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Regime-based debt sustainability analysis: Evidence from euro area economies

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Abstract

This paper empirically studies non-linearities in debt sustainability analysis by resorting to the modern estimation technique of panel smooth transition regression (PSTR). We assess euro area debt sustainability by analysing the reaction of the primary balance to changes in public debt, relative to GDP respectively, in annual frequency from 2000-2019 in a panel framework. The PSTR allows to estimate the existence of a threshold in the behaviour of the reaction function, refrains from the country-wise perspective (pooling) and applies a regime-switching model to detect non-linearities. Data is segregated into different regimes endogenously via a logistics regression. Our results show that there are two different regimes in the euro area: a high and a low debt regime. The estimated reaction coefficient for the low debt regime is statistically insignificant, whereas it is positive and statistically significant for the high debt regime. Further, for a sub-sample of highly indebted economies we find a statistically significant negative (positive) reaction coefficient for the low (high) debt regime.

JEL: H63, E62

Keywords: Debt Sustainability, Fiscal Response Function, Panel Smooth Transition Regression

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1 Introduction

Debt sustainability discussions revived with the European debt crisis starting in 2008/2010 and have received additional prominence with the recent Covid-19 pandemic forcing governments world-wide to stabilise their economies with immense recovery programs that are primarily financed by credits, amid situations already characterised by high debt ratios. Thus, analysing sustainability of fiscal policies and the budget positions is an important and essential task. Government interventions by way of fiscal policy have implications for the present budget as well as for future decisions. Fiscal policy is one of the most powerful instruments in the design of public policy and to achieve political goals. Hence, in order to be able to conduct independent government spending and tax decisions debt sustainability is as relevant as ever.

Introduced by Bohn (1995, 1998) the fiscal response function, which estimates the reaction of the primary balance to changes in public debt relative to the gross domestic product (GDP), is a well-established approach to asses debt sustainability. If the government reacts to a higher debt ratio by actively adjusting its discretionary fiscal policy in terms of a higher primary surplus, the fiscal policy under consideration is considered as sustainable. That relationship is commonly tested empirically in a single equation regression model and, more recently, also in a standard panel set-up. The theoretical background and a formal model for debt sustainability analysis and the fiscal response function can be found in Greiner and Fincke (2015), for instance.

There have been several applications of this approach in the economics literature, see for example Afonso (2005), Afonso and Jalles (2019) and Beqiraj et al. (2018) presenting a nice overview of the literature. However, most papers analyse linear single equation models or standard fixed effects panel regression models. With this paper, we question these usual approaches and the idea that the response of the discretionary fiscal policy is uniform across the distribution of the debt to GDP ratios. Rather, we research whether there are non-linearities and heterogeneity in the reaction of governments to rising public debt, implying that the level of the explanatory variable influences the relationship and the shape of the response. Particularly, we find that the reaction of the primary balance does vary with the size of the debt ratio, meaning that in situations with low debt to GDP ratios the response turns out to be different from that in situations with high debt ratios, even within one country.

Some studies allowing for non-linearities and heterogeneity in debt sustainability analysis can be found in the economics literature. For instance, the notion of "fiscal fatigue" has been introduced by Gosh et al. (2013) who detect a reversal in the behaviour of the primary balance as debt ratios become very high and the response peters out and becomes negative. Other applications in this direction are by Checherita-Westphal and Zdarek (2017) as well as Fournier and Fall (2015), Legrenzi and Milas (2011) and Owusu et al. (2021) for instance.

With this paper we want to contribute to this line of research and to extend it by resorting to the modern estimation technique of panel smooth transition regression (PSTR) applied to euro area economies. PSTR has been established by Gonzalez et al. (2017) and allows to detect a threshold in the reaction function. It refrains from a country-wise perspective and applies a regime-switching model to detect non-linearities in the data generating process. Data is segregated into different regimes endogenously via a logistics regression, i.e. using a data driven approach. Legrenzi and Milas (2011) applied a similiar approach based on single equation / country studies for the highly indebted economies Greece, Italy, Ireland, Portugal and Spain, based on historical data time series some starting in the 1850s.

With PSTR we apply a new and modern methodology to analyse the fiscal response that overcomes the pooling problem present in the panel data context, where one coefficient fits all and the data is separated country-wise, and it distinguishes debt regimes due to the data generating process across all observed states of the debt to GDP ratio. The advantage is obvious: the number of regimes is determined by the data, the coefficients are estimated for each regime (for example high or low debt) and the reaction coefficients yield a better fit than the pooled ones in usual panel regression models. In addition, they are not determined by particular economies, but, by the respective debt situations. Individual countries are not restricted to stay in the same group or category but can switch between the groups depending on the heterogeneity of their fiscal behaviour. This implies that in one country different responses are feasible according to the different debt situations, i.e. the response is different in low debt situations compared to high debt situations. Specifically, the regression coefficients are allowed to switch smoothly between two regimes characterised by low or high values of the transition variable. Smooth transitioning from one regime to another one is feasible and can be justified by the fact that there are usually some time lags in policy decisions (policy inertia). Hence, the effect is expected to change gradually over the time period rather than sharply. Therefore, PSTR models are more appealing and better reflect reality than other threshold or regime based models.

