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This study analyzes the interdependence of monetary policy and the stock market in a structural VAR model. We argue that commonly used short- and long-run restrictions on the interaction of both variables might not hold and propose an estimator not requiring any of these restrictions on the interaction of monetary policy and the stock market. The proposed estimator combines a data driven and restriction based identification approach. In particular, the estimator allows the researcher to order and identify some shocks recursively, while other shocks can remain unrestricted and are identified based on independence and non-Gaussianity. We find that a positive stock market shock contemporaneously increases the nominal interest rate, while a contractionary monetary policy shock leads to lower stock returns on impact. Furthermore, we present evidence that monetary policy is non-neutral with respect to long-run real stock prices.

JEL Codes: C32, E52, E44

Keywords: SVAR, identification, non-Gaussian, recursive, stock market, monetary policy

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1 Introduction

Simultaneously identifying monetary policy and stock market shocks in a SVAR is an ongoing challenge for econometricians. Identifying both shocks requires to impose an a priori structure. Most of the literature covers one of two extreme cases; I) identifying all shocks based on restrictions concerning the short- or long-run interaction, or II) data driven approaches without restrictions, but based on heteroskedastic or non-Gaussian shocks. We argue that neither of the two extreme cases is suited for the application at hand. In particular, we show that commonly used short- and long-run restrictions on the interaction of monetary policy and the stock market are questionable. However, also purely data driven estimators do not yield conclusive insights into the interaction of both variables, since these estimators depend on latent, volatile, or hardly observable features which results in a poor small sample performance of the estimator.

The estimator proposed in this study combines the traditional identification approach based on restrictions with the more recent data driven approach based on non-Gaussianity. Our estimator allows the researcher to rely on recursiveness restrictions if possible and to be agnostic on the interaction of the variables and rely on data driven estimates when necessary. The estimator is applied to analyze the interaction of monetary policy and the stock market. We find evidence against commonly used short- and long-run restrictions and demonstrate that a purely data driven estimator leads to imprecise estimates, which barely allow any conclusions on the interaction of monetary policy and the stock market.

In the literature, the interaction of monetary policy and the stock market has been estimated based on short-run restrictions (see e.g. Laopodis (2013)), and based on long-run restrictions (see Bjørnland and Leitemo (2009) or Kontonikas and Zekaite (2018)). The estimation based on short-run restriction in Laopodis (2013) yields evidence that real stock prices are persistently negative after a tightening of monetary policy, which is at odds with the long-run restrictions used in Bjørnland and Leitemo (2009). However, the estimation based on long-run restrictions by Bjørnland and Leitemo (2009) suggests that any zero restriction on the interaction of monetary policy and the stock market is incorrect and is thus at odds with the short-run restrictions used in Laopodis (2013). Therefore, the results from the restriction based approaches contradict each other. We argue that neither the short- nor the long-run restrictions are plausible. In particular, stock market shocks can contain news about future business cycle fluctuations (see e.g. Beaudry and Portier (2006)) and assuming that the central bank does not react simultaneously to these shocks is debatable. Moreover, recent studies (see for instance Moran and Queralto (2018), Bianchi et al. (2019) and Jordà et al. (2020)) find evidence against the long-run neutrality of monetary policy, which casts doubt on long-run restrictions used to identify monetary policy shocks.

Due to the unavailability of short- and long-run restrictions, several authors used data driven approaches to estimate the interaction of monetary policy and the stock market (see e.g. Lanne et al. (2017) or Lütkepohl and Netšunajev (2017)). These approaches do not require any restrictions on the interaction of the variables, but instead exploit a structure imposed on the statistical properties of the shocks. Lütkepohl and Netšunajev (2017) estimate the interaction of monetary policy and the stock market based on time-varying volatility and find a negative impact of a tightening of monetary policy on stock prices. However, the authors are unable to clearly label a stock market shock. Moreover, a tightening of monetary policy appears to have an unexpected initial positive impact on output and inflation and therefore even the labeling of the monetary policy shock is debatable. Lanne et al. (2017) estimate a SVAR based on non-Gaussianity and find that a tightening of monetary policy has an immediate negative impact on financial conditions. However, they are also unable to label any other shock and in particularly cannot label a stock market shock.

We argue that neither the traditional restriction based approaches nor the more recent purely data driven approaches yield conclusive insight into the interaction of monetary policy and the stock market. The restriction based approaches fail due to the unavailability of sufficiently many short- or long-run restrictions and the data driven approaches fail, since they impose such little structure that finite sample estimates become highly volatile, up to the point that it becomes difficult to even label the shocks.

The key to gain insight into the interaction of monetary policy and the stock market is a combination of the traditional restriction based and the more recent data driven approach. The estimator proposed in this study allows to divide the variables of the SVAR into a first block of recursively ordered variables and a second block of non-recursive variables. Only the non-recursive block relies on data driven estimates based on non-Gaussian and independent shocks. The more recursiveness restrictions the researcher applies, the less the estimator depends on moments beyond the variance. In a Monte Carlo simulation we show how the performance of a purely data driven estimator based on non-Gaussianity deteriorates with a decreasing sample size and an increasing model size. However, the simulation also shows that exploiting the partly recursive order can stop the performance decline. Therefore, the estimator proposed in this study allows the researcher to rely on an arbitrary number of recursiveness restrictions, which reduces the dependence of the estimator on moments beyond the variance and thereby increases the finite sample performance of the estimator.

In our application the variables output, investment and inflation are assumed to be rigid and are restricted such that they cannot respond to stock market and monetary policy shocks within the same quarter. However, interest rates and stock returns remain unrestricted and can simultaneously respond to all shocks. We apply the proposed partly recursive estimator and find a simultaneous contractionary response of the Federal Funds Rate to positive stock market shocks and an immediate negative stock return response to contractionary monetary policy shocks. Moreover, we present evidence that monetary policy has a long-run effect on stock prices. Additionally, we estimate an unrestricted SVAR solely based on independent and non-Gaussian shocks. Overall, the unrestricted estimation confirms the results of our partly recursive estimation, however, the confidence intervals are larger and it becomes increasingly difficult to explain the estimated interaction of stock prices and interest rates. The application illustrates that the partly recursive, partly non-Gaussian identification scheme introduced in the present study serves as a helpful addition to the econometrician's tool box when faced with situations, where only a few restrictions on the interaction of the variables are available.

The remainder of this article is structured as follows. Section 2 shows that commonly used identification schemes in the related literature come with caveats that render them not applicable to analyze the interaction of monetary policy and the stock market. Section 3 derives our estimator for partly recursive, partly non-Gaussian SVAR models and contains a Monte-Carlo study illustrating how exploiting the partly recursive order increases the finite sample performance of the estimator. In Section 4 we use the proposed partly recursive, partly non-Gaussian SVAR estimator to analyze the interaction of the stock market and monetary policy. Section 5 concludes.

2 Monetary policy and the stock market

2.1 The unavailability of common identifying restrictions

In this section, we use a simple asset pricing model to illustrate that there is no indisputable answer about the short- and long-run interaction of stock markets and monetary policy. We keep the model intentionally simple to show that only a small deviation in basic assumptions can cast common short- or long-run restrictions inappropriate.

Consider that households can save by buying firm stocks of firm i at price $v_{i,t}$, yielding dividend $d_{i,t+1}$ in the next period or by a non-contingent bond b_t^f yielding a guaranteed real interest at rate r_t. The no-arbitrage condition then is

$$1 + r_t = E_t \frac{v_{i,t+1} + d_{i,t+1}}{v_{i,t}}. \tag{1}$$

From this, one can acquire the central asset pricing equation of the form

$$v_{i,t} = E_t \sum_{s=1}^{\infty} \frac{d_{i,t+s}}{\prod_{j=1}^{s}(1 + r_{t+j-1})}, \tag{2}$$

so the current stock price is the expected discounted sum of future dividends. On the firm side assume a continuum of infinitely small firms with mass 1 and dividends of firm i are given by

$$d_{i,t+s} = y_{i,t+s} - j_{i,t+s} + b_{i,t+1+s}^f - (1 + r_{t+s-1})b_{i,t+s}^f - \bar{w}\bar{n}, \tag{3}$$

where $y_{i,t}$ is output, $j_{i,t}$ investment in the physical capital stock, $b_{i,t}^f$ are debt sales (where $\int_0^1 b_{i,t}^f di = b_t^f$), $\bar{w}$ the constant real wage and $\bar{n}$ labor input, also assumed constant for simplicity. We assume further an accumulation of physical capital $k_{i,t}$ of the form

$$k_{i,t+1} = (1 - \delta)k_{i,t} + j_{i,t}, \quad \delta \in (0, 1). \tag{4}$$

The production function reads

$$y_{i,t} = Ak_{i,t}^{\alpha}(Z_t\bar{n})^{1-\alpha}, \quad \alpha \in (0, 1), \tag{5}$$

with A an exogenous scaling factor and Z_t an aggregate productivity factor exogenous to the individual firm. Consequently, the firm maximization problem reads

$$\max_{\left\{k_{i,t+1+s}, b^f_{i,t+s}\right\}} \sum_{s=0}^{\infty} E_t \Lambda_{t+s} d_{i,t+s}, \tag{6}$$

with Λ_t the firm's discount factor and subject to (4)-(5). The optimality conditions yield the common interest rate parity condition of the form

$$E_t A \alpha k_{i,t+1}^{\alpha-1} (Z_{t+1}\bar{n})^{1-\alpha} + (1-\delta) = 1 + r_t, \tag{7}$$

which says that in equilibrium the interest rate on foreign capital and the return on capital investment will coincide. Now inserting (3)-(5) into (2) yields

$$v_{i,t} = E_t \sum_{s=1}^{\infty} \frac{A k_{i,t+s}^{\alpha} (Z_{t+s}\bar{n})^{1-\alpha} - k_{i,t+s+1} + (1-\delta) k_{i,t+s} + b^f_{i,t+1+s} - (1 + r_{t+s-1}) b^f_{i,t+s} - \bar{w}\bar{n}}{\prod_{j=1}^{s} (1 + r_{t+j-1})}. \tag{8}$$

