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Imprint

Ruhr Economic Papers

Published by

RWI – Leibniz-Institut für Wirtschaftsforschung
Hohenzollernstr. 1-3, 45128 Essen, Germany

Ruhr-Universität Bochum (RUB), Department of Economics
Universitätsstr. 150, 44801 Bochum, Germany

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Editors

Editorial Office

Ruhr Economic Papers #766

Responsible Editor: Manuel Frondel

ISSN 1864-4872 (online) – ISBN 978-3-86788-894-3

Ruhr Economic Papers #766

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Bibliografische Informationen der Deutschen Nationalbibliothek

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available on the Internet at http://dnb.dnb.de

RWI is funded by the Federal Government and the federal state of North Rhine-Westphalia.

http://dx.doi.org/10.4419/86788894
ISSN 1864-4872 (online)
ISBN 978-3-86788-894-3

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A Focused Information Criterion for Quantile Regression: Evidence for the Rebound Effect

Abstract

In contrast to conventional model selection criteria, the Focused Information Criterion (FIC) allows for the purpose-specific choice of model specifications. This accommodates the idea that one kind of model might be highly appropriate for inferences on a particular focus parameter, but not for another. Using the FIC concept that is developed by BEHL, CLAESKENS, and DETTE (2014) for quantile regression analysis, and the estimation of the rebound effect in individual mobility behavior as an example, this paper provides for an empirical application of the FIC in the selection of quantile regression models.

JEL Classification: C3, D2

Keywords: Information criteria; fuel efficiency; price elasticities

August 2018

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1 Peter Behl, RUB; Holger Dette, RUB; Manuel Frondel, RWI and RUB; Colin Vance, RWI and Jacobs University, Bremen. *– This work has been supported by the Federal Ministry of Education and Research (BMBF) within project LICENSE (grant 01UT1701A) and by the Collaborative Research Center "Statistical Modelling of Nonlinear Dynamic Processes" (SFB 823) of the German Research Foundation (DFG), within projects A3, "Dynamic Technology Modelling", and C1, "Model Choice and Dynamic Dependence Structures".* *– All correspondence to: Manuel Frondel, RWI, Hohenzollernstr. 1-3, 45128 Essen, Germany, e-mail: manuel.frondel@rwi-essen.de*

1 Introduction

Common model selection methods, such as the AKAIKE (1974) criterion (AIC) and the SCHWARZ (1978) criterion (SIC), do not require the specification of any purpose of inference. This also holds true for alternative model selection methods, such as goodness-of-fit tests, which are proposed by, among many others, DETTE (1999), DETTE, PODOLSKIJ and VETTER (2006), and PODOLSKIJ and DETTE (2008). However, conditional on the underlying purpose, some specifications might be better suited than others in terms of estimation efficiency. Recognizing this argument, CLAESKENS and HJORT (2003) designed the Focused Information Criterion (FIC) for the targeted search of parametric regression models that are estimated using maximum-likelihood methods, thereby explicitly taking the purpose of inference into account (BEHL, CLAESKENS, DETTE, 2014).

This is of high relevance in many fields of applied research, such as estimating the well-known direct rebound effect, which captures the behaviorally induced offset in the reduction of energy consumption following efficiency improvements (e. g. SORRELL, DIMITROUPOULOS, 2008; FRONDEL, PETERS, VANCE, 2008). To this end, alternative focus parameters, that is, population parameters defined irrespective of any model specification, are estimated in the context of individual transportation: First, the efficiency elasticity of mobility demand s:[1]

$$\eta_{\mu}(s) := \frac{\partial \ln s}{\partial \ln \mu}\,, \tag{1}$$

reflecting the relative change in mobility demand s due to a percentage increase in efficiency μ (see e. g. BERKHOUT *et al.*, 2000), and, second, the negative of the fuel price elasticity of mobility demand, $\eta_{p_e}(s)$:

$$\eta_{p_e}(s) := \frac{\partial \ln s}{\partial \ln p_e}\,, \tag{2}$$

[1]For the specific example of individual conveyance, parameter μ designates fuel efficiency, which can be measured in terms of vehicle kilometers per liter of fuel input.

with p_e denoting the fuel price per liter.