We assess euro area debt sustainability by analysing the reaction of the primary balance to changes in public debt, relative to GDP respectively, in annual frequency over the time period 2000-2019 in a panel framework. Our analysis relies on AMECO data for 27 euro area economies from 2000 to 2019. The estimation results show that there are two different regimes in the euro area: a high and a low debt regime. These distinctions also reveal heterogeneous behaviour of the primary balance across the distribution of the debt to GDP ratios. The coefficients are positive for both regimes, the coefficient in the low debt regime, however, is not statistically significant. For a sub-sample of highly indebted countries we find a statistically significant negative (positive) reaction coefficient for the low (high) debt regime. Thus, debt sustainability seems to be given in the high debt regime. Several robustness tests support our findings.

The rest of this paper is organised as follows: Section 2 briefly discusses the theoretical background, section 3 introduces the PSTR methodology and section 4 presents the estimation outcomes. Finally, section 5 summarises the main results.

2 Theoretical Background

The analysis of debt sustainability by means of the fiscal reaction function studies discretionary fiscal policy decisions by estimating the response of the primary surplus to changes in the public debt, relative to GDP.

Based on the public budget of an economy that consists of government revenues, mainly taxes, and of public spending we could assume that a government determines the primary surplus (i.e. surplus net of interest payments) to GDP ratio, $s(t) = S(t)/Y(t)$, such that it is a positive linear function of the public debt to GDP ratio, $b(t) = B(t)/Y(t)$, and of a term that is independent of public debt, $\psi(t)$ (see Bohn, 1995, 1998, Greiner and Fincke, 2015, Afonso and Jalles, 2019). Thus, the primary surplus ratio can be written as

$$s(t) = \beta(\cdot)\, b(t) + \psi(t), \tag{1}$$

where $\beta(\cdot)$ is the reaction coefficient determining how strong the primary surplus reacts to changes in the public debt ratio and that is allowed to be variable - here being estimated as a function depending on the distribution of the debt to GDP ratios. The parameter $\psi(t) \in \mathbb{R}$ is affected by other economic variables, like transitory government spending (such as social, climate or educational outlays etc.). As regards $\psi(t)$, we posit that it is bounded by above and by below by a certain finite number that is constant over time. Additionally, $\psi(t)$ cannot be completely controlled by the government: it can influence that coefficient only to a certain degree because $\psi(t)$ is also affected by the business cycle, for example, that affects the economy temporarily.

With a theoretical model it can be shown that a strictly positive reaction coefficient on average, such that $\lim_{t\to\infty} \int_{t_0}^{t} \beta(\mu) d\mu = \infty$, implies that the debt policy of a government is sustainable, cf. for instance Greiner and Fincke (2015). It must be pointed out that the reaction coefficient $\beta(\cdot)$ can be time-varying or a non-linear function of public debt. It may even be negative for some time periods, however, on average that coefficient must be positive. Another shortcoming of the analysis is that it implicitly assumes that the primary surplus relative to GDP can grow without an upper bound. However, a positive but small reaction coefficient on average does not necessarily guarantee a bounded debt to GDP ratio.

To finish our theoretical considerations, we want to point out that we allowed the reaction coefficient to be varying/non-linear. And in fact, times series analyses provide strong empirical evidence that this coefficient is not constant over time (see Greiner and Fincke, 2015, chap. 2.2-2.5 or Owusu et al., 2021). It should be noted, too, that a linear model with time-varying coefficients can be seen as an approximation of a non-linear model and the approximation is good if the parameter changes smoothly (cf. Granger, 2008).

The latter is of interest because the question arises which factors are responsible for variations in the coefficients. When analysing the response of the primary surplus to variations in public debt, there is evidence that the reaction depends on the magnitude of the public debt ratio. Our analysis takes into account these consideration and detects the heterogeneity data driven and determines the development across the distribution of the debt ratios, independent of the countries.

