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A traffic flow breakdown externality induced by stochastic road capacity

By Kathrin Goldmann* and Gernot Sieg*

Traffic jams occur even without bottlenecks, simply because of interaction between vehicle drivers on the road. From a driver point of view, the instability of free flow arises stochastically. Because the probability of a traffic jam increases with a rising road saturation, there is a traffic flow breakdown externality. Ignoring the stochastic nature of traffic brake downs results in congestion charges that are too small.

Keywords: hypercongestion, congestion costs, stochastic capacity, external costs

I. Introduction

Freeway capacity has been defined as the maximum flow rate that can reasonably be expected to traverse a facility under prevailing roadway, traffic and control conditions. The traditional view (Small and Chu, 2003; Button, 2004) is, that, with an increasing number of vehicles on the road, vehicles affect each others speeds and slow each other down. As more traffic enters the road, average speed falls but, up to a point, the flow will continue to rise, because the effect of additional vehicles outweighs the reduction in average speed. This is the congested branch of the speed-flow curve. At the point where increased demand does not increase traffic flow any further, the roads capacity is reached. The flow becomes unstable, with the characteristic stop-and-go conditions which are typical of traffic jams. This traffic state is called hypercongestion in economics. The reasons for traffic jams could be on the demand side (on-ramps with high inflows, fluctuations in demand) or on the supply side (traffic accidents, construction sites, tunnels or inhomogeneous road design).

However, Sugiyama et al. (2008) showed, that even in the absence of supply side reasons, traffic jams (hypercongestion) can occur. For this to happen, it is sufficient that drivers on a street interact with each other to make the traffic flow unstable. There may be deterministic reasons like tailgating, to fast reactions to speed changes, slow overtaking by trucks, slow reactions because of inattentiveness or queue-jumping, but in the system, these driving errors occur stochastically (Schönhof and Helbing, 2007). Some of these factors culminate in a traffic jam, but some do not. The probability of their causing a traffic jam increases with the saturation of the highway. Therefore, capacity is stochastic (Elefteriadou et al., 1995; Brilon et al., 2005).

A driver only considers his own costs, but not the time losses other drivers have due to increased traffic. We determine the external costs imposed on other drivers. A stochastic capacity approach enables us to establish a model that

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includes hypercongestion without explicitly modelling queues.

Drivers can be faced with a free flow or congested traffic on good days and hypercongested traffic on bad days. Average travel speed and average travel times differ greatly between the two states. For those reasons, a driver entering the road in order to travel a certain distance faces a stochastic travel time, depending on the number of other vehicles on the road. We identify a so far academically ignored externality of an additional driver on the road. The driver increases the probability of hypercongestion, a state with inefficiently long travel times.

Verhoef (1999) shows that hypercongestion is dynamically infeasible when considering capacity as deterministic. In order to depict hypercongestion in a static model with continuous demand, inflows onto the road have to exceed the maximum possible inflow at some point in the past, which is inconsistent with the concept of maximum deterministic capacity. Small and Chu (2003) suggest that hypercongestion on a highway entails a queue of cars waiting in front of a bottleneck. Therefore, density within the queue does not exert an effect on the outflow rate from the bottleneck and on travel time, but only on the number of cars waiting. Following this interpretation, Small and Chu (2003, p. 326) state that hypercongestion is irrelevant to users who care only about total travel time.

A few papers have used the bottleneck model (Vickrey, 1969; Small, 2015) for analyzing hypercongestion, by postulating that bottleneck capacity varies with the length of the queue. Yang and Huang (1997) identify a dynamic externality, that is, how an additional car influences the queue length and therefore the bottleneck efficiency, but do not consider inefficient hypercongestion states such as stop-and-go traffic, but only queues. Consequently, Yang and Huang (1998) include a queuing externality in the congestion charge and suggest calculating the flow-dependent travel time and the queuing delay separately, with the former being predicted by an analytical delay formula, and the latter determined from network equilibrium conditions. The bottleneck model was extended to stochastic capacity by Lindsey (1994), Arnott et al. (1999) and Fosgerau (2010). Demand and bottleneck capacity is assumed to fluctuate from day to day, but as soon as a given day proceeds, they remain constant for the travel period on this day. For this reason Lindsey (1994) states, that the model can adequately display capacity fluctuations due to road work, weather conditions and major truck accidents, but not temporary capacity fluctuations.

