while it´s real value almost exploded since its invention. Hence, although speculation still causes extreme volatility, parallel cryptocurrencies like the bitcoin might emerge as a serious alternative to holding national currency in future, in particular with the low interest policy going on or even being perpetuated.

Another collecting tank for excess saving is land, as Homburg (1991;2015) has accurately shown (an argument which is amazingly ignored by Weil 2008). Unlike our modelling of money, the intrinsic value of land immediately comes from its contribution as a production factor. In particular, with a constant GDP-share of land rent, its value obviously increases by the growth rate n. Moreover, in the absence of any special risks or preferences, holding land must yield the same return as any other asset. Hence, for the total value of land B we have

$$(39) \qquad\qquad iB \overset{!}{=} \Pi + nB$$

Regarding equation (3) for the land rent, in our model we thus have

$$(40) \qquad\qquad B = \frac{(1-\beta-\gamma)Y}{i-n}$$

from which it immediately follows that, with i approaching n, land value strives to infinity, as already Turgot has known (and Homburg has formally proved).

Formally, the respective individual budget restrictions with land are identical with (32) and (33), only with (L/p) being substituted by B. However, because land does not directly appear in the utility function, we actually need equation (40) in order to determine it`s real value. Otherwise, $B$ would simply drop out of the calculation and was thus ineffective like Samuelson`s printed slips of paper.

Hence, like in the case of neutral money and for the same reason, with land as an alternative store of wealth the interest rate can never undergo the growth rate. Indeed, from (39) it follows that the interest rate is $i = \Pi/B + n$, i.e. with a positive land rent it must even exceed the growth rate by the respective return on land.

This argument is occasionally thrown into doubt by the objection that land was only an imperfect substitute for other forms of asset investment. Firstly, its value was highly dependent on local conditions, and secondly, it was generally taxed more heavily than financial assets, because land is an ideal item from the view of optimum taxation theory (von Weizsäcker 2015, pp.199). However, even with such additional risks, the land-option still prevents a negative interest rate. Assume e.g. that, unlike financial investments, the land rent is subject to a special tax rate (or an equivalent risk) $0 < b < 1$. Then our interest formula simply reads $i = \Pi(1-b)/B + n$ which is still above n. While the emergence of such a tax or risk will surely reduce the absolute value of land, this is only a one-off write down which does not affect its annual increase by n. Hence, as long as the tax rate does not amount to unity, the interest rate still cannot fall below the growth rate, and dynamic inefficiency cannot occur.

6. Summary

According to the results of our simple model, the natural interest rate i turns out to be anything but a useless or even ideological concept. It is even superior to the golden rule as a benchmark for efficient capital allocation, if the Kaldor-Hicks criterion is adopted to intergenerational conflicts. We have also shown that neither public goods nor the existence of money necessarily disturbs the natural interest rate, so the concept is not at all confined to pure barter economies without a public sector. In theory at least, there are relatively simple rules which the government would have to follow in order to preserve the natural interest rate. Concerning public goods, our model supports the classical rule of thumb after that (present) consumption goods should be financed by taxes, while public investment should be financed by public debt. Moreover, we have argued that neutral money can preserve the natural interest rate, while credit money generally tends to disturb it. This is a strong argument in favor of helicopter money, which does not affect the natural capital market equilibrium.

Generally, we can brake a lance for OLG modelling, which is erroneously alleged to violate the first fundamental theorem of welfare economics in our view. Our model clearly supports

the Turgot-Homburg argument that an interest rate below the growth rate is impossible with land as an alternative store of wealth, with perfect capital markets at least. We have also argued that the same is true with neutral money, thereby generalizing the well- known fact that stable money acts as a "lower zero bound" to the interest rate. Hence, from the theoretical viewpoint at least, the so-called savings glut hypothesis must be rejected if either land, neutral money, or other assets are available the value of which tends to change with the economy´s growth rate. Only if such alternatives do not exist, either because of imperfections or because of legal obstacles such as banning cash, dynamic inefficiency can occur. Likewise, only then an expansionary monetary policy can indeed enforce negative interest rates resp. interest rates below the growth rate, thereby voluntarily creating dynamic inefficiency.

From the political perspective, the arguments in favor of more public debt in order to absorb excess savings appear quite dubious. First, an only temporarily low interest rate does not at all prove dynamic inefficiency, in particular not in an emerging world economy. The whole concept is only meaningful in steady states by definition, so it is hard to verify empirically by nature. Second, even if dynamic inefficiency was existent, one has to examine very carefully if it is really the result of excess savings or rather the result of excess (credit-) money supply. Although this is an empirical issue in the end, our model clearly shows that money is not generally neutral but can indeed permanently affect the interest rate. Because credit money can be viewed as negative public debt, its reduction (in favor of more helicopter money as the case may be) was a natural alternative to further increasing public debt.

While this may appear all intuitive and clear, we have of course to regard the limitations of our model. First, with only two periods and perfect markets throughout, it is far from reality. Second, it is surely too simple to allow for direct empirical adoptions. Third, we do not have incorporated uncertainty or risk. Fourth, we have confined us to a closed economy with identical individuals and firms. Last not least, our model is mainly concerned with steady states and does not allow for dynamics, although we have partly taken transition periods into account.

Admittedly, these are quite strong limitations. On the other hand, the underlying temporal capital theory is mostly based on quite simple models as well. In particular, it is not even clear how dynamic inefficiency should be defined when the interest rate does not only mirror the temporal consumption sacrifice but covers also risk and pure profits. With such modifications, one cannot simply adopt the common "i < n" definition but would have to show in detail whether and why a social planner could do better than the market. Last not least, in order to draw any political consequences from respective models, one would also have to scrutinize both the state of knowledge and the benevolence of the social planners themselves. Hence there is plenty room left for further research, even on the purely theoretical level, not to mention empirical evidence.

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[i] From $\delta Y / \delta K = (1+i)K$ it follows that $\beta \equiv (\delta Y / \delta K)/(Y / K) = (1+i)K / Y$. As is well known, in the special case of a Cobb-Douglas production function β is simply the exponent of capital.

[ii] Note that this line of argument does not hold for the case $i^* < n$, because then an additional unit of C_2 in future does not involve any sacrifice of C_1 today. Hence, in that case, dY / dK cannot be interpreted as MRT. (The respective figure would look like figure 1, with the intersections $1 + i^*$ and $1 + n$ being interchanged. In the limiting case $i^* = n$ the MRT line is still steeper than the MRS line, but just tangents it at one point.)

[iii] With $\tau_2 = 0$, from the public budget restriction it immediately follows that $\tau_1^*(1-\beta)Y^* = I^* + Di$, and from the budget restrictions of the elderly it follows that $S_1 = S_1^*$. Hence, because $S_1 = K^* + D$ and $S_1^* = K^* + I^*$, we have $D^* = I^*$.

[iv] The figure relates to an example with $U = C_1^{1/3} C_2^{1/3} (L/p)^{1/3}$, $Y = K^{1/3}$ and $\overline{M} = 1$.