3 Methodology

Regarding the methodology, we apply the Panel smooth transition regression model according to Gonzalez et al. (2017). A two regime PSTR is specified as

$$y_{it} = \mu_i + \lambda_t + \beta_0 x_{it} + \beta_1 x_{it} g(q_{it}; \gamma, c) + u_{it} \tag{2}$$

where i is the individual within the panel whilst t is the time dimension. The variable y represents the response variable, μ represents the individual effect which we assume to be fixed, hence correlating with the regressors, λ is the time effect, x denotes the covariates which are assumed to be exogenous and β represents the coefficients or parameters to be estimated. The function $g(q_{it}; \gamma, c)$ is the transition function which is observable, continuous and bounded. The variable q_{it} is the transitional variable on which the regime switching is conditioned, c is a vector of location parameter whilst γ measures the slope of the transition function. The transition function is captured by a logistics regression function (see Teräsvirta, 1994 and 1998) so that it is bounded between 0 and 1 as seen below

$$g(q_{it}; \gamma, c) = (1 + exp(-\gamma \prod_{j=1}^{m} (q_{it} - c_j))^{-1} \tag{3}$$

where $\lambda > 0$ determines the smoothness of the transition from one regime to another. The location parameter c captures the threshold between the two extreme regimes with transition functions $g(q_{it}; \gamma, c) = 0$ and $g(q_{it}; \gamma, c) = 1$. The index m determines the number of regimes and could be more than one depending on the variations in the parameter. For instance when $m = 1$, the model is characterised by two extreme regimes associated with high and low values of the transition variable (q_{it}). In that case the coefficients from equation 2 switches between β_0 and $\beta_0 + \beta_1$ and the change is centred around c_1. When $m = 2$ the transition function attains its minimum at $(c_1 + c_2)/2$ and attains its maximum at 1 for both low and high values of the transition variable. It is not difficult to see that for $\gamma \rightarrow 0$ the transition function becomes a constant and the model reduces to a linear panel fixed effects model with a homogeneous slope for any positive value of m.

Building and applying the PSTR model consists of three main procedures, namely specification, estimation and model evaluation. Model specification entails specifying a linear model with a homogeneous slope and testing the hypothesis of linearity against an alternative heterogeneous slope in the likeness of PSTR. If the PSTR model is not identified, it implies that the true data generating process is homogeneous. The linearity test is, therefore, a significant step in the modeling process. In case homogeneity is rejected, the appropriate transition variable q_{it} and, hence, the transition function $g(q_{it}; \gamma, c)$ is determined then.

Linearity/homogeneity tests can be carried out by imposing either $H_0 : \gamma = 0$ or $H_0 : \beta_1 = 0$. Due to the issue of unidentified nuisance parameters in the PSTR model, a homogeneity test requires expanding equation 2 by first-order Taylor expansion around $\gamma = 0$ and re-parameterising to yield

$$y_{it} = \mu_i + \beta_0^* x_{it} + \beta_1^* x_{it} q_{it} + .. + \beta_m^* x_{it} q_{it}^m + u_{it}^* \tag{4}$$

where $u_{it}^* = u_{it} + R_m \beta_1 x_{it}$ so that r_m is the reminder of the Taylor expansion. $\beta_1^*, \ldots, \beta_m$ are the vectors of parameters. Hence, the standard linearity test implies testing for the null of $H_0^* : \beta_1^* = \ldots = \beta_m^*$ from equation 4 by Lagrangian Multiplier (LM) test based on the F-distribution and Chi-square distribution. Furthermore, if the PSTR is identified, we proceed with a test to determine the optimal number of regimes also known as sequence of homogeneity test. Under this test, we refer to equation 4 and assume $m = 3$ so that we test for significance of a model with 1, 2 and 3 transition functions, respectively.

Specifically, we apply the LM to test the hypothesis $H_0^* : \beta_3^* = \beta_2^* = \beta_1^* = 0$, $H_{03}^* : \beta_3 = 0$, $H_{02}^* : \beta_2^* = 0 | \beta_3^* = 0$ and finally $H_{01}^* : \beta_1^* = 0 | \beta_3^* = \beta_2^* = 0$. The null with the strongest rejection is selected as the appropriate model and hence the optimal number of regimes. See Gonzalez et al. (2017) and Teräsvirta (1994) for a detailed discussion of the test.

Regarding the second step of parameter estimation ($\phi = \beta_0, \beta_1, \gamma, c$), a combination of fixed effects procedure and Non-linear Least Squares (NLS) is used. Firstly, the individual fixed effects in the panel are eliminated by within transformation after which the transformed data is estimated by non-linear least squares due to the non-linearity induced by the transition function.

The selection of appropriate values of γ, c for the NLS optimisation is done by choosing starting values of the parameters and using a grid search across the parameters of the transition function such that the parameters which yield the minimum sum of squared errors are selected. This is done via an algorithm since a manual grid search over a wide range of values could be computationally demanding.

The final step of model evaluation entails a specification test also known as the test of no remaining heterogeneity to ensure the model is correctly specified. The test is a generalisation of the linearity or homogeneity test discussed above. In other words, we test if there exists additional unmodified non-linearity in equation 2. A second transition function is added to the model and a hypothesis of its significance is tested. We undertake a LM test of no extra additive linearity to ensure that the final estimated model is correctly specified. Additionally, we also conduct residual diagnostics checks to ensure that our residuals are not prone to typical long panel data issues such as cross-sectional dependence.