Imposing the limiting condition $\lim_{T\to\infty} b_T = 0$ then leads to future debt sales dropping out from the asset pricing equation, as dividends cannot be debt-financed indefinitely. As becomes evident, the dynamics of the numerator are then entirely driven by the evolution of capital. Using equation (7) then allows to find the evolution of capital as

$$k_{i,t+1} = E_t \left[\left(\frac{\alpha A}{r_t + \delta} \right)^{\frac{1}{1-\alpha}} \bar{n} Z_{t+1} \right]. \tag{9}$$

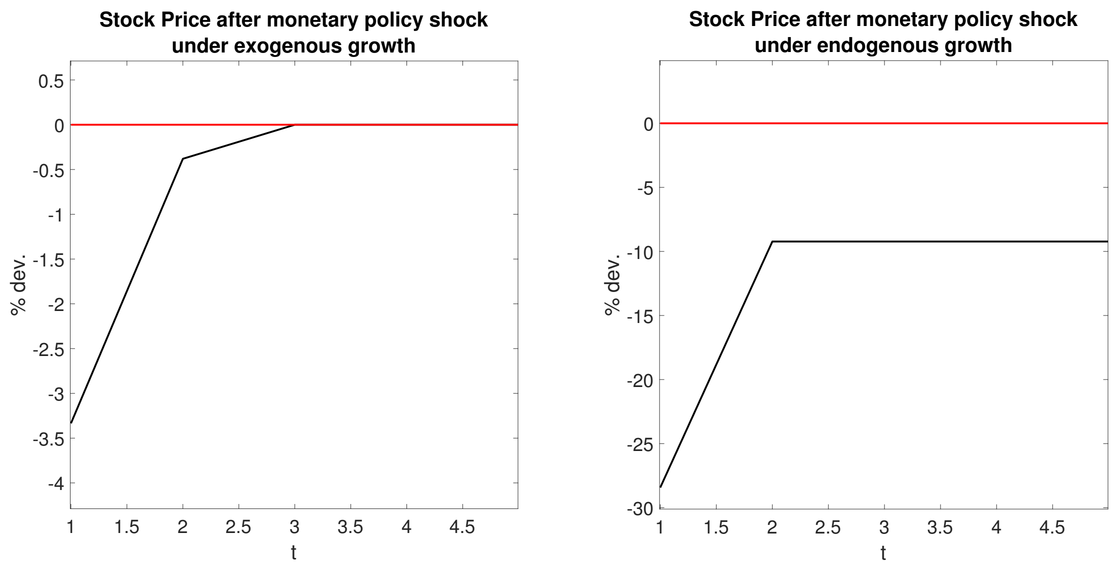
Now consider that the real interest rate increases once such that $r'_t > r^*_t$ and for the rest of the time $r'_{t+s} = r^*_{t+s}, \forall s > 0$ (primes denote variables after the shock, asterisks variables without the shock). The resulting response of dividends and stock prices now crucially depends on what we assume about the productivity factor Z_t:

1.) Exogenous growth: Assume a neoclassical growth model with decreasing marginal returns to capital, so Z_t is some exogenously growing variable.

2.) Endogenous growth: Assume an endogenous growth model, for instance a standard learning-by-doing technology with $Z_t = \int_0^1 k_{i,t-1} di = K_{t-1}$.

Figure 1 shows the effect of an interest rate shock on stock prices for an exogenous and endogenous growth model[1]. Assuming sticky prices, thus nominal and real variables move in the same

Figure 1: Simulated response of real stock prices to a one-time exogenous interest rate increase of about one percentage point as implied by the exogenous and endogenous growth model.



direction in the short-run, we can interpret the exogenous real interest rate increase as equivalent to a monetary policy shock. Both models imply an immediate reaction of stock prices to the monetary policy shock. However, in the exogenous growth model with decreasing returns to capital, stock prices revert back to their long-run level, while under endogenous growth with the learning-by-doing technology, the decrease in stock prices is permanent. This is because in the first case the lower capital stock implies a higher marginal return of capital in the future, which drives back capital to its old steady state, while in the second case it does not, because the lower aggregate capital stock implies lower capital investment return for the individual firm.

Furthermore, interpret a stock price shock as news about higher future productivity that is not realized today like in Beaudry and Portier (2006). For instance assume A is no longer a constant,

[1]For simplicity we assume $\bar{n} = 1$, the initial debt $b_t^f = 0$, $\bar{w} = 0$ and use a standard calibration of $\alpha = \frac{1}{3}$, $\delta = 0.1$ and setting $A = 0.46$ to ensure a long-run output growth rate of about 3%.

but time dependent. Assume now that in the next period $A'_{t+1} > A^*_{t+1}$. From equation (8) it becomes evident that an increase in future dividends leads to an increase in stock prices now. Because of $A'_{t+1} > A^*_{t+1}$, we also know that $y'_{t+1} > y^*_{t+1}$. A central bank aiming at flattening business cycle fluctuations would immediately adjust its policy rate. Consequently, stock prices will contemporaneously react to monetary policy shocks, as will monetary policy to stock market shocks.

Now the econometrician's task would be to let the data decide which of the two theoretical approaches is correct. Of course, we need to make some assumptions to identify the structural shocks. However, we know that a monetary policy shock will immediately influence stock prices and vice versa, so we cannot impose a short-run restriction. Imposing a long-run restriction means to ex ante decide that the model with decreasing returns is the right one and not the endogenous growth model and strips us of the ability to let the data decide. Thus we are in need of a data driven identification approach, which is the objective of the present paper.

2.2 Monetary policy and the stock market SVAR models

As a second step we review the approaches of the related literature to estimate the interaction of monetary policy and the stock market in a SVAR. We show that there is a lack of a compelling estimation approach that is both feasible, but not too restrictive for the problem at hand.

In a SVAR a vector of time series is explained by its past values and a linear combination of structural shocks

$$y_t = A_1 y_{t-1} + ... + A_p y_{t-p} + u_t, \tag{10}$$

$$u_t = B\varepsilon_t, \tag{11}$$

with an n-dimensional vector of macroeconomic variables y_t, parameter matrices $A_1, ..., A_p$, a non-singular matrix B, the n-dimensional vector of structural shocks ε_t and the n-dimensional vector of reduced form shocks u_t. Here, the vector of structural shocks will contain a monetary policy and a stock market shock. The goal is to identify both shocks and estimate their impact on the macroeconomic variables. The VAR imposes only little a priori structure, however, without further assumptions the structural shocks are not identified.

In general, the probably most frequently used identifying assumption for a SVAR is a recursive ordering, meaning zero restrictions on the impact of some shocks, such that each variable is simultaneously only influenced by shocks ordered in rows below the variable. However, in the case of monetary policy and the stock market zero restrictions on the interaction of both variables are hardly credible. In particular, stock prices can contain news about future productivity, see Beaudry and Portier (2006). Therefore, a positive stock price shock might indicate a future boom accompanied by inflationary pressure and a stabilizing central bank would respond immediately. Consequently, a zero restriction on the response of monetary policy to stock market shocks is difficult to defend. Nevertheless, zero restrictions on the interaction of monetary policy and the stock market have been used to estimate the interaction of both variables, see e.g. Laopodis (2013). However, these estimates only reflect the interaction of monetary policy and the stock market if the identifying assumption is correct, which is at best questionable.

Due to the unavailability of credible short-run restrictions on the interaction of monetary policy and the stock market, several authors identify the shocks based on restrictions of the long-run interaction of both variables (see e.g. Bjørnland and Leitemo (2009) or Kontonikas and Zekaite (2018)). In particular, the authors assume long-run neutrality of monetary policy, meaning the monetary policy shock by construction has no long-run impact on real stock prices. Bjørnland and Leitemo (2009) find that monetary policy and the stock market interact simultaneously. In particular, a tightening of monetary policy leads to an immediate decrease of stock prices and a positive stock market shock leads to an immediate tightening of monetary policy. Again, these results only reflect the true interaction of both variables if the identifying long-run restriction is correct. In contrast to the short-run restriction used in Laopodis (2013), the long-run restriction used in Bjørnland and Leitemo (2009) is at least based on an underlying theory yielding long-run neutrality of monetary policy. However, as shown in Section 2.1, a slight modification of the theory from exogenous to endogenous growth already breaks the long-run neutrality result. In fact, recent studies (see e.g. Moran and Queralto (2018), Bianchi et al. (2019) and Jordà et al. (2020)) consistently find that monetary policy affects real variables much longer than usually assumed[2]. These results cast doubt on the long-run restriction and the corresponding estimated interaction of monetary policy and the stock market.

Rigobon and Sack (2004) propose an estimator which does not require any restrictions on the short- or long-run interaction of the stock market and monetary policy. Instead, it is based on heteroskedastic shocks and requires to a priori specify periods of different variances of the monetary policy shocks. The identification is thus based on a stochastic property of the structural shocks and not on a restriction on the impact of the shocks. Specifying volatility regimes of monetary policy may be straight-forward on a daily basis (by choosing all days with FOMC announcements), however, with lower frequency data it becomes increasingly difficult. Therefore, the estimator becomes infeasible in a typical macroeconomic application with monthly, quarterly or even lower frequency data.