While $\eta_\mu(s)$ is the most natural definition of the direct rebound effect, the negative of the fuel price elasticity $\eta_{p_e}(s)$ is frequently the preferred measure for various reasons (FRONDEL, RITTER, VANCE, 2012), most notably because of the likely endogeneity of the efficiency variable μ. For instance, if a more efficient car is purchased in response to a job change that results in a longer commute, fuel efficiency would not be exogenous (see e. g. SORRELL, DIMITROUPOULOS, SOMMERVILLE, 2009:1361).

Using both these rebound definitions as focus parameters and the FIC developed by BEHL, CLAESKENS and DETTE (2014) for quantile regression analysis, this paper provides for an empirical application of the FIC in the selection of quantile regression models, thereby building on FRONDEL, RITTER and VANCE (2012), who investigate the heterogeneity of the rebound effect in individual mobility behavior on the basis of quantile regressions.It will become evident from our empirical illustration that model selection may depend on both the focus parameter and the percentiles of the dependent variable under scrutiny.

Because of its usefulness in balancing modeling bias against estimation variability, the FIC has been increasingly applied in the realm of statistics (see e. g. CLAESKENS, CROUX, VAN KERCKHOVEN, 2007, CLAESKENS, HJORT, 2008, and HJORT, CLAESKENS, 2006), but this concept appears to be virtually unknown in the economics literature. The contributions of BEHL et al. (2012, 2013) represent the sole exceptions for the literature on economic modeling, while the analysis of BROWNLEES and GALLO (2008) is a rare example originating from financial economics.

The general idea underlying the FIC, which ultimately results from estimating the mean squared error of the focus parameter estimators (CLAESKENS, HJORT, 2003:902), is to study perturbations of a parametric model, with the known parameter vector $\gamma^0 := (\gamma_1^0, ..., \gamma_q^0)^T$ as the point of departure. A variety of models may then be considered that depart from γ^0 in some or all of q directions: $\gamma \neq \gamma^0$. On the basis of parameter estimates of the altogether 2^q (sub-)models, that candidate model will be selected for which the FIC is minimal for a given focus parameter $\Lambda = \Lambda(\gamma)$.

By minimizing the FIC, one captures the trade-off between modeling bias, which, by definition, is zero for the most general model for which $\gamma_i \neq \gamma_i^0$ for $i = 1, ..., q$, and relative estimation variability, which, by definition, is zero for the most restricted model for which $\gamma_i = \gamma_i^0$ for $i = 1, ..., q$. For the sake of simplicity, in our empirical example on how to estimate the direct rebound effect, we will confine ourselves to $q = 1$. That is, we choose between just $2^q = 2$ model specifications, where, for instance, the unrestricted specification includes the variable fuel efficiency μ, while the restricted specification does not.

The following Section 2 provides for a concise introduction into the concept of the FIC. Section 3 presents the regression method, followed by the presentation of the empirical example in Section 4. The last section summarizes and concludes.

2 The Example of the Rebound Effect

To illustrate the concept of the FIC with the empirical example of the heterogeneity in individual mobility behavior, we choose both the negative of the fuel price elasticity of transport demand, $-\eta_{p_e}(s)$, for the identification of the rebound effect, as well as the efficiency elasticity $\eta_\mu(s)$.

To capture heterogeneity in the rebound response, we estimate the conditional quantile function (CQF) of the logged monthly vehicle kilometers traveled, $\ln(s)$, for a given percentile $\tau \in (0, 1)$, using quantile regression methods developed by KOENKER and BASSETT (1978):

$$\begin{aligned} Q_\tau(\ln(s_i)|p_{e_i},\mathbf{z}_i) &= \alpha(\tau)+\alpha_{p_e}(\tau)\ln(p_{e_i})+\mathbf{z}_i^T\boldsymbol{\alpha}_{\mathbf{z}}(\tau) \\ &= (\boldsymbol{\alpha}^0(\tau))^T\mathbf{x}_i\,, \end{aligned} \tag{3}$$

where $\ln(p_e)$ designates logged fuel prices, $\ln(\mu)$ denotes logged efficiency and $\mathbf{x}_i := (1,\ln(p_{e_i}),\mathbf{z}_i,\ln(\mu_i))^T$, with T indicating the transposition of a vector. $\mathbf{z}$ is a vector of control variables, such as household income, employment status of adult household members and number of children, and $\boldsymbol{\alpha}^0(\tau)$ is defined by $\boldsymbol{\alpha}^0(\tau) := (\boldsymbol{\xi}(\tau),\gamma^0)^T$ with $\boldsymbol{\xi}(\tau) := (\alpha(\tau),\alpha_{p_e}(\tau),\boldsymbol{\alpha}_{\mathbf{z}}(\tau))^T$. As efficiency μ is not included as a regressor in model (3), $\gamma^0=\alpha_\mu=0$.