4 Empirical Results

Before we present the estimation and a discussion of the results, we first depict the data we used for our study. All data was obtained from the European commission AMECO website, see AMECO (2021). The data consists of 18 euro area countries with the exception of Lithuania which was excluded because of missing data. Regarding the variables, we use the cyclically adjusted primary balance as published by AMECO (2021) as the dependent variable. Our covariate of interest is the lagged debt to GDP ratio we denote as laggeddebtratio, i.e. b_{t-1}. Further, we control for international trade using net export as a proxy and inflation is controlled for, too. Finally, motivated by Barro's (1979) tax smoothing hypothesis, we include a business cycle variable we denote as YVAR and a spending gap variable henceforth known as GVAR. YVAR is obtained as the deviation between actual GDP and its long term trend (potential GDP). Similarly, GVAR is computed as the deviation between actual real government spending and its long term trend. Potential GDP and long term spending are estimated using the HP filter. As a robustness test, we estimate these variables by an alternative filter according to Hamilton (2018) and we show that the results are very similar and hence robust. All data span a period of 20 years (from 2000 to 2019).

Table 1 provides summary statistics for the data set. It can be observed that the average debt to GDP ratio for the euro area during the period stood at about 65% whilst the maximum hovers about 180%. Net export and business cycle variable (YVAR) had the most variability among the countries, which is not so surprising given the fact that the monetary union is made up of bigger economies such as Germany and smaller economies like Estonia. Table 2 provides more insights regarding the distribution of the euro area debt ratio at the country level. Smaller eastern European countries namely; Estonia, Slovakia, Slovenia, and Finland achieved the lowest debt ratio over the period. Conversely, high debt exceeding 100% was recorded for some western European countries notably Greece, Italy, Ireland, Portugal and Spain. Among them, Italy and Greece had persistently high debt growth with an average debt ratio of about 118% and a low interquartile range implying that debt is consistently at a high level with low variation. Hence, it is not surprising that these countries experienced debt crisis.

Figure 1 provides a visual plot of the euro area debt ratio for the period. The dotted red line depicts structural breaks for each country using the test according to Zeileis et. al (2003) [1] It can be observed that debt series are characterised by two different behaviour; before the structural break and after the break. This lends some support for conditioning our regime-dependent model on the debt ratio for the euro area.

Table 1: Summary statistics

Statistic	pbratio	laggeddebtratio	GVAR	Net_export	Inflation	YVAR
N	360	360	360	360	360	360
Mean	0.0001	0.651	0.000	14.019	0.909	−0.000
St. Dev.	0.031	0.372	7.280	40.666	0.118	17.696
Min	−0.277	0.038	−36.706	−62.146	0.491	−106.608
Pctl(25)	−0.013	0.409	−1.566	−0.996	0.837	−2.804
Pctl(75)	0.017	0.908	0.713	14.615	1.000	2.337
Max	0.097	1.862	52.882	231.246	1.134	96.162

Next, we proceed with the model specification by applying two main econometric tests discussed in the preceding section. Firstly, we conduct a homogeneity or linearity test to ascertain if the PSTR is identified. The number of regimes is subsequently determined if linearity is rejected. Our transition variable is the lagged debt to GDP ratio. Table 3 presents the results of the linearity test. The null hypothesis indicates linearity of the

[1] The structural break test is based on F-test for potential breaks or model stability. Since the individual series is not so long, we consider 1 breakpoint. Break dates are report in table 11 in the appendix.

Figure 1: Euro area debt-ratio with 1 structural break (break date highlighted in dash red). Structural break test based on Zeileis et al (2003).