In general, identification based on time-varying volatility does not require to a priori specify volatility periods (see e.g. Rigobon and Sack (2003), Lanne et al. (2010), Lütkepohl and Netšunajev (2017) or Lewis (2019)). In fact, a latent volatility process can be used for identification without imposing much structure on the latent process. However, Lütkepohl and Netšunajev (2017) argue that reliable estimators based on GARCH or Markov switching processes are only available in small models and few volatility states. The intuition is simple: The more (correct) structure we impose on the latent process, the more precise the corresponding estimate. Therefore, Lütkepohl and Netšunajev (2017) propose an estimator which imposes a parametric smooth transition function between two states of the variance-covariance matrix of the reduced form shocks. The estimator is applied to analyze the interaction of monetary policy and the stock market. The authors find a small simultaneous negative response of the stock market to a tightening of monetary policy. However, a tightening of monetary policy is also found to lead to an initial increase of inflation and output. Due to the counterintuitive response of output and inflation to the shock, the authors admit that labeling the shock as a monetary policy shock in a "conventional" sense may be misleading. Additionally, the authors cannot label a stock market shock and hence it remains unclear how monetary policy reacts to a stock market shock.

[2]For instance, Moran and Queralto (2018) and Bianchi et al. (2019) find that the impulse response of TFP is significantly positive even 15 years after a negative monetary policy shock has hit the economy. Again as in the previous section, higher productivity goes hand in hand with higher expected dividends. Therefore, stock prices should not only decrease immediately, but permanently in response to an unexpected tightening of monetary policy.

Another branch of the SVAR literature uses non-Gaussian and independent shocks for identification (see e.g. Lanne et al. (2017), Gouriéroux et al. (2017), Lanne and Luoto (2019) and Keweloh (2020)). Theses approaches are also data driven and do not require to impose any short- or long-run restrictions. Instead, these approaches require that the structural shocks are mutually independent and at most one shock is allowed to be Gaussian. Intuitively, non-Gaussian shocks do contain information in moments beyond the variance which allows to identify the simultaneous interaction. In a short application Lanne et al. (2017) use a data driven identification approach imposing non-Gaussian and independent shocks to estimate the interdependence of monetary policy and the stock market. The authors find that an unexpected tightening of monetary policy has an immediate negative impact on financial conditions. However, they are unable to label a stock market shock. Therefore, it again remains unclear how stock market shocks influence monetary policy.

To sum up, the commonly used short- and long-run restriction regarding the interaction of monetary policy and the stock market have implications on the underlying data generating process. Until now there is no consensus on which theoretical model is correct and the estimation should not depend on an a priori restriction to one or another model, but rather be able to decide which fits the data best. On the other side, there are identification approaches that do not rely on short- or long-run restrictions, but they are either not able to be generalized to a broader macroeconomic setup or become less feasible the more variables are included into the VAR. Ideally, the SVAR estimator should allow to factor in a priori restrictions that we are certain about, but also allow a data driven identification, when we are not certain about the underlying theory. In the following section we propose an estimator that fulfills these criteria.

3 A partly recursive, partly non-Gaussian SVAR estimator

A non-Gaussian SVAR with independent shocks can be estimated based on restrictions governing the interaction of the variables or based on information contained in moments beyond the variance and without any assumptions on the interaction of the variables. At first glance, in a non-Gaussian SVAR and from an asymptotic point of view, the traditional identification approach based on restrictions appears to be unnecessarily restrictive. However, we show that in a small sample the performance of a data driven estimator based on non-Gaussianity quickly deteriorates with an increasing model size, while the performance of a restriction based estimator is less affected by the model and sample size. In macroeconomic applications, we can oftentimes derive some credible restrictions based on economic theory. However, in many cases we cannot derive sufficiently many restrictions to identify the SVAR based on second moments and the researcher is forced to rely on additional less credible restrictions or to use an unreliable data driven estimator.

The estimator proposed in this section combines the traditional restriction based approach with the more recent data driven approach based on non-Gaussianity. Our estimator allows the researcher to rely on recursiveness restrictions if possible and to be agnostic on the interaction of the variables and relying on data driven estimates when necessary. In particular, the proposed estimator allows to order some, but not all, shocks recursively. While the impact of the recursive shocks is estimated based on second moments, the impact of the non-recursive shocks is estimated based on non-Gaussianity. We show that in comparison to an unrestricted estimator solely based on non-Gaussian and independent shocks, exploiting the partly recursive structure i) improves the finite sample performance of the estimator, ii) reduces the burden of labeling the shocks, and iii) relaxes the non-Gaussianity and independence assumptions.

3.1 Derivation of the estimator

Consider a partly recursive SVAR, meaning there exists $m \in \mathbb{N}$ with $0 \leq m \leq n$ and

$$B = \begin{bmatrix} b_{11} & 0 & & \dots & & 0 \\ \vdots & \ddots & \ddots & & & \vdots \\ b_{m1} & \dots & b_{mm} & 0 & \dots & 0 \\ b_{m+1,1} & \ddots & b_{m+1,m} & b_{m+1,m+1} & & b_{m+1,n} \\ \vdots & & & \vdots & & \vdots \\ b_{n1} & \dots & b_{nm} & b_{n,m+1} & & b_{nn} \end{bmatrix}. \tag{12}$$

Therefore, the first m variables are ordered recursively, meaning they cannot contemporaneously be influenced by structural shocks in rows ordered below. However, the last $n-m$ variables are not ordered recursively and can contemporaneously be influenced by all structural shocks.

Since the matrix B is only partly recursive, it cannot be identified solely by second moments. However, the partly recursive structure can be combined with estimators based on independent and non-Gaussian shocks.

The partly recursive SVAR can be estimated in three steps. For simplicity, consider a SVAR with four variables and the following partly recursive structure

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}. \tag{13}$$

The recursive part can be written as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}, \tag{14}$$

which is a simple recursive SVAR and can be identified and estimated based on second moments (e.g. by applying the Cholesky decomposition to the variance-covariance matrix of the reduced form shocks, see Kilian and Lütkepohl (2017)). The non-recursive part can be written as

$$\begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} + \begin{bmatrix} \nu_3 \\ \nu_4 \end{bmatrix}, \tag{15}$$

with

$$\begin{bmatrix} \nu_3 \\ \nu_4 \end{bmatrix} = \begin{bmatrix} b_{33} & b_{34} \\ b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}. \tag{16}$$

Using the estimated structural shocks $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ from the first step allows to estimate the lower-left block of B in equation (15) by OLS. The residuals ν in equation (15) represent the variation in u_3 and u_4 which is unexplained by the structural shocks in the recursive block and can be explained by the shocks in the non-recursive block with equation (16) which yields a non-recursive SVAR. The structural shocks of the non-recursive block are globally identified up to labeling if the shocks of the block are mutually independent and at most one shock is Gaussian. The non-recursive lower-right block of B can then be estimated by an estimator based on non-Gaussian and independent shocks, see e.g. Lanne et al. (2017), Gouriéroux et al. (2017), Lanne and Luoto (2019) or Keweloh (2020).

If the SVAR is only block recursive, such that there exists $m \in \mathbb{N}$ with $0 \leq m \leq n$ and

$$B = \begin{bmatrix} b_{11} & ... & b_{m1} & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots & & \vdots \\ b_{m1} & \dots & b_{mm} & 0 & \dots & 0 \\ b_{m+1,1} & \ddots & b_{m+1,m} & b_{m+1,m+1} & & b_{m+1,n} \\ \vdots & & & \vdots & & \vdots \\ b_{n1} & \dots & b_{nm} & b_{n,m+1} & & b_{nn} \end{bmatrix}, \quad (17)$$

the approach proposed above yields inconsistent estimates for the upper-left and lower-left block of B, but remains consistent for the lower-right block[3].

The partly recursive, partly non-Gaussian estimator can also be calculated in a single step. For example, a partly recursive version of the GMM estimator proposed in Keweloh (2020) can be obtained by including the second-order moment conditions of all shocks and the higher-order moment conditions associated with the shocks in the non-recursive block. Some estimators based on non-Gaussianity rely on an initial whitening step, see e.g. the PML estimator proposed in Gouriéroux et al. (2017) or the whitened GMM estimators proposed in Keweloh (2020). In the preliminary whitening step the reduced form shocks are transformed into uncorrelated shocks with unit variance and in the second step the optimization is performed over orthogonal matrices,

[3]Falsely imposing a recursive order in equation (14) yields inconsistent estimates of the upper-left block of B. Additionally, using the shocks of the first step, here $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$, to estimate equation (15) will also yield inconsistent estimates of the lower-left block of B. However, if the shocks in the non-recursive block, here ε_3 and ε_4, have no simultaneous impact on the variables in the first block, the shocks $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ obtained from the first step are equal to a linear combination of the true shocks ε_1 and ε_2. Therefore, the residuals ν in equation (15) still represent the variation in u_3 and u_4 which is unexplained by the structural shocks in the recursive block and hence, the non-recursive SVAR in equation (16) remains valid. The proposed estimator thus allows to identify and consistently estimate the impact of a non-recursive block of variables, as long as equation (17) holds, meaning that all the shocks in the second and non-recursive block have no simultaneous impact on the variables in the first block of variables.

which correspond to rotations of the transformed reduced form shocks[4]. Whitening is equivalent to an optimization subject to the constraint that the estimated structural shocks are uncorrelated with unit variance in the given sample, compare Keweloh (2020). However, in the partly recursive SVAR defined in equation (12), the first m columns of B are uniquely determined by the whitening constraint, imposing that the estimated structural shocks have to be uncorrelated with unit variance. Therefore, a whitened estimator with partly recursive constraints by definition only relies on second moments to identify and estimate the impact of the shocks in the recursive block, see Appendix A.1 for more details.