Contrasting with specification (3), where efficiency μ is omitted, the rebound effect is frequently estimated from a wider model that includes efficiency variable μ:

$$\begin{aligned} Q_\tau(\ln(s_i)|p_{e_i},\mu_i,\mathbf{z}_i) &= \alpha(\tau)+\alpha_{p_e}(\tau)\ln(p_{e_i})+\boldsymbol{\alpha}_{\mathbf{z}}^T(\tau)\mathbf{z}_i+\alpha_\mu(\tau)\ln(\mu_i) \\ &= (\boldsymbol{\alpha}^{full})^T\mathbf{x}_i\,, \end{aligned} \tag{4}$$

where $\boldsymbol{\alpha}^{full} := (\boldsymbol{\xi}(\tau),\gamma(\tau))^T$ and $\gamma(\tau) := \alpha_\mu(\tau)$.

Adopting the terminology of CLAESKENS and HJORT (2003), specification (3) is called the narrow or null model, as efficiency variable μ is lacking, whereas it is included in the full model (4). Using the terminology introduced in the previous section, by estimating the full model, we depart from $\alpha_\mu=\gamma^0=0$ in just $q=1$ direction: $\alpha_\mu(\tau)=\gamma(\tau)\neq 0$.

In our example, in which model (3) is nested in specification (4), the quantile regression formulae for the FIC adopt a straightforward shape that strongly resembles those for specifications that are estimated using maximum-likelihood methods (see e. g. BEHL et al., 2013). While for the derivation of the FIC formula, the local asymptotics framework is essential, see e.g. BEHL, CLAESKENS, and

DETTE (2014), following BEHL, CLAESKENS, and DETTE (2014), the FIC for the null model is given by

$$\mathrm{FIC}^0 := \boldsymbol{\omega}_I^T \mathbf{B}\mathbf{B}^T \boldsymbol{\omega}_I, \tag{5}$$

where for $q = 1$ bias vector $\mathbf{B} := \sqrt{n}(\gamma - \gamma^0)$ degenerates to a scalar, $B = \sqrt{n}\gamma(\tau)$, and vector $\boldsymbol{\omega}_I$ is defined by

$$\boldsymbol{\omega}_I := I_{10} I_{00}^{-1} \frac{\partial \Lambda}{\partial \boldsymbol{\xi}} - \frac{\partial \Lambda}{\partial \gamma} = I_{10} I_{00}^{-1} \frac{\partial \Lambda}{\partial \boldsymbol{\xi}} - \frac{\partial \Lambda}{\partial \alpha_\mu}.$$

I_{10} and I_{00} belong to the information matrix

$$\mathbf{I} := \begin{pmatrix} I_{00} & I_{01} \\ I_{10} & I_{11} \end{pmatrix}, \tag{6}$$

whose components are defined as follows:

$$\begin{aligned} I_{00} &:= \lim_{n\to\infty} \tfrac{1}{n} \textstyle\sum_{i=1}^n f_i(\varepsilon_i|\mathbf{x}_i)\, \mathbf{x}_i^0 (\mathbf{x}_i^0)^T, & I_{01} &:= \lim_{n\to\infty} \tfrac{1}{n} \textstyle\sum_{i=1}^n f_i(\varepsilon_i|\mathbf{x}_i)\, \mathbf{x}_i^0 \ln(\mu_i), \\ I_{10} &:= \lim_{n\to\infty} \tfrac{1}{n} \textstyle\sum_{i=1}^n f_i(\varepsilon_i|\mathbf{x}_i)\, (\mathbf{x}_i^0)^T \ln(\mu_i), & I_{11} &:= \lim_{n\to\infty} \tfrac{1}{n} \textstyle\sum_{i=1}^n f_i(\varepsilon_i|\mathbf{x}_i)\, \ln(\mu_i)^2, \end{aligned}$$