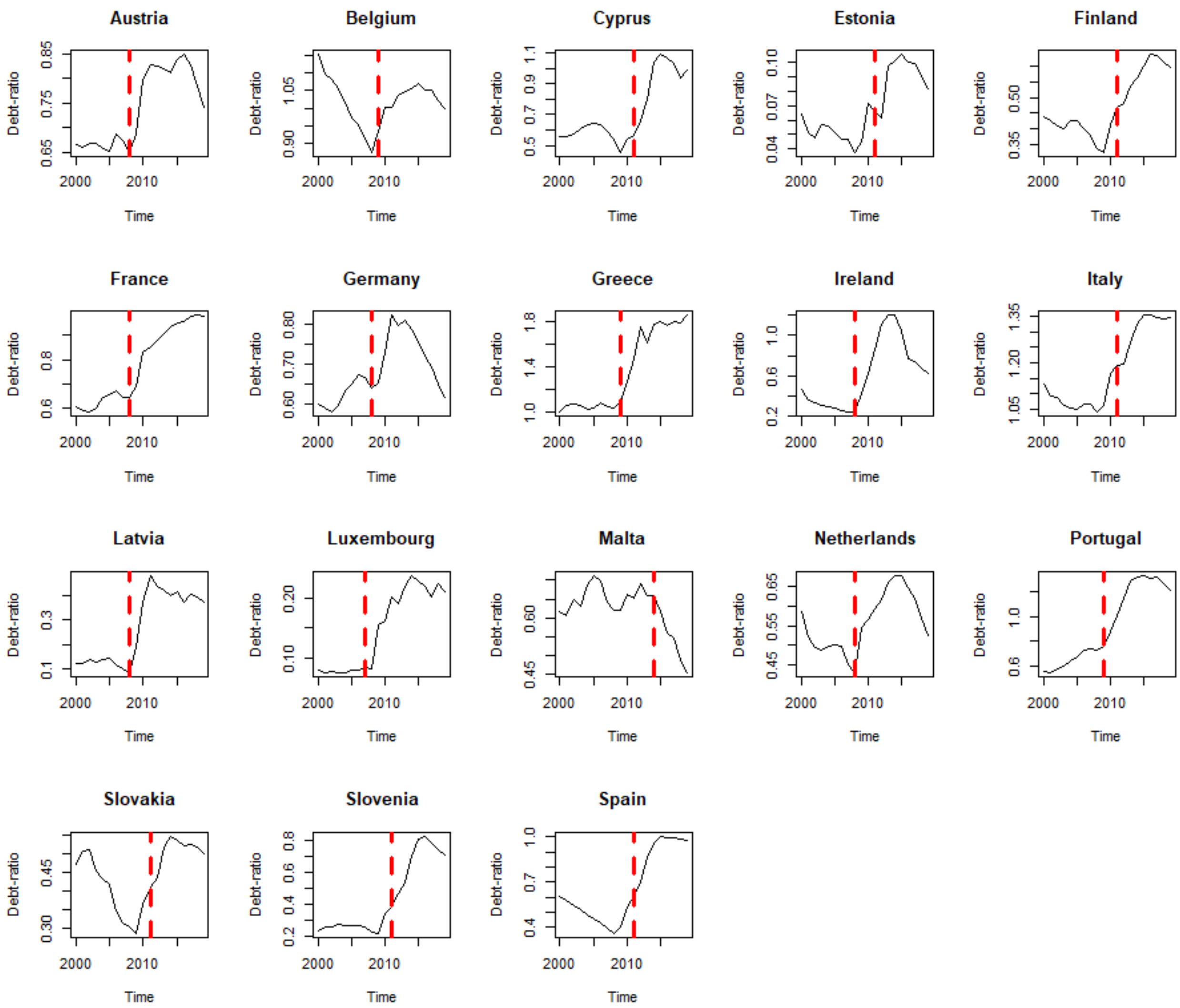


Table 2: Individual country summary statistics - Debtratio

Country	Min	Pctl(25)	Median	Mean	Pctl(75)	Max	Std
Austria	0.650	0.667	0.714	0.740	0.821	0.849	0.0782
Belgium	0.873	0.991	1.03	1.02	1.05	1.15	0.0664
Cyprus	0.455	0.563	0.636	0.727	0.949	1.09	0.214
Estonia	0.0377	0.0502	0.0627	0.0689	0.0927	0.106	0.0227
Finland	0.326	0.407	0.434	0.476	0.571	0.636	0.0994
France	0.583	0.645	0.759	0.779	0.938	0.983	0.156
Germany	0.579	0.629	0.661	0.684	0.737	0.824	0.0777
Greece	0.989	1.05	1.18	1.37	1.77	1.86	0.358
Ireland	0.236	0.304	0.541	0.602	0.790	1.20	0.335
Italy	1.04	1.07	1.15	1.18	1.33	1.35	0.124
Latvia	0.0846	0.128	0.277	0.267	0.401	0.479	0.145
Luxembourg	0.0743	0.0794	0.158	0.147	0.212	0.237	0.0671
Malta	0.452	0.614	0.637	0.623	0.660	0.713	0.0677
Netherlands	0.430	0.499	0.557	0.559	0.618	0.678	0.0747
Portugal	0.542	0.663	0.817	0.929	1.27	1.33	0.308
Slovakia	0.286	0.398	0.462	0.445	0.517	0.546	0.0836
Slovenia	0.218	0.261	0.309	0.441	0.701	0.826	0.229
Spain	0.358	0.471	0.592	0.667	0.962	1.01	0.241

model. In all three models ($m = 1, m = 2$ or $m = 3$), we notice that the null is rejected at a high significance level, irrespective of whether we consider the LM test based on the chi-square distribution or F-distribution. This indicates that a non-linear model with more that one regime is suitable.

Furthermore, we resort to the sequence of homogeneity test to ascertain the appropriate number of regimes for the model. From the table 4, the null H^*_{01} is the regime with the most severe rejection since its p-value is the lowest. Hence, a model with one transition (two regimes) is more suitable for the data[2].