Exploiting the partly recursive structure yields several advantages compared to an unrestricted estimator solely identified by independence and non-Gaussianity. First, the Monte Carlo study in Section 3.2 shows that exploiting the partly recursive order and thus decreasing the dependence of the estimator on higher moments leads to an increase of the small sample performance of the estimator. Second, every identification approach requires to impose an a priori structure. In particular, if no restrictions on the interaction of the variables are imposed, the researcher has to impose that all shocks are independent and at most one shock is allowed to be Gaussian. Sometimes there is clear evidence in favor of non-Gaussianity, as for instance in the case of financial shocks, but sometimes there is not. For instance it is unclear if inflation shocks are Gaussian or not. By moving the inflation shock into the recursive block, we do not need to impose any non-Gaussianity assumptions on the inflation shock and instead can rely on the standard argument of rigid prices. Third, a data driven identification scheme based only on non-Gaussian and independent shocks only identifies the shocks up to labeling. Therefore, the researcher has to decide which impulse response belongs to which shock. The task of labeling the shocks becomes increasingly difficult the more shocks are identified by this procedure, especially if the impulse responses of the variables are quite similar with respect to two or more shocks. Imposing a partly recursive structure alleviates this burden on the econometrician, since the shocks in the recursive block are already labeled by the identifying assumptions of the partly recursive order.

In summary, we propose an estimator for partly recursive, partly non-Gaussian SVAR models. Exploiting the partly recursive structure allows to relax the independence and non-Gaussianity

[4]Optimizing over orthogonal matrices is computationally simple, since it can be pulled back to an optimization problem over the euclidean space, see Lezcano-Casado and Martínez-Rubio (2019). In Appendix A.1 we propose a similar transformation for the optimization problem over orthogonal matrices with partly recursive constraints.

assumptions, it decreases the dependence on higher moments and it simplifies the task of labeling the estimated shocks.

3.2 Finite sample performance

In the following Monte Carlo study, we show that data driven estimators based on non-Gaussianity suffer from a curse of dimensionality, i.e. the bias and variance increases quickly with an increasing model size and a decreasing sample size. However, we show that exploiting the partly recursive structure can stop the curse of dimensionality.

We simulate a partly recursive SVAR with $n = 4$ and $n = 2$ variables. The structural shocks are drawn from a t-distribution with seven degrees of freedom[5] and the mixing matrices B are given by

$$B = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0.5 & 0.5 & 1 & 0 \\ 0.5 & 0.5 & 0.5 & 1 \end{bmatrix}. \tag{18}$$

The Monte Carlo study analyzes the impact of imposing a partly recursive order on the PML estimator proposed by Gouriéroux et al. (2017), where the shocks have been correctly specified as t-distributed shocks with seven degrees of freedom. In the small SVAR with $n = 2$ we impose no recursive order. In the large SVAR with $n = 4$ one estimator is calculated without imposing a recursive order and a second estimator is estimated which uses the restriction that the first two shocks are ordered recursively. In an online Appendix we provide additional simulations with the GMM estimators proposed in Keweloh (2020), different shock distributions, different mixing matrices B (in particular a block-recursive mixing similar to equation (17)), different restrictions, and larger models. However, none of the conclusions drawn in this section is sensitive to the alternative simulations.

Table 1 shows the mean and standard deviation of each estimated element of B. The simulation shows how the performance of estimates based entirely on non-Gaussianity decreases with an

[5] The shocks have been normalized to unit variance by multiplying each shock with $1/\sqrt{(v/(v-2))}$ and $v = 7$.

Table 1: Finite sample performance

	$n=2$ PML	$n=4$ PML	$n=4$ partly recursive PML
$T=150$	$\begin{bmatrix} \underset{(1.37)}{0.97} & \underset{(6.7)}{0.0} \\ \underset{(7.26)}{0.48} & \underset{(3.0)}{0.97} \end{bmatrix}$	$\begin{bmatrix} \underset{(2.08)}{0.92} & \underset{(6.81)}{0.01} & \underset{(6.79)}{0.01} & \underset{(7.29)}{-0.0} \\ \underset{(7.55)}{0.46} & \underset{(3.76)}{0.92} & \underset{(8.09)}{0.01} & \underset{(8.95)}{-0.0} \\ \underset{(9.12)}{0.46} & \underset{(9.66)}{0.47} & \underset{(5.48)}{0.92} & \underset{(10.1)}{-0.0} \\ \underset{(11.17)}{0.46} & \underset{(12.02)}{0.47} & \underset{(12.1)}{0.46} & \underset{(8.57)}{0.9} \end{bmatrix}$	$\begin{bmatrix} \underset{(0.94)}{1.0} & \underset{(0.0)}{0.0} & \underset{(0.0)}{0.0} & \underset{(0.0)}{0.0} \\ \underset{(1.28)}{0.5} & \underset{(0.95)}{0.99} & \underset{(0.0)}{0.0} & \underset{(0.0)}{0.0} \\ \underset{(1.56)}{0.5} & \underset{(1.23)}{0.5} & \underset{(1.35)}{0.96} & \underset{(6.96)}{-0.01} \\ \underset{(1.78)}{0.5} & \underset{(1.48)}{0.5} & \underset{(7.36)}{0.49} & \underset{(3.15)}{0.96} \end{bmatrix}$
$T=500$	$\begin{bmatrix} \underset{(1.1)}{0.99} & \underset{(5.87)}{0.0} \\ \underset{(6.22)}{0.5} & \underset{(2.46)}{0.99} \end{bmatrix}$	$\begin{bmatrix} \underset{(1.47)}{0.98} & \underset{(6.32)}{0.0} & \underset{(6.4)}{-0.0} & \underset{(6.32)}{0.0} \\ \underset{(6.65)}{0.49} & \underset{(3.12)}{0.98} & \underset{(7.83)}{-0.0} & \underset{(8.01)}{0.0} \\ \underset{(8.32)}{0.49} & \underset{(8.12)}{0.49} & \underset{(4.56)}{0.98} & \underset{(8.91)}{-0.0} \\ \underset{(9.58)}{0.49} & \underset{(10.16)}{0.49} & \underset{(9.43)}{0.49} & \underset{(6.21)}{0.98} \end{bmatrix}$	$\begin{bmatrix} \underset{(0.97)}{1.0} & \underset{(0.0)}{0.0} & \underset{(0.0)}{0.0} & \underset{(0.0)}{0.0} \\ \underset{(1.23)}{0.5} & \underset{(1.01)}{1.0} & \underset{(0.0)}{0.0} & \underset{(0.0)}{0.0} \\ \underset{(1.46)}{0.5} & \underset{(1.27)}{0.5} & \underset{(1.12)}{0.99} & \underset{(5.68)}{-0.0} \\ \underset{(1.72)}{0.5} & \underset{(1.47)}{0.5} & \underset{(5.93)}{0.5} & \underset{(2.61)}{0.99} \end{bmatrix}$
$T=5000$	$\begin{bmatrix} \underset{(1.0)}{1.0} & \underset{(4.57)}{0.0} \\ \underset{(4.8)}{0.5} & \underset{(2.17)}{1.0} \end{bmatrix}$	$\begin{bmatrix} \underset{(0.98)}{1.0} & \underset{(4.68)}{0.0} & \underset{(4.55)}{0.0} & \underset{(4.41)}{0.0} \\ \underset{(4.98)}{0.5} & \underset{(2.18)}{1.0} & \underset{(5.67)}{0.0} & \underset{(5.65)}{0.0} \\ \underset{(5.88)}{0.5} & \underset{(5.95)}{0.5} & \underset{(3.34)}{1.0} & \underset{(6.79)}{0.0} \\ \underset{(6.89)}{0.5} & \underset{(6.9)}{0.5} & \underset{(6.95)}{0.5} & \underset{(4.53)}{1.0} \end{bmatrix}$	$\begin{bmatrix} \underset{(0.96)}{1.0} & \underset{(0.0)}{0.0} & \underset{(0.0)}{0.0} & \underset{(0.0)}{0.0} \\ \underset{(1.24)}{0.5} & \underset{(0.98)}{1.0} & \underset{(0.0)}{0.0} & \underset{(0.0)}{0.0} \\ \underset{(1.49)}{0.5} & \underset{(1.21)}{0.5} & \underset{(1.01)}{1.0} & \underset{(4.47)}{0.0} \\ \underset{(1.68)}{0.5} & \underset{(1.5)}{0.5} & \underset{(4.66)}{0.5} & \underset{(2.17)}{1.0} \end{bmatrix}$

Monte Carlo simulation with sample sizes 150, 500, and 5000 each with 5000 iterations. The simulated SVAR has $n = 2$ or $n = 4$ variables and the diagonal of the mixing matrix B is equal to 1, the lower-left triangular of B is equal to 0.5 and the upper-right triangular of B is equal to 0. The structural shocks are drawn from a t-distribution with $v = 7$ degrees of freedom and have been normalized to unit variance shocks by multiplying each shock with $1/\sqrt{(v/(v-2))}$. The SVAR is estimated by the PML estimator proposed by Gouriéroux et al. (2017), where the shocks have been correctly specified as t-distributed shocks with seven degrees of freedom. The last column shows the PML estimator with the restriction that the first two shocks are ordered recursively. The table shows the mean of $\hat{b}_{ij}$ and in parentheses the standard deviation of $\sqrt{T}(\hat{b}_{ij} - b_{ij})$ of all estimated elements $\hat{b}_{ij}$.

increasing model size. Moreover, we find that this curse of dimensionality is more pronounced in smaller samples. Note that we do not find such a performance decrease due to additional shocks in a recursive SVAR, see the online Appendix. Therefore, the simulation illustrates how in a typical macroeconomic application, which rarely contains more than a few hundred observations, data driven estimates based on non-Gaussianity become less reliable the more variables and shocks the SVAR contains.

However, the simulation also shows that exploiting the partly recursive structure stops the deterioration of the performance induced by a larger model. In the large SVAR with $n = 4$ the first two columns of the partly recursive PML estimator are fixed by the whitening step and are thus entirely determined by second moments[6] and only the unrestricted elements of the last two columns depend on higher moments. However, these unrestricted elements perform very similar to the estimates of the small model with $n = 2$. Therefore, the simulation shows that including a priori information on the recursive order can break the curse of dimensionality of the data driven estimator based on non-Gaussianity.