where $\mathbf{x}_i^0 := (1, \ln(p_{e_i}), \mathbf{z}_i)^T$ and $f_i(\varepsilon_i|\mathbf{x}_i)$ denotes the unknown conditional density of the error term $\varepsilon_i := \ln(s_i) - (\boldsymbol{\alpha}^{full})^T \mathbf{x}_i$, which has to be estimated by smoothing techniques that are explained in the subsequent section.[2]

The FIC depends on focus parameter Λ via $\boldsymbol{\omega}_I$, which also simplifies to a scalar, as our focus parameter Λ is given by

$$\Lambda(\boldsymbol{\alpha}^{full}(\tau)) = -\alpha_{p_e}(\tau) = -\eta_{p_e}(s). \tag{7}$$

[2]It bears noting in this context that the intention of the expression $f(y;\theta,\gamma+\delta/\sqrt{(n)})$ is to use a large sample framework that gives squared bias and variance of the same asymptotic size. If one were to assume that e. g. $f(y;\theta,\gamma+\delta)$ is the true model, the bias would always dominate, as the sample-size grows, which would lead to always choosing the biggest model. In contrast, the local model of the FIC yields a fruitful approach for modeling the trade-off between bias and variance, which may be used to approximate the true MSE in practical situations outside this framework. For a thorough discussion of this framework, see the discussions in CLAESKENS and HJORT (2003) and the rejoinder to this article.

Hence, $\frac{\partial \Lambda}{\partial \alpha_\mu} = 0$ and

$$\frac{\partial \Lambda}{\partial \xi} = (0, -1, \mathbf{0})^T. \tag{8}$$

From the definition of $\boldsymbol{\omega}_I$ and derivative (8), it follows that $\boldsymbol{\omega}_I$ equals the negative of the second element of matrix $I_{10} I_{00}^{-1}$.

As becomes evident from the formula for $\boldsymbol{\omega}_I$, information matrix $\mathbf{I}$ is a key element for the calculation of the FIC in quantile regression analysis, whereas for specifications that are estimated by maximum-likelihood methods, the well-known Fisher information measure represents such a key element (BEHL et al., 2012). Information matrix $\mathbf{I}$ also plays an important role for the asymptotic covariance matrix $\mathbf{V}$ defined by

$$\mathbf{V} := \tau(1-\tau)\mathbf{I}^{-1}\mathbf{V}_{\mathbf{x}}\mathbf{I}^{-1}, \tag{9}$$

where $\mathbf{V}_{\mathbf{x}}$ is a covariance matrix that is based on the vector $\mathbf{x}_i^{full}$ of the explanatory variables of the full model:

$$\mathbf{V}_{\mathbf{x}} := \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \mathbf{x}_i (\mathbf{x}_i)^T.$$

For the FIC formula for the full model, we need the inverse of the asymptotic covariance matrix $\mathbf{V}$:

$$\mathbf{V}^{-1} = \frac{1}{\tau(1-\tau)} \mathbf{I}\mathbf{V}_{\mathbf{x}}{}^{-1}\mathbf{I} = \begin{pmatrix} J_{00} & J_{01} \\ J_{10} & J_{11} \end{pmatrix},$$

with the dimensions of the block matrices J_{00}, J_{01}, J_{10}, and J_{11} equaling those of matrices I_{00}, I_{01}, I_{10}, and I_{11}, respectively. On this basis, the FIC formula for the full model reads:

$$\mathrm{FIC}^{full} = \boldsymbol{\omega}_J^T \mathbf{V} \boldsymbol{\omega}_J, \tag{10}$$

where $\mathbf{V}$ captures the relative estimation variability and $\boldsymbol{\omega}_J$ is defined similar to

$\boldsymbol{\omega}_J$:

$$\boldsymbol{\omega}_J := J_{10}J_{00}^{-1}\frac{\partial\Lambda}{\partial\boldsymbol{\xi}} - \frac{\partial\Lambda}{\partial\alpha_\mu}.$$

In the one-dimensional case $q = 1$ investigated here, due to $\frac{\partial\Lambda}{\partial\alpha_\mu} = 0$ and $\frac{\partial\Lambda}{\partial\boldsymbol{\xi}} = (0, -1, \mathbf{0})^T$, $\boldsymbol{\omega}_J$ degenerates to the negative of the second element of the vector $J_{10}J_{00}^{-1}$.