The bathtub model (Arnott, 2013; Fosgerau and Small, 2013; Fosgerau, 2015; Arnott et al., 2016) analyzes urban hypercongestion. A backward-bending fundamental diagram of traffic flow also applies at the level of an urban neighborhood, which meets certain conditions (Daganzo et al., 2011). As a result, urban congestion can be analyzed in aggregated form, using a speed-flow relationship.

However, stochastic traffic flow breakdowns are not covered by the abovementioned models. Furthermore, in contrast to urban centers, in our model, traffic flow is unidirectional. To obtain a model that is theoretically consistent, we focus on a predetermined number of homogenous drivers aiming to travel at the same speed on a circuit or roundabout, that is a circular street without beginning or end. Unidirectional traffic flows on a road without bottlenecks can be observed on many highway sections and therefore, our model can best be applied to hypercongestion situations on highways.

II. A model of congestion costs

Sugiyama et al. (2008), Nakayama et al. (2009) and Tadaki et al. (2013) performed traffic experiments on a circuit to investigate the emergence of a jam without a bottleneck. Their experiments are depicted in Figure 1. Tadaki et al. (2013) let 10 to 40 homogenous cars enter a circuit of 314 m length one by one, with the driver of the first car requested to drive slowly until all the cars have entered. After that, all drivers should drive at a homogenous target speed of 30 km/h. When the number of cars N is low, $10 \leq N \leq 25$, they observe free flow. If the number of cars exceeds 32, the flow jams. However, if the number of cars is within a medium range, $26 \leq N \leq 31$, they detect metastable phases - stop-and-go traffic - in which cars stop or nearly stop in jam clusters, alternating between escaping from the jam cluster and again catching up with it.

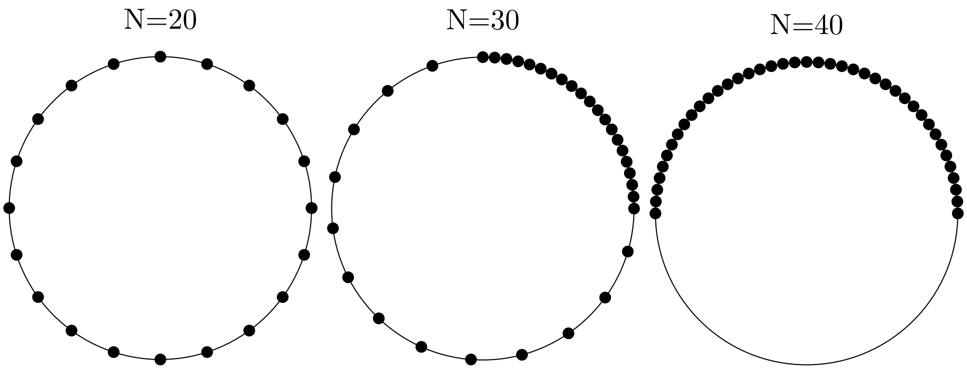

Figure 1. Depiction of traffic experiments on a circuit initially performed by Tadaki et al. (2013)

The expected travel speed of a driver depends on the traffic situation on the circuit. If the number of vehicles is small, drivers enjoy free flow and can travel at the speed they want. If the number of vehicles is large, drivers are stuck in a traffic jam and travel speed is low. However, in between, there is an area where we detect both states alternating. To calculate the expected travel speed, we consider the two traffic states identified by Tadaki et al. (2013), free-flow and jammed. Depending on the number of vehicles N on the circuit, both states alternate, and from the point of view of a vehicle driver, the traffic is either fluid and the average travel speed is $\overline{v}(N)$ or jammed at a speed of $\underline{v}(N)$. The probability of jammed traffic is $p(N)$ and also depends on the number of other drivers on the circuit, because the more vehicles, the larger the probability that the traffic flow breakes down. A driver on the circuit expects a travel speed[1] of

$$E(v) = p(N)\underline{v}(N) + (1 - p(N))\overline{v}(N). \tag{1}$$

If we increase the number of vehicles on the circuit, the expected speed changes

[1] In this calculation, we assume that the probability is calculated in such a way that both the free flow and the jammed traffic holds for a period of time that is long enough to travel the considered distance, for example, a whole circuit.

and the marginal effect is

$$\frac{dv}{dN} = \underbrace{(1-p)\frac{d\overline{v}}{dN} + p\frac{d\underline{v}}{dN}}_{\text{expected capacity effect}} + \underbrace{\frac{dp}{dN}(\underline{v}(N) - \overline{v}(N))}_{\text{traffic flow breakdown effect}} \quad . \tag{2}$$

The first two terms on the right hand side of this equation represents the expected loss of speed if an additional driver enters the circuit. The third term describes the effect caused by an increased breakdown probability. The third effect is usually ignored if capacity is considered to be deterministic. Because p is an increasing function in N (Brilon et al., 2005), and the free flow speed is larger than that in jam clusters, the traffic breakdown effect is negative. If capacity is considered deterministic, the expected loss of speed of an additional vehicle is underestimated.