In macroeconomic applications, we oftentimes face relatively large models but only small samples with at best a few hundred observations. In this case, purely data driven estimates based on non-Gaussianity become volatile and in a given application it can become difficult to draw any conclusions on the interaction of the variables or even label the shocks. However, econometricians have put much work into deriving and defending restrictions on the interaction of macroeconomic variables and the simulation shows, how including traditional zero restrictions increases the finite sample performance of a data driven estimator based on non-Gaussianity. Therefore, we argue that in a given application, the researcher should include restrictions when possible and rely on a data driven estimation when necessary.

4 The interdependence of monetary policy and stock market in U.S. data

In this section, we apply the proposed estimator to analyze the interaction of monetary policy and the stock market. Our SVAR contains a first block of macroeconomic variables and a second block of financial variables. We first apply our partly recursive SVAR estimator and impose that the first block is ordered recursively, however, the second block containing the monetary policy and stock market shock remains unrestricted. Afterwards, we apply an unrestricted purely data driven estimator, to check on the validity of our recursive ordering for the first block of variables. Both estimators indicate that a tightening of monetary policy leads to an immediate and permanent decrease in stock prices, while a positive stock market shock leads to an immediate increase

[6] The first two columns of the unrestricted PML estimator depend on higher moments. Comparing the first two columns of the unrestricted and partly recursive estimator shows the possible performance gains of decreasing the dependence of the estimator on higher moments. However, note that this difference is driven by the degree of non-Gaussianity of the shocks and more or less Gaussian shocks would result in a smaller or larger difference.

in interest rates. Additionally, the unrestricted estimation indicates that the macroeconomic variables do not simultaneously respond to stock market and monetary policy shocks and hence it supports the partly recursive order. However, the unrestricted and purely data driven estimation leads to large confidence intervals and the dynamics which potentially explain the interaction of monetary policy and the stock market remain hidden. In contrast to that, the partly recursive estimator yields smaller confidence bands and we find that a tightening of monetary policy is followed by a recession which explains the decrease in stock returns, while a stock market shock behaves equivalent to a news shock and indicates a future business cycle expansion with an increase in output and inflation, which explains the response of monetary policy.

We consider a SVAR in five variables and quarterly U.S. data from 1983Q1 to 2019Q1 of the form

$$\begin{bmatrix} y_t \\ I_t \\ \pi_t \\ i_t \\ s_t \end{bmatrix} = \alpha + \gamma t + \sum_{i=1}^{p} A_i \begin{bmatrix} y_{t-1} \\ I_{t-1} \\ \pi_{t-1} \\ i_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} u_t^y \\ u_t^I \\ u_t^\pi \\ u_t^i \\ u_t^s \end{bmatrix}, \tag{19}$$

where y denotes output growth, I investment growth, π the inflation rate, i the federal funds rate and s real stock returns[7]. Moreover, we set $p = 2$ as suggested by the Akaike and Schwarz-Bayesian information criterion. The linear time trend t is added to catch an eventual long-term decline in the interest rate as discussed by for instance Carvalho et al. (2016).

Appendix A.2 contains multiple robustness checks covering the exclusion of the time trend, inclusion of further variables, exclusion of the financial crises starting in 2008, different lag structures, estimating a specification with all variables in levels rather than growth rates or using different non-Gaussian estimators. Our main results remain unchanged: Stock prices and the nominal

[7]The inflation rate is defined as the quarter to quarter growth rate in the quarterly chain-type GDP price index retrieved from the FRED. The GDP growth rate is given by the quarterly log-difference of real GDP retrieved from the FRED. Real investment growth is given by the quarterly growth rate of real gross private domestic investment obtained from the FRED. The nominal interest rate is defined as the Federal Funds Rate (FFR), where the effective FFR (retrieved from FRED) is replaced by the shadow FFR provided by Wu and Xia (2016) for the Zero Lower Bound observations during the Great Recession. Stock returns are defined as the quarterly log-difference in real stock prices, where real stock prices are given by the S&P 500 index (retrieved from macrotrends.net) divided by the chain-type GDP price index.

interest rate both react immediately to monetary policy and stock market shocks, indicating that these variables cannot be ordered recursively. However, across specifications we cannot fully reject the long-run neutrality assumption, but also do not find much evidence for its validity.

4.1 Partly recursive estimation

We first assume that real investment growth, real output growth and inflation are ordered recursively and behave sluggishly, meaning they cannot react to monetary policy or stock market shocks within the same quarter. However, interest rates and stock returns are unrestricted and can contemporaneously respond to all shocks. Therefore, the simultaneous relationship is given by

$$\begin{bmatrix} u_t^y \\ u_t^I \\ u_t^\pi \\ u_t^i \\ u_t^s \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 \\ b_{21} & b_{22} & b_{23} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^I \\ \varepsilon_t^\pi \\ \varepsilon_t^s \\ \varepsilon_t^i \end{bmatrix}, \tag{20}$$

The estimator proposed in Section 3 allows to identify the impact of the monetary policy shock ε_t^i and the stock market shock ε_t^s without committing to any further restrictions if the monetary policy and stock market shocks are independent and at least one of the two shocks is non-Gaussian. Non-Gaussianity is a commonly found feature of financial variables, see e.g. Mittnik et al. (2000) or Kim and White (2004). Table 2 shows the skewness, kurtosis and the Jarque-Bera test for normality of the reduced form shocks. We find strong evidence that the reduced form

Table 2: Non-Gaussianity reduced form

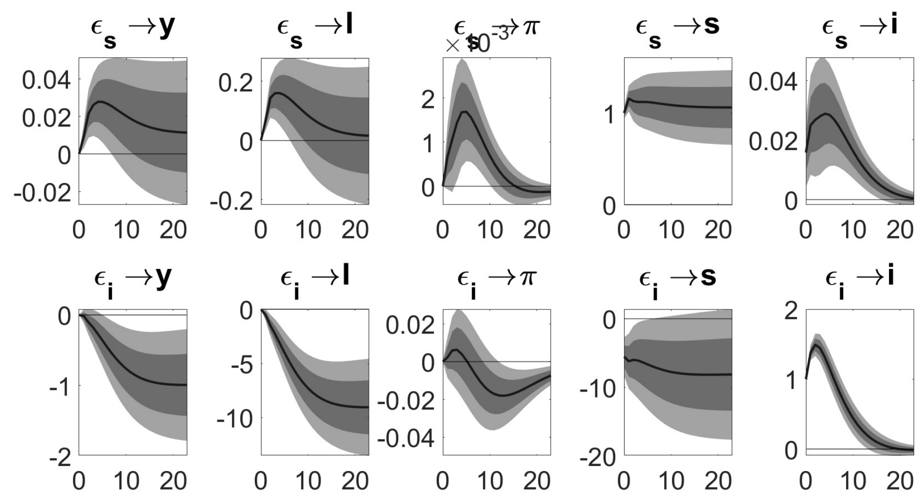
	u^y	u^I	u^π	u^i	u^s
Skewness	−0.73	0.10	−0.03	−0.58	−1.13
Kurtosis	5.13	3.75	2.84	4.33	14.09
JB-Test	0.00	0.11	0.50	0.00	0.00

Skewness, kurtosis and the p-value of the Jarque-Bera test for normality of the reduced form shocks.

shocks in the second block are non-Gaussian, while the result for the first block is mixed, since for only one of three shocks the Gaussianity hypothesis can be rejected at the 10% level. This finding is consistent with our assumed partly recursive structure. If the structural monetary policy and/or stock market shock are non-Gaussian, we would expect to find non-Gaussianity in the second block containing the reduced form interest rate and stock return shocks. Furthermore, if the assumption of a partly recursive order in equation (20) is correct, the non-Gaussianity of the structural monetary policy and stock market shock does not affect the reduced form shocks in the first block, which is consistent with the result reported in Table 2.

The simultaneous interaction of the non-recursive block containing the monetary policy and stock market shock is then estimated by the fast SVAR-GMM estimator proposed in Keweloh (2020). Figure 2 shows the corresponding impulse response functions (IRF) where the stock market shock has been normalized to a one percent increase of the stock price and the monetary policy shock has been normalized corresponding to a one percentage point increase of the interest rate. The responses of stock returns and real GDP growth are integrated to show the associated level effects. Exploiting the partly recursive order makes labeling trivial. There is only one shock which leads

Figure 2: Impulse Responses to normalized shocks in stock returns (s) and monetary policy (i). I denotes investment growth, y output growth and π the inflation rate. Confidence bands are 68% and 80% bootstrap bands. 5000 replications are used in the bootstrap algorithm. The columns y, I and s show the cumulative responses.



to an increase of the interest rate together with a decrease of output and a medium-run decrease of inflation, which is what we would expect from a monetary policy shock. The remaining shock is then labeled as the stock market shock.

We find a simultaneous interaction between the stock market and monetary policy. In particular, a stock market shock increasing stock prices by 1% leads to an interest rate increase of about nearly 0.03 percentage points within the first five quarters and a monetary policy shock increasing interest rates by 1 percentage point leads to an immediate decrease of stock prices by over 5% on impact. The estimated simultaneous interaction is qualitatively comparable to the results in Bjørnland and Leitemo (2009), although our impulse responses show weaker shock responses compared to them.

Consistent with the news literature around Beaudry and Portier (2006) we find that a positive stock market shock is followed by a future business cycle expansion with an increase in the real output growth rate and a positive inflation rate. Therefore, even if the central bank is not interested in stock prices in the first place, a stock market shock can indicate a future business cycle expansion with inflationary pressure, which explains the estimated positive response of the interest rate to the stock market shock. Additionally, we find that a contractionary monetary policy shock induces a recession with a decrease in output and and prices. The future recession and an efficient stock market, which immediately incorporates all available information, then explains the initial negative response of stock prices to the monetary policy shock.