The FIC formula (10) reflects the fact that for the full model, there is no modeling bias by definition: $\mathbf{B} = \mathbf{0}$, whereas relative estimation variability $\mathbf{V}$ vanishes by definition for the null model and, hence, does not emerge from formula (5). Instead, modeling bias $\mathbf{B}$ becomes the pivotal factor in FIC formula (5) for the null model. In short, the FIC formulae for the null and full models reveal the trade-off between modeling bias and estimation variability.

3 Estimation Method

For obtaining estimates of FIC^0 and FIC^{full}, linear minimization problems have to be solved, as is typical for quantile regression methods (KOENKER, 2005). For instance, estimates of parameter vector $\boldsymbol{\alpha}^{full}(\tau)$ result from the following minimization problem:

$$\min_{\boldsymbol{\alpha}_{full}} \left(\sum_{r_i>0} \tau \cdot r_i + \sum_{r_i<0} (1-\tau) \cdot |r_i| \right), \tag{11}$$

where underpredictions $r_i := \ln(s_i) - Q_\tau(\ln(s_i)|p_{e_i}, \mu_i, \mathbf{z}_i) = \ln(s_i) - (\boldsymbol{\alpha}^{full})^T\mathbf{x}_i > 0$ are penalized by τ and overpredictions $r_i < 0$ by $1-\tau$. This is reasonable, as for large τ one would not expect low estimates $\hat{Q}_\tau$ and vice versa, so that these incidences have to be penalized accordingly.

Just as ordinary least squares methods fit a linear function to the dependent variable by minimizing the expected squared error, quantile regression methods fit a linear model by minimizing the expected absolute error, using the asymmetric loss function $\rho_\tau(r) := 1(r > 0) \cdot \tau \cdot r + 1(r \leq 0) \cdot (1 - \tau) \cdot |r|$, where the indicator function $1(r > 0)$ indicates positive residuals r and $1(r \leq 0)$ non-positive residuals. $\rho_\tau(r)$ is called 'check' function, as its graph looks like a check mark.

For $\tau = 0.5$, in particular, the parameter estimates result from the minimization of the sum of the absolute deviations of r_i. This special case of a median regression is perfectly in line with the well-known statistical result that it is the median that minimizes the sum of the absolute deviations of a variable, whereas it is the mean that minimizes the sum of squared residuals, being a special case of OLS estimation. It is also well known that the median is more robust to outliers than the mean. This property translates to both median and quantile regressions in general, which have the advantage that they are more robust to outliers than mean (OLS) regression methods.

Conditional on p_e, μ, and $\mathbf{x}$, the conditional quantile functions (CQFs) given by (3) and (4) depend on the distribution of the corresponding error terms ε_i via the inverse distribution function $F^{-1}_{\varepsilon_i}(\tau)$. In the special case of homoscedasticity, that is, if the error terms ε_i were to be independent and identically distributed (iid) and, hence, the density of the errors and their inverse distribution function would not vary across observations ($f_i(\varepsilon_i) = f(\varepsilon_i)$ and likewise $F^{-1}_{\varepsilon_i}(\tau) = F^{-1}_{\varepsilon}(\tau)$), the CQFs would exhibit common slopes, differing only in the intercepts $\alpha(\tau)$. In this case, there is no need for quantile regression methods if the focus is on marginal effects and elasticities, such as $\eta_{p_e}(s)$, as these are given by the invariant slope parameters, e. g. $\alpha_{p_e}(\tau) = \alpha_{p_e}$. In general, however, the CQFs Q_τ will differ at different values τ in more than just the intercept and may well be even non-linear in $\mathbf{x}$.