Figure 2 depicts a plot of the transition function based on a logistic regression function using lagged debt to GDP ratio. We are about to see the two extreme regions bounded between zero and one and the monotonic switch between the two extreme regimes where zero depicts the regime of low debt whilst one is the extreme regime of high debt. The transition is rather smooth between the two regimes. We are also able to see the distribution of the data (plotted on the transition function) between the two extreme regimes. In table 5, we show the distribution/characteristics of our transition function. It can be observed that there is a switch from lower to higher values between the 25th and 75th percentile, especially after 2008. Hence, the transition function depicts a time-varying behaviour. The drastic increase in the average value of the transition function coincides

[2] For the econometric theory behind selection of the appropriate regimes, see Teräsvirta (1994)

Table 3: Homogeneity tests (Transition variable lagged debt ratio)

	LM_χ		LM_F	
Regimes	test	p-value	test	p-value
$m = 1$	39.37	(0.000)	7.63	(0.000)
$m = 2$	65.77	(0.000)	6.284	(0.000)
$m = 3$	75.55	(0.000)	4.743	(0.000)

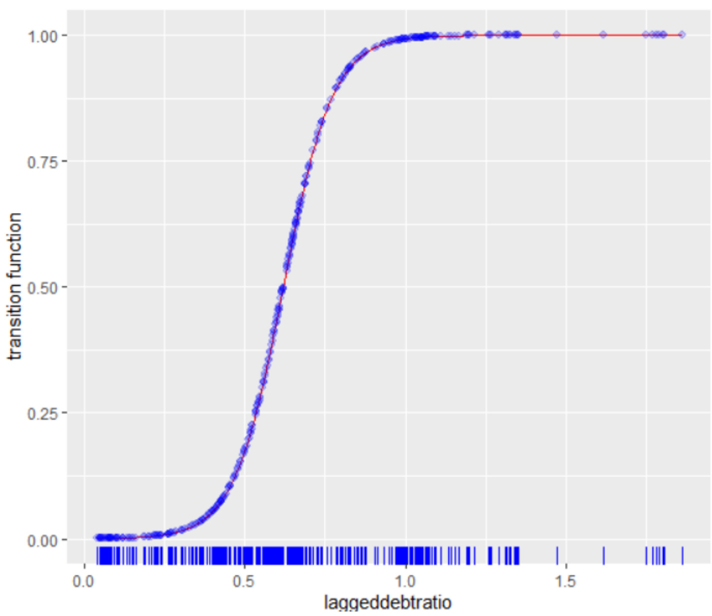
Results of Langrangian multiplier test of homogeneity/linearity based on chi-square (LM_χ) and F distribution (LM_F). Null hypothesis of homogeneous coefficient is tested against an alternative hypothesis of heterogeneous coefficients (PSTR).

Table 4: Sequence of homogeneity tests (Transition variable: lagged debt ratio)

	$LM\chi$		LM_F	
m	test	p-value	test	p-value
H^*_{03}	11.97	(3.524e-02)	2.254	(4.878e-02)
H^*_{02}	29.64	(1.736e-05)	1.5.665	(4.909e-05)
H^*_{01}	39.37	(2.004e-07)	7.633	(8.002e-07)

Results of LM sequence of homogeneity test based on chi-square and F distribution. The hypothesis is based on the following; $H^*_0 : \beta^*_3 = \beta^*_2 = \beta^*_1 = 0$, $H^*_{03} : \beta_3 = 0$, $H^*_{02} : \beta^*_2 = 0| = \beta^*_3 = 0$ and finally $H^*_{01} : \beta^*_1 = 0| = \beta^*_3 = \beta^*_2 = 0$

Figure 2: Transition function - Laggeddebtratio



with the global financial crisis. This is plausible since the financial crises required some policy reaction via expansionary fiscal policies from fiscal authorities in the spirit of the new Keynesian economics. Hence, the transition function can be characterised by two main regimes: namely, the regime before the crisis and after the crisis which supports our earlier econometric specification test.

Table 5: Characteristic of transition variable - lagged debt ratio

Year	Median	25th Percentile	75th Percentile
2000	0.0978	0.352	0.484
2001	0.0441	0.289	0.446
2002	0.0333	0.310	0.542
2003	0.0268	0.314	0.518
2004	0.0310	0.360	0.613
2005	0.0264	0.386	0.652
2006	0.0147	0.360	0.729
2007	0.0119	0.258	0.663
2008	0.00956	0.172	0.595
2009	0.0154	0.194	0.707
2010	0.0447	0.415	0.932
2011	0.129	0.531	0.956
2012	0.126	0.728	0.988
2013	0.253	0.919	0.991
2014	0.398	0.911	0.996
2015	0.446	0.930	0.997
2016	0.374	0.903	0.995
2017	0.331	0.862	0.994
2018	0.243	0.744	0.991
2019	0.186	0.640	0.992

Time varying summary statistics of the transition function $g(q_{it}; \gamma, c)$ in a PSTR. The transition variable is laggeddebt to GDP ratio.