Unlike Bjørnland and Leitemo (2009) we do not impose long-run neutrality of monetary policy w.r.t. stock prices. Based on the IRF we even reject the null hypothesis that monetary policy is neutral w.r.t stock prices in the long-run. Instead, we find that a contractionary monetary policy shock leads to a permanently lower output, investment and stock prices level. Thus, remembering our simple model from section 2, our data driven approach would actually favor the endogenous growth and not the neoclassical model.

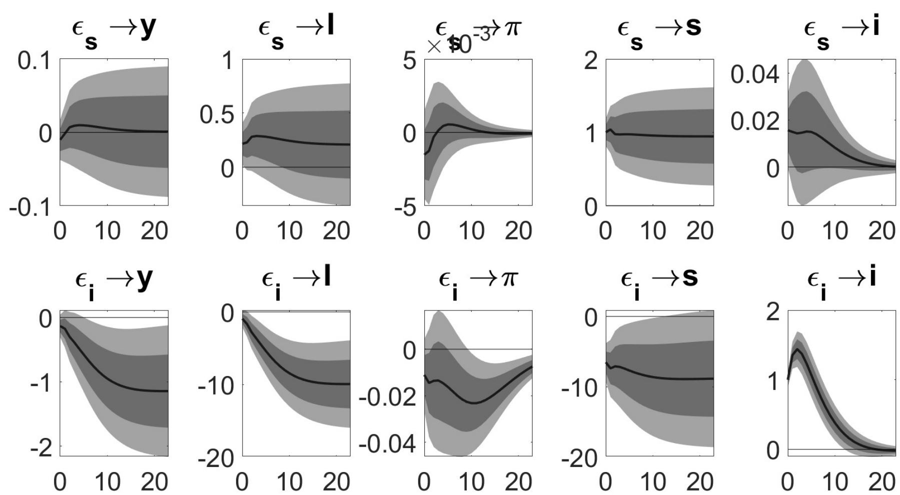
4.2 Unrestricted estimation

We now check the recursiveness assumption of the variables in the first block. Therefore, we use no restrictions on the simultaneous interaction and allow all variables to interact simultaneously. The estimation of the simultaneous interaction is purely data driven and based on the fast SVAR-GMM estimator proposed in Keweloh (2020).

We focus on the interaction of monetary policy and the stock market and therefore only label a monetary policy and a stock market shock. The shocks have been labeled such that the monetary policy shock ϵ_i is the shock with the highest correlation with the reduced form shock u_i, and the stock market shock ϵ_s is the shock with the highest correlation with the reduced form shock u_s[8].

Figure 3 shows the normalized IRF. The unrestricted estimation confirms our finding on the

Figure 3: Impulse Responses to normalized shocks in stock returns and monetary policy. Confidence bands are 68% and 80% bootstrap bands. 5000 replications are used in the bootstrap algorithm. The columns I, y and s show the cumulative responses.



interaction of monetary policy and the stock market; I) a tightening of monetary policy induces a recession with a decrease in output, investment, inflation, and stock prices, and II) a positive stock market shock is accompanied by an immediate increase in interest rates. Turning to the validity of the recursivness assumptions used in the partly recursive estimation, we find that investment, GDP and inflation do not significantly respond simultaneously to monetary policy and stock market shock (only the response of investments to stock market shocks is significant, but close to insignificant, at the 20% level).

Consistent with the finding in the Monte Carlo simulation in Section 3.2, we find that the confi-

[8]The IRF of all variables and shocks can be found in the Appendix. The IRF shows that the shock labeled as the monetary policy shock is the only shock which leads to an increase in the interest rate accompanied by a decrease in GDP, investment and a long-run decrease in inflation, which reinforces our labeling.

dence intervals are larger compared to the partly recursive estimation in Section 4.1. In particular, a stock market shock appears to have almost no significant impact on investment, GDP or inflation, thus making it difficult to explain the response of the interest rate. The application illustrates how a data driven approach can be combined with traditional zero restrictions to impose more structure on the SVAR and thereby decrease the variance of the estimator and gain deeper insights into the transmission of stock market and monetary policy shocks.

5 Conclusion

The present paper proposes a partly recursive, partly non-Gaussian SVAR estimator, which generalizes between the traditional restriction based and the more recent data driven identification approach based on non-Gaussianity. We show that purely data driven estimators based on non-Gaussianity suffer from a curse of dimensionality in small samples and large models. Exploiting the partly recursive order can break the curse of dimensionality and increase the finite sample performance. The estimator is applied to analyze the interaction of monetary policy and stock markets. We find that contractionary monetary policy shocks have a contemporaneous negative impact on stock prices, while stock market shocks have an on impact positive effect on the nominal interest rate. Additionally, we provide evidence that cast doubt on the long-run neutrality of monetary policy w.r.t stock prices used for identification by the literature in the past. In this setup where both short- and long-run restrictions are questionable, the proposed estimator allows to estimate the interaction of the stock market and monetary policy without imposing any restrictions on the interaction of both variables.

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A Appendix

A.1 Appendix - White SVAR estimators with partly recursive constraints

Let $Q(B, u)$ be the objective function of a non-Gaussian SVAR estimator. Moreover, define the unmixed innovation $e(B) = B^{-1}u$. A whitened SVAR estimator then requires that

$\frac{1}{T}\sum_{t=1}^{T} e_t(B)e_t'(B) = I$, such that in a given sample the unmixed innovations are mutually uncorrelated with unit variance.

Estimating a n dimensional SVAR with m partly recursive constraints and T observations yields the following optimization problem

$$\hat{B} := \underset{B \in \mathbb{R}^{n\times n}}{\arg\min}\, Q(B, u) \tag{21}$$
$$s.t. \quad b_{i,j} = 0, \quad \text{for } i < j \text{ and } i \leq m.$$

A whitened SVAR estimator has an additional constraint

$$\hat{B} := \underset{B \in \mathbb{R}^{n\times n}}{\arg\min}\, Q(B, u) \tag{22}$$
$$s.t. \quad b_{i,j} = 0, \quad \text{for } i < j \text{ and } i \leq m \tag{23}$$
$$\frac{1}{T}\sum_{t=1}^{T} e_t(B)e_t'(B) = I. \tag{24}$$

However, due to the whitening constraint (24) the optimization problem (22) is difficult to solve numerically.

First, we ignore the partly recursive constraint (23) and consider a white SVAR estimator with the corresponding optimization problem

$$\hat{B} := \underset{B \in \mathbb{R}^{n\times n}}{\arg\min}\, Q(B, u) \tag{25}$$
$$s.t. \quad \frac{1}{T}\sum_{t=1}^{T} e_t(B)e_t'(B) = I$$

The constrained optimization problem (25) can be transformed into an unconstrained optimization problem over orthogonal matrices. Let $VV' = \frac{1}{T}\sum_{t=1}^{T} u_t u_t'$ be the Cholesky decomposition of the sample variance-covariance matrix of the reduced form shocks. For simplicity, we ignore the indeterminacy of sign and permutation. It holds that $\hat{B} = V\hat{O}$ with

$$\hat{O} := \underset{O \in \mathbb{O}^{n\times n}}{\arg\min}\, Q(VO, u), \tag{26}$$

where $\mathbb{O}^{n \times n}$ denotes the set of $n \times n$ dimensional orthogonal matrices. The optimization problem over orthogonal matrices in equation (26) has no constraints and can be pulled back to an optimization problem over the euclidean space, see Lezcano-Casado and Martínez-Rubio (2019). Therefore, let $exp(\cdot)$ denote the matrix exponential function, let $s(\cdot)$ be the function which maps a vector into a lower skew-symmetric matrix. It then holds that

$$\hat{O} := \underset{\theta \in \mathbb{R}^{\frac{n(n-1)}{2}}}{\arg\min}\ Q(V\mathcal{O}(\theta), u), \tag{27}$$

where $\mathcal{O}(\theta) = exp(s(\theta))$ maps the $\frac{n(n-1)}{2}$ dimensional vector θ into an orthogonal matrix.

Similar to the case without the partly recursive constraints, the optimization problem (22) with the partly recursive constraints (23) can be transformed into an optimization problem over orthogonal matrices such that $\hat{B} = V\hat{O}$ with

$$\hat{O} := \underset{O \in \mathbb{O}^{n \times n}}{\arg\min}\, Q(VO, u), \tag{28}$$

$$s.t. \quad (VO)_{i,j} = 0, \quad \text{, for } i < j \text{ and } i \leq m \tag{29}$$

Let $d = \frac{(n-m)(n-m-1)}{2}$ and define the mapping between a d dimensional vector into an orthogonal matrix which preserves the partly recursive constraint (29)

$$\mathcal{O}_m : \mathbb{R}^d \to \mathbb{O}^{n \times n},\, \theta \longmapsto \begin{bmatrix} I_m & 0 \\ 0 & exp(s(\theta)) \end{bmatrix}, \tag{30}$$

where I_m denotes and m dimensional identity matrix. The optimization problem (28) can now be pulled back to an unconstrained optimization problem over the euclidean space

$$\hat{O} := \underset{\theta \in \mathbb{R}^d}{\arg\min}\, Q(V\mathcal{O}_m(\theta), u), \tag{31}$$

which simplifies the numerical optimization problem.