It also bears noting that in the special case of homoscedasticity, the asymptotic covariance matrix $\mathbf{V}$ would collapse to

$$\mathbf{V} = \frac{\tau(1-\tau)}{f^2(F^{-1}(\tau))} {\mathbf{V}_\mathbf{x}}^{-1}. \tag{12}$$

This strongly resembles the covariance matrix of an ordinary least squares estimator given by $\sigma^2 \mathbf{V_x}^{-1}$. Note that in formula (12), the term $\tau(1-\tau)$ reflects the asymptotic variance of the check function ρ_τ. This term takes its maximum for $\tau = 0.5$, but gets small for percentiles close to 0 and 1. In this case, the term $\tau(1-\tau)$ may be dominated by the factor $f^2(F^{-1}(\tau))$, leading to less precise parameter estimates, whereas the variance of the parameter estimates gets smaller for quantiles close to the median.

An important step in obtaining estimates of FIC^0 and FIC^{full} is to find suitable estimators for the matrix $\mathbf{I}$. To this end, smoothing techniques can be applied. Behl, Claeskens, and Dette (2014), as well as Kim and White (2003), propose to use the estimator

$$\widehat{\mathbf{I}} = \frac{1}{2\hat{c}_n n} \sum_{i=1}^{n} \mathbf{1}_{\{-\hat{c}_n \leq \hat{\varepsilon}_i \leq \hat{c}_n\}} \mathbf{x}_i \mathbf{x}_i^T, \tag{13}$$

where $\hat{c}_n$ denotes a bandwidth that has to be determined by data-driven procedures, such as Cross Validation, and n denotes sample size.

4 Empirical Illustration

The data used in this illustrating example is drawn from regular surveys on the mobility behavior of German households (MOP, 2016). Households that participate in a survey are requested to fill out a questionnaire eliciting general household information, such as household income and the number of employed household members, person-related characteristics, and relevant aspects of everyday travel behavior. In addition, for a period of six weeks in the spring, households are requested to record detailed travel information for every car in the household, such as the price paid for fuel with each visit to a gas station, the liters of fuel consumed, and the kilometers driven. (For more details on the database, see Frondel, Ritter, and Vance, 2012.)

We use this travel survey information to derive both the regressors and the dependent variable s, which is given by the total monthly distance driven in kilometers. On the basis of survey information that covers thirteen years, spanning 1997 through 2009, a period during which real fuel prices rose 1.97% per annum on average, we first estimate the rebound effect via focus parameter $\Lambda(\tau) = -\eta_{p_e}(s) = -\alpha_{p_e}$ using quantile regression methods, thereby obtaining estimates of the rebound that depend on the percentile τ (Table 1). The FIC uniformly recommends employing the full model (4) for the estimation of the direct rebound on the basis of focus parameter $\Lambda(\tau) = -\eta_{p_e}(s)$. These results illustrate that a more advanced model than that used by FRONDEL, RITTER and VANCE (2012) should be applied.

Table 1: **Quantile Regression Estimates on the Rebound Effect given by Focus Parameter $\Lambda(\tau) = -\eta_{p_e}(s)$ resulting from the null model (3) and the full model (4).**

	Null Model (3)		Full Model (4)	
τ	$\widehat{\Lambda}^0(\tau)$	FIC	$\widehat{\Lambda}^{full}(\tau)$	FIC
0.1	0.898 (0.114)	55.03	0.869 (0.114)	**53.47**
0.3	0.714 (0.076)	25.25	0.686 (0.076)	**23.41**
0.7	0.551 (0.068)	20.00	0.493 (0.068)	**18.87**
0.9	0.561 (0.080)	26.46	0.551 (0.080)	**26.09**
Number of obs.:	4,097		4,097	

Note: Standard errors are in parentheses.

In line with FRONDEL, RITTER, and VANCE (2012), who estimated the rebound effect on the basis of the null model, we also find for the full model substantially smaller rebound effects for households with a high travel intensity. This outcome is in accord with intuition: To the extent that those who drive more are more dependent on car travel, we would expect them to exhibit less responsiveness to changes in fuel prices than those who drive less. Yet another source of heterogeneity in the rebound estimates is the kind of model specification: although the discrepancies across the null and the full model are not statistically significant, the magnitudes of the rebound estimates differ to some degree, indicating that model selection is of relevance in our example.