Next, we proceed to estimate our model characterised by two regimes. Table 6 presents the estimation results. It can be observed that the reaction coefficient is positive and statistically significant only in the high debt regime while it is positive but not significant in the low debt regime, indicating sustainable fiscal behaviour only for high debt situations in the euro area. Spending deviations $GVAR$ exceed a statistical significant negative effect on the primary balance in both regimes, while inflation is only significant in high debt situations and trade effects for low debt situations. Interestingly, the effect of the business cycle $YVAR$ varies: while in the low debt regime there is a positive effect, indicating counter-cyclical behaviour, for the high debt regime the impact on the primary balance turns out to become pro-cyclical. The data-driven threshold value for discriminating between the high and the low debt regime is 62%, which is about the reference value according to the Maastricht Treaty.

Table 6: Estimation results PSTR 1

Variables	Coefficient	Standard error
Regime I - Low debt ratio		
Laggeddebt ratio	0.0142	0.0158
GVAR	-0.0031***	0.0002
YVAR	0.0002***	0.000
Inflation	0.0029	0.0037
Net export	0.0004***	0.000
Regime II - High debt ratio		
Laggeddebt ratio	0.0639***	0.0058
GVAR	-0.0007***	0.000
YVAR	-0.0002***	0.000
Inflation	-0.0572***	0.0022
Net export	0.00007	0.00004
Threshold value(c)	0.62	0.020
Number of observations	360	

Estimation of $y_{it} = \mu_i + \lambda_t + \beta_0 x_{it} + \beta_1 x_{it} g(q_{it}; \gamma, c) + u_{it}$ using PSTR. Where *** indicates statistical significance of at most 5% in the strict sense.

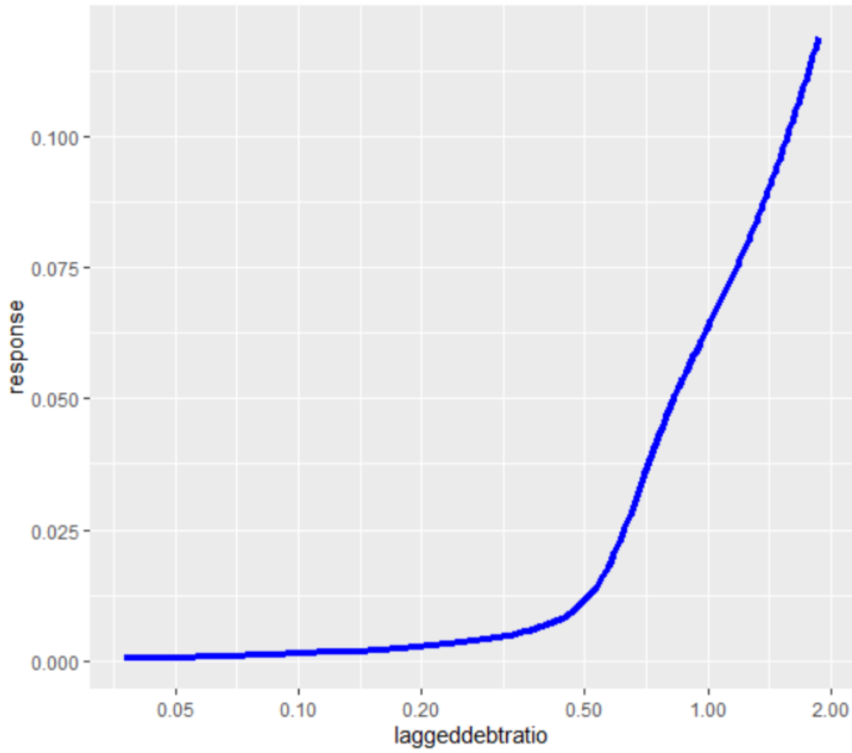
In figure 3, we fit our PSTR to the data and visualise the relationship with the response variable (primary balance). The graph supports a two regime model perfectly so that in the first regime with debt less than 60%, we notice a fairly flat fit indicating a weak response of the primary balance to increment in the debt to GDP ratio. Conversely, a debt ratio of above 60 percent warrants a strong response by way of increasing the primary balance which is supportive of our estimation results. The threshold value of about 62% indicates a turning point of the reaction function that can be seen in the graph.

Next, we proceed to conduct a misspecification test also known as the test of no-remaining heterogeneity. By this test, we examine if the two-regime PSTR is an adequate model for our data set with the lagged debt ratio as the transition variable or if there exists additional unmodelled heterogeneity. Results from table 9 suggest that the estimated model with one transition is appropriate since the p-values of the LM test (both chi-square and F-distribution) indicate a rejection of the null hypothesis of an additional transition function (regime).