We now show that in a SVAR with a whitening constraint, the first m columns of the B matrix and therefore the first m recursively ordered shocks are determined by second moments due to the whitening constraint. Put differently, no information in moments beyond the variance can affect the estimated impact of the first m recursively ordered shocks since it is entirely determined by the whitening constraint. For simplicity, consider the four dimensional example with $m = 2$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix} \tag{32}$$

which can be written as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \tag{33}$$

$$\begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} + \begin{bmatrix} \nu_3 \\ \nu_4 \end{bmatrix} \tag{34}$$

$$\begin{bmatrix} \nu_3 \\ \nu_4 \end{bmatrix} = \begin{bmatrix} b_{33} & b_{34} \\ b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}. \tag{35}$$

In a whitened SVAR, the unmixed innovations have to satisfy the condition $\frac{1}{T} \sum_{t=1}^{T} e_t(B) e'_t(B) = I$. In particular, the matrix B has to satisfy

$$\frac{1}{T} \sum_{t=1}^{T} e_{1,t}(B) e_{1,t}(B) = 1 \tag{36}$$

$$\frac{1}{T} \sum_{t=1}^{T} e_{2,t}(B) e_{2,t}(B) = 1 \tag{37}$$

$$\frac{1}{T} \sum_{t=1}^{T} e_{1,t}(B) e_{2,t}(B) = 0. \tag{38}$$

However, equation (33) is a recursive SVAR which is uniquely determined by the variance and covariance conditions (36)-(38). Therefore, in a whitened SVAR the parameters b_{11}, b_{21}, and b_{22} and hence the first m estimated structural shocks, here $\hat{e}_1$ and $\hat{e}_2$, are uniquely determined by second moments. Note that this solution is equal to the solution obtained by applying the Cholesky decomposition to the variance covariance matrix of the reduced form shocks. Moreover, the whitening constraint implies

$$\frac{1}{T}\sum_{t=1}^{T}\hat{e}_{1,t}v_{3,t}(B) = 0 \tag{39}$$

$$\frac{1}{T}\sum_{t=1}^{T}\hat{e}_{2,t}v_{3,t}(B) = 0 \tag{40}$$

$$\frac{1}{T}\sum_{t=1}^{T}\hat{e}_{1,t}v_{4,t}(B) = 0 \tag{41}$$

$$\frac{1}{T}\sum_{t=1}^{T}\hat{e}_{2,t}v_{4,t}(B) = 0. \tag{42}$$

Replacing ε_1 and ε_2 with $\hat{e}_1$ and $\hat{e}_2$ in equation (34) and exploiting the four conditions (39)-(42) implies that the parameters b_{31}, b_{32}, b_{41}, and b_{42} are again uniquely determined by second moments. Therefore, the estimated impact of the first m recursively ordered shocks is uniquely determined by second-order moment conditions derived from the whitening constraint.

A.2 Appendix - Application

This section contains supplementary material and robustness checks for the application presented in Section 4. The estimated interaction of stock markets and monetary policy is found to be robust to all applied robustness checks.

Table 3 shows some descriptive statistics of the variables used in the SVAR.

Table 3: Descriptive statistics

	Mean	Median	Mode	Std. deviation	Variance	Skewness	Kurtosis	Range
y	0.71	0.74	-2.19	0.61	0.37	-0.83	6.46	4.44
I	1.1	0.96	-11.56	3.16	9.97	-0.28	5.3	21.28
π	2.28	2.09	0.27	0.87	0.76	0.36	2.72	4.33
i	1.56	2.11	-26.45	6.5	42.25	-1.08	5.88	43.95
s	3.69	4.02	5.25	3.44	11.84	-0.03	2.07	14.31

Table 4 shows the skewness, kurtosis and p-value of the Jarque-Bera test of the estimated monetary policy and stock market shock in the partly recursive SVAR in Section 4.1.

Table 5 shows the skewness, kurtosis and p-value of the Jarque-Bera test of all estimated struc-

Table 4: Moments of estimated structural shocks (partly recursive SVAR)

	ε^i	ε^s
Skewness	-0.5809	-1.1274
Kurtosis	4.3264	11.0881
JB-Test	0.00	0.00

Skewness, Kurtosis and p-value of the Jarque-Bera test of the estimated monetary policy shock (ε^i) and stock market shocks (ε^s) from our partly recursive SVAR estimator in Section 4.1.

tural shocks in the non-recursive SVAR in Section 4.2.

Table 5: Moments of estimated structural shocks (non-recursive SVAR)

	ε^y	ε^I	ε^π	ε^i	ε^s
Skewness	-1.0395	0.6256	-0.0878	-0.7813	-0.3616
Kurtosis	7.1174	4.0654	3.0879	5.2690	15.0522
JB-Test	0.00	0.01	0.50	0.00	0.00

Skewness, Kurtosis and p-value of the Jarque-Bera test of all estimated structural shocks in the non-recursive SVAR estimation in Section4.2.

Table 6 shows the correlation between the estimated structural shocks from the non-recursive SVAR in Section 4.2 and the reduced form shocks.

Table 6: Correlation of reduced form and estimated structural shocks

	u^y	u^I	u^π	u^s	u^i
ε^y	0.9	0.52	0.25	0.41	0.42
ε^I	0.39	0.73	-0.32	-0.35	-0.02
ε^π	-0.16	0.21	0.9	-0.23	0.15
ε^s	-0.09	0.37	-0.12	0.72	0.18
ε^i	-0.09	-0.13	-0.07	-0.37	0.88

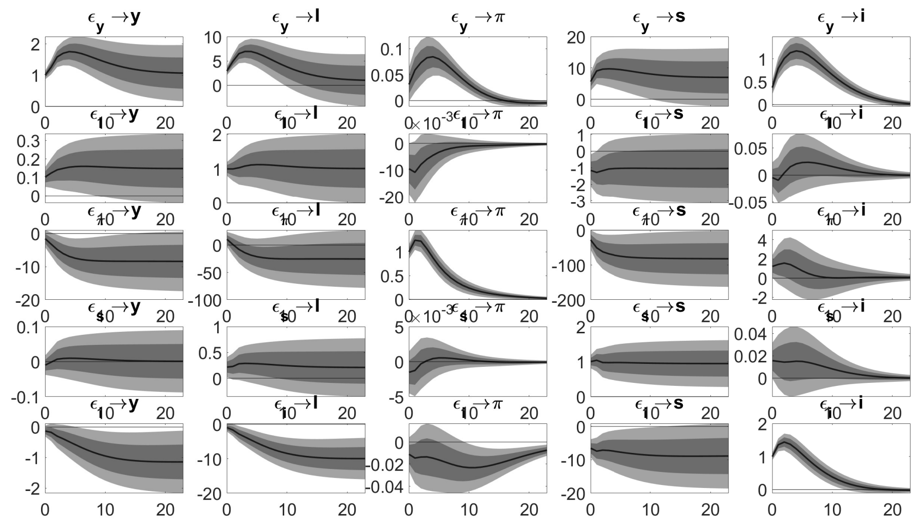
Correlation of estimated structural shocks and reduced form shocks from Section 4.2 .

Figure 4 shows the full set of impulse responses estimated by the non-recursive SVAR in Section 4.2. As it becomes evident the qualitative results of the point estimates are similar to the ones regularly found in the literature. However, the confidence bands are generally very broad and thus it is generally difficult to get any further conclusive answers about the real behavior of the variables following a certain structural shock.

We proceed by further checking on the robustness of the results presented in Section 4. All robustness checks exploit the partly recursive order described in equation (20).

First we replace output growth, inflation growth and stock returns by the respective log-levels to

Figure 4: Impulse responses estimated by the full moment based SVAR estimator. Responses are to normalized structural shocks in output growth, investment growth, inflation, stock returns and the monetary policy rate. Confidence bands are 68% and 80% bootstrap bands. 5000 replications are used in the bootstrap algorithm. The columns y, I and s show the cumulative responses.



see if the levels contain more information compared to the growth rates that influence our results. As can be seen from Figure 5 the qualitative results from our main paper are robust to this change of specification. However, now the long-run negative impact of monetary policy shocks on stock prices becomes insignificant and is less pronounced. Nevertheless, the long-run response of stock prices to a monetary policy shock is associated with a high uncertainty. Therefore, even if the long-run neutrality of monetary policy w.r.t. stock prices holds, estimates based on long-run restrictions might be unreliable due to the volatile long-run response.

We now check if our results are dependent on our estimation technique for the non-recursive block. Thus we employ the PML estimator proposed by Gouriéroux et al. (2017) to estimate the non-recursive block. Figure 6 shows the results. As it becomes evident the change of the estimation technique does not change our results from section 4: The interest rate increases in response to a stock market shock, stock prices immediately decrease after a monetary policy shock and stay permanently below the level without the shock.

Third, we increase the number of lags to $p = 8$. As becomes evident from Figure 7, the estimated

Figure 5: Impulse Responses to normalized shocks in stock returns and monetary policy with log-levels for output, investment and stock prices. Confidence bands are 68% and 90% bootstrap bands.