In this respect, we now employ a subsample of the MOP survey data for the years 1998 and 1999 and alternatively estimate the rebound effect on the basis of either $\Lambda(\tau) = -\eta_{p_e}(s)$ or $\Lambda(\tau) = \eta_\mu(s)$ and three models, model (3), model (4) and model (14), with model (14) being an alternative specification to the null model (3):

$$Q_\tau(\ln(s_i)|\mu_i, \mathbf{z}_i) = \gamma(\tau) + \gamma_\mu(\tau)\ln(\mu_i) + \mathbf{z}_i^T \boldsymbol{\gamma}_{\mathbf{z}}(\tau). \tag{14}$$

Apparently, as model (14) does not include the fuel price variable p_e, the focus parameter $\Lambda(\tau) = -\eta_{p_e}(s)$ cannot be estimated therefrom. Likewise, the focus parameter $\Lambda(\tau) = \eta_\mu(s)$ cannot be estimated from null model (3), as it does not include the efficiency variable μ.

While for focus parameter $\Lambda(\tau) = -\eta_{p_e}(s)$ the recommendations of the FIC are the same for all percentiles (Table 2), they differ for focus parameter $\Lambda(\tau) = \eta_\mu(s)$. For $\tau = 0.9$, for instance, the FIC prefers the alternative null model (14), whereas for $\tau = 0.1$, as well as for $\tau = 0.3$, it recommends selecting the full model (4). Most importantly, there are also divergent recommendations across focus parameters: While for low percentiles the FIC unanimously selects the full model (4), for higher percentiles the FIC recommends choosing model (14) for focus parameter $\Lambda(\tau) = \eta_\mu(s)$.

5 Summary and Conclusion

The well-known rebound effect captures the behaviorally induced offset in the reduction of energy consumption following efficiency improvements. To investigate the heterogeneity of the direct rebound effect in mobility demand across different percentiles of the distribution of distance traveled, we have used quantile regression methods and the Focused Information Criterion (FIC) introduced by BEHL, CLAESKENS and DETTE (2014) for quantile regression analysis. Our aim was to choose between competing model specifications, for instance, specifications in which the likely endogenous variable energy efficiency μ is either omitted or included.

Table 2: Quantile Regression Estimates of Rebound Effect given by either the Focus Parameter $\Lambda(\tau) = -\eta_{p_e}(s)$ or $\Lambda(\tau) = \eta_{\mu}(s)$.

	$\Lambda(\tau) = -\eta_{p_e}(s)$		$\Lambda(\tau) = \eta_{\mu}(s)$	
	$\widehat{\Lambda}(\tau)$	FIC	$\widehat{\Lambda}(\tau)$	FIC
$\boldsymbol{\tau} = \mathbf{0.1}$:				
Model (3)	1.381	91.41	–	–
Model (4)	0.708	**55.91**	0.443	**61.23**
Model (14)	–	–	0.418	75.35
$\boldsymbol{\tau} = \mathbf{0.3}$:				
Model (3)	0.853	79.02	–	–
Model (4)	0.566	**31.89**	0.368	**54.91**
Model (14)	–	–	0.397	61.37
$\boldsymbol{\tau} = \mathbf{0.7}$:				
Model (3)	0.645	58.91	–	–
Model (4)	0.556	**26.34**	0.405	53.49
Model (14)	–	–	0.456	**21.26**
$\boldsymbol{\tau} = \mathbf{0.9}$:				
Model (3)	0.933	48.82	–	–
Model (4)	0.819	**36.46**	0.293	64.91
Model (14)	–	–	0.331	**28.11**

Note: These results are based on MOP survey information for the years 1997 and 1998 and a total of $n = 344$ observations.

The FIC is conceived for targeted model searches, whereas conventional model selection criteria, such as the Akaike criterion (AIC), are mainly designed to find a model that is optimal in a general sense, regardless of a specific purpose of the data analysis. The empirical example presented in the previous section has illustrated that, first, the recommendations of the FIC differ across percentiles and, second, they may differ across alternative focus parameters to identify the rebound effect. Given that the FIC is precisely designed for purpose-specific model selection, we follow this recommendation, arguing that whenever a model is to be chosen that is optimal for the estimation of a certain parameter of interest, such as the rebound effect here, the FIC is a good choice.

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