Travel time costs c depend on the speed, which in turn depends on the number of vehicles on the circuit, and the the expected travel time costs C of a driver are

$$C(N) = p(N)c(\underline{v}(N)) + (1 - p(N))c(\overline{v}(N)) \tag{3}$$

and when we assume homogenous drivers, these costs are the average costs of all N drivers on the circuit. Social costs are $SC = N \cdot C(N)$ and marginal social costs are $MSC = C + N \cdot C'$. If we were to pay drivers a wage for successfully completing circuits, and if we allowed free entry to the circuit, drivers would take part if their travel time cost C for a circuit is less than the amount we pay. In this decision $N \cdot C'$ is the external effect (on other drivers), which is not taken into account by individual drivers. The marginal external travel time costs are:

$$N\frac{dC}{dN} = N\left[(1-p)\cdot\frac{dc}{dv}\frac{d\overline{v}}{dN} + p\cdot\frac{dc}{dv}\frac{d\underline{v}}{dN} + \frac{dp}{dN}\left(c(\underline{v}(N)) - c(\overline{v}(N))\right)\right]. \tag{4}$$

Considering a distance of a and a time value of t, $c(v) = ta/v$ and $dc/dv = -ta/v^2$, equation 4 can be written as:

$$N\frac{dC}{dN} = N\left[(1-p)\cdot\frac{(-ta)}{\overline{v}^2}\frac{d\overline{v}}{dN} - p\frac{ta}{\underline{v}^2}\cdot\frac{d\underline{v}}{dN} + \frac{dp}{dN}\left(\frac{ta}{\underline{v}} - \frac{ta}{\overline{v}}\right)\right]. \tag{5}$$

To summarize, marginal external travel costs on the circuit equal

$$N\frac{dC}{dN} = \underbrace{-Nta\left[(1-p)\frac{1}{\overline{v}^2}\frac{d\overline{v}}{dN} + p\frac{1}{\underline{v}^2}\frac{d\underline{v}}{dN}\right]}_{\text{expected capacity effect}} + \underbrace{Nta\frac{dp}{dN}\left(\frac{1}{\underline{v}} - \frac{1}{\overline{v}}\right)}_{\text{traffic flow breakdown effect}} \tag{6}$$

Because the traffic flow breakdown effect is positive, we can state:

PROPOSITION 1: *Ignoring the stochastic nature of traffic flow breakdowns by considering capacity as deterministic, underestimates the congestion charge needed to internalize hypercongestion.*

If we assume that $p(N) = 0$ if N is small and that $p = 1$ if N is large, the formula holds for the complete range of N drivers. It is worth noting that the number of vehicles on the circuit is fixed and therefore, density on the circuit is constant and does not increase when traffic flow breaks down. Therefore, if we consider traffic on the circuit in equation 6, N can be replaced by the density.

III. Conclusion

Hypercongestion can occur as a transient response to a demand spike (Arnott, 1990, p. 200), or as a transient reduction in capacity (due, for example, to a traffic accident), or as a queue in front of a bottleneck (Small and Chu, 2003). Bottleneck models have also been modified with stochastic demand and capacity. We focus on hypercongestion that occurs in a non-linear system without identifiable reasons, and therefore assume stochastic road capacity without bottlenecks. Departing from traffic experiments and a constant number of drivers, we set up a model with random traffic jam formations. By doing so, congestion and hypercongestion costs can be calculated. We identify a previously ignored externality. An additional vehicle not only reduces the speed of the other vehicles, but also increases the probability of a traffic jam developing. This is an important effect that needs to be quantified when calculating the external costs of hypercongestion.

We do not analyze transitions between different traffic regimes, because we regard the traffic situation as a chain of different states, but our approach can easily be expanded to more than two traffic states. Our model is a static one based on average speed and flow data with a constant density on the circuit. Applying the approach to other roads, for example highways, calculating congestion costs or charges requires a knowledge of speed-flow functions and flow-dependent breakdown probabilities. This information can be extracted from high-frequency traffic data. Furthermore, in contrast to a circuit, on highways, the flow usually drops when traffic breakes down. This phenomenon can also easily be included in an empirical application of our model.

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