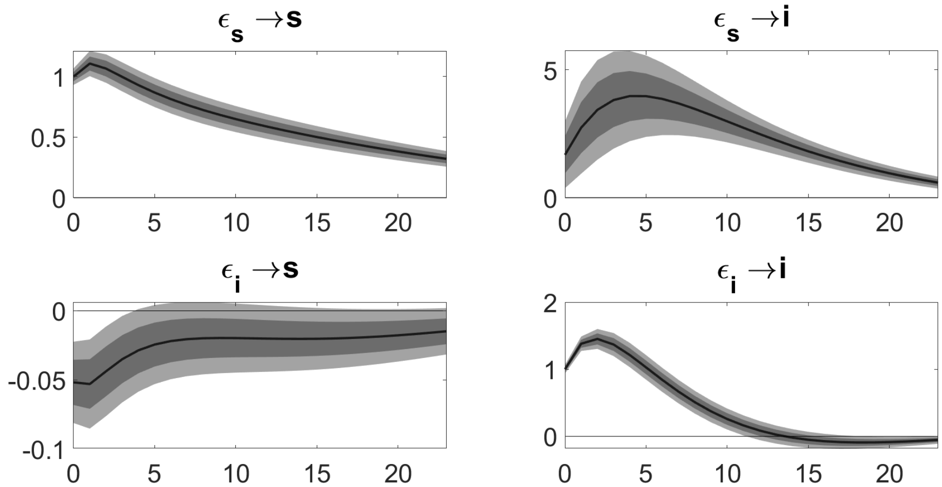
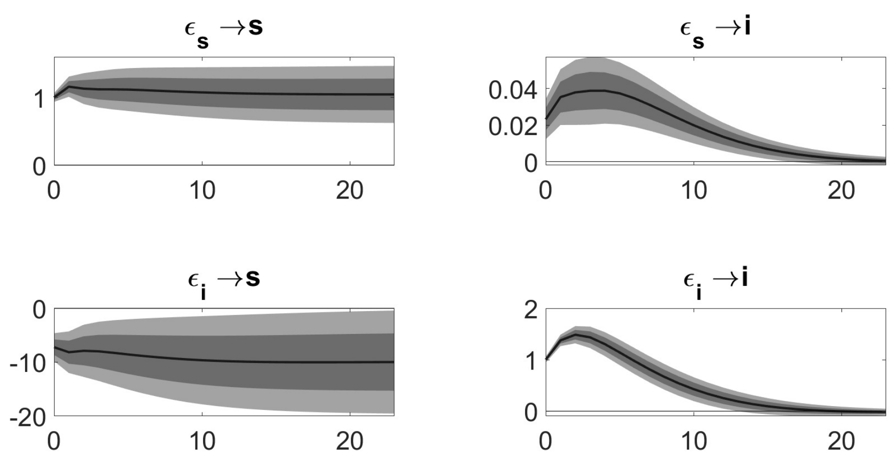
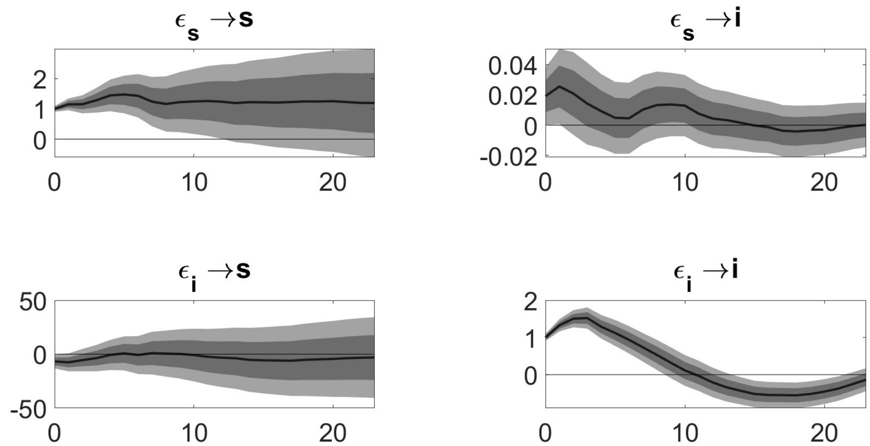


Figure 6: Impulse Responses to normalized shocks in stock returns and monetary policy using the PML estimator (see Gouriéroux et al. (2017)) for the non-recursive part. Confidence bands are 68% and 80% bootstrap bands. The column s shows the cumulative response of stock returns to show the effect on aggregate stock prices.



simultaneous interaction is again similar to our baseline specification. However, the confidence

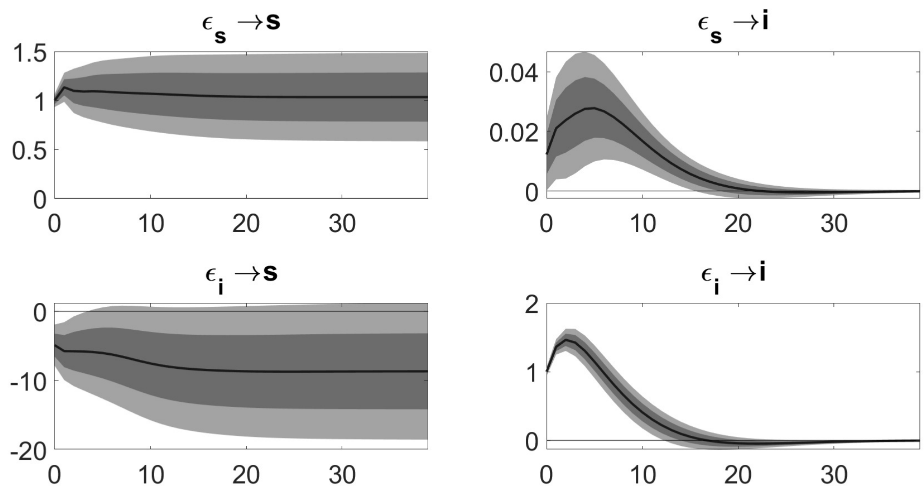
Figure 7: Impulse Responses to normalized shocks in stock returns and monetary policy with a lag order of $p = 8$. Confidence bands are 68% and 80% bootstrap bands. In contrast to Section 4 the observation period is restricted to 1983Q1-2019Q1. The column s shows the cumulative response of stock returns to show the effect on aggregate stock prices.



band in this case is quite broad and there is not much to conclude from the impulse response of stock prices to a monetary policy shock regarding the long-run behavior. Consequently, we cannot reject the long-run neutrality of monetary policy with respect to stock prices, but on the other side there is not much evidence for it either as due to the broad confidence bands many other long-run outcomes are possible.

Fourth, we consider the inclusion of commodity price inflation (named dcomm), defined as the logarithmic difference in the producer price index (also taken from the FRED). For instance, Bjørnland and Leitemo (2009) argue that the inclusion of commodity price inflation helps to reduce the price puzzle and thus should be included into the SVAR specification. We assume that commodity price inflation shocks can be identified recursively and are ordered third in the recursive block, so commodity price inflation can react immediately to real output growth and inflation shocks, but not to stock market and monetary policy shocks. Figure 8 shows the resulting IRFs. As it can be seen, the inclusion of commodity price inflation has no impact on the estimated interaction of monetary policy and stock markets compared to Section 4 and thus

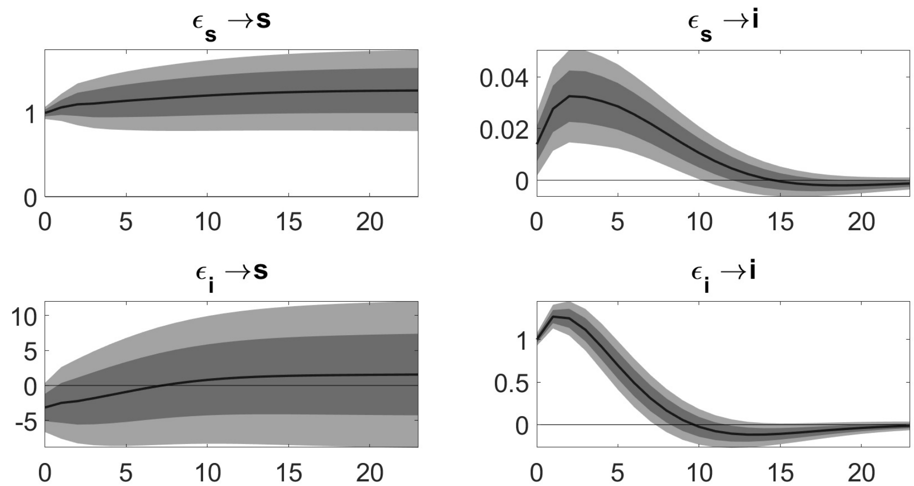
Figure 8: Impulse Responses to normalized shocks in stock returns and monetary policy. Confidence bands are 68% and 80% bootstrap bands. In addition to the price level, real output growth rate, stock returns and nominal interest rate, the commodity price inflation is included. Commodity price inflation shocks are identified recursively, where commodity price inflation is ordered third in the recursive block. The column s shows the cumulative response of stock returns to show the effect on aggregate stock prices.



we omit commodity price inflation from the main paper's specification. The long-run neutrality cannot be rejected based on the 80% confidence band, but can based on the 68% confidence band. So we again conclude that we cannot for sure reject it, but the evidence in favor of it is quite weak.

Fifth, we exclude all observation from 2007Q4 onward from the sample. For instance Kontonikas and Zekaite (2018) and Chatziantoniou et al. (2013) argue that the financial crisis starting in 2008 might have led to major disruptions in the relationship between monetary policy and stock prices. Figure 9 shows the resulting IRFs. As it can be seen from Figure 9 our main results remain unchanged. The only difference is that now the response of stock prices to a monetary policy shock is not negative in the long-run, but turns out to be positive after about 10 quarters. However, this finding again is associated with a large confidence band making the response insignificant in total judging by the 80% confidence band and soon after the shock hits the economy judging by the 68% confidence band. Long-run neutrality is part of the confidence band, but only one of several outcomes, so its validity remains unclear.

Figure 9: Impulse Responses to normalized shocks in stock returns and monetary policy. Confidence bands are 68% and 80% bootstrap bands. In contrast to Section 4 the observation period is restricted to 1983Q1-2007Q3. The column s shows the cumulative response of stock returns to show the effect on aggregate stock prices.



At last, we check on the relevance of the time trend included in our specification. Figure 10 shows the impulse responses of the stock price and FFR to a stock market and monetary policy shock under the specification without a linear time trend. As it turns out, the main qualitative and quantitative insights remain unchanged. However, the confidence band of the stock price response to a monetary policy shock is a bit broader, thus the response becomes insignificant earlier and there is no conclusive answer about the long-run behavior. As the time trend seems to contribute to a more precise estimate, we choose to leave it in our base specification.

Figure 10: Impulse Responses to normalized shocks in stock returns and monetary policy. Confidence bands are 68% and 80% bootstrap bands. In contrast to Section 4 the linear time trend is omitted from the specification. The column s shows the cumulative response of stock returns to show the effect on aggregate stock prices.