We also present some residual diagnostics checks, where the residual is plotted against the fitted variables to check for potential heteroskedasticity. The right panel in figure 5 in the appendix shows that the residuals are centred around zero with no specific patterns and, hence, the absence of heteroskedasticity. The quantile-quantile plot on the left panel of figure 5 shows that the residuals are fairly distributed with few outliers.

Since the countries belong to an economic and monetary union, the likelihood of cross-

Figure 3: Fitting PSTR



sectional correlation is quite high since such countries could take similar policy decision, for instance responding to shocks similarly. Cross-sectional dependence affects the size properties of most econometrics test and, hence, renders inferences invalid (Banerjee et al., 2004). Figure 6 depicts a residual plot of the PSTR model where we look for potential cross-sectional dependence in the country-specific residual plot. One can observe that the residuals are mostly unique (do not look alike) which rules out the possibility of cross sectional dependence from our estimated PSTR model. Finally, we visualise our fitted model according to PSTR and compare it against the actual primary balance in figure 7 in the appendix. It can be observed that both plots are similar which suggests that our estimated model provides a good approximation to the actual data and, hence, a good fit.

4.1 Sensitivity analysis and Robustness checks

We conduct a sensitivity analysis to ascertain how changes in the control variables affect our debt sustainability parameter. From table 10 (specification A) in the appendix, we begin by estimating a model with only lagged debt as the regressor after which we add the other variables to the model. Results point to a lack of sustainability in the low debt regimes, whilst fiscal sustainability is given in the high debt regime. The same can be said about specifications B to D which all confirm that varying the control variables does not change the main message. At low debt levels, the reaction function is not statistically significant to infer sustainability. However, as debt crosses a particular threshold, we notice a strong response in terms of adjustment of primary balance to infer sustainability. We also conduct a robustness test where we consider an alternative to the HP filter used for detrending the GDP and real government spending. In a seminal paper, Hamilton (2018) criticised the HP filter by pointing out its limitations. Firstly, he identified discrepancies between filtered values in the middle of the sample and at the end of the sample. Secondly, the HP induces a spurious dynamic relationship which does not have a basis relating to the data generating process. We use the filter proposed by Hamilton (2018) to construct GVAR and YVAR variables and re-estimated the model. It can be observed from table 7 that the results are similar to our main results in table 6. Fiscal sustainability is not given in the first regime of low debt whilst the second regime of high debt is found to be sustainable by way of a positive and statistically significant reaction coefficient of the lagged debt ratio. This points to the robustness of our model results to changes in the cyclical variables.

Table 7: Robustness Test - Using alternative filter for GVAR and YVAR

Variables	Coefficient	Standard error
Regime I - Low debt ratio		
Laggeddebt ratio	0.01196	0.0098
GVAR	0.0005***	0.00004
YVAR	-0.0003	0.0000
Inflation	0.00723	0.004238
Net export	0.00001***	0.000
Regime II - High debt ratio		
Laggeddebt ratio	0.0740**	0.01257
GVAR	-0.0002***	0.000
YVAR	0.0002***	0.000
Inflation	-0.0674***	0.0119
Net export	0.0002***	0.000
Threshold value(c)	0.651	0.041
Number of observations	306	

Estimation of $y_{it} = \mu_i + \lambda_t + \beta_0 x_{it} + \beta_1 x_{it} g(q_{it}; \gamma, c) + u_{it}$ using PSTR. Where *** indicates statistical significance of at most 5% in the strict sense. potential GDP and potential spending used for computing YVAR and GVAR respectively were constructed using the filter according to Hamilton (2018).

4.2 Estimation of sub-sample

In this subsection, we consider the estimation of a special group of countries in the euro area with relatively high debt to GDP ratios. These are countries which mainly run into high debt problems necessitating a bail-out intervention by the European Union, the European Central Bank and the IMF. They include Greece, Ireland, Italy, Portugal and Spain also referred to as GIIPS groups of countries. Using the same sample period 2000-2019, we estimate a PSTR model for this group of countries.

The results from table 8 indicate a different pattern compared to the whole euro area: the reaction coefficient is negative and statistically significant in the first regime indicating non-sustainable behaviour if the debt ratio is low. Thus, those economies played a Ponzi game: Instead of raising the primary surplus in order to bear the higher debt service as public debt rises, they had raised the deficit still further. However, the reaction coefficient becomes positive and significant in high debt situations showing a switch towards a sustainable fiscal policy design in high pressure debt situations. This is also supported by the magnitude of the coefficient: compared to the whole euro area results in table 6 the size of this coefficient almost doubled. The plots of the debt ratios for this sub-sample group of countries in figure 1 support the outcome: high and increasing debt ratios until and within the crisis and stabilisation afterwards, which could be assigned to the sustainable behaviour.

Further, public spending has again a significant negative effect in both regimes, while trade exerts a positive effect on the primary balance, and the business cycle effect indicates to be rather small and counter-cyclical, respectively. The effect of inflation changes as it has a positive effect in low debt regimes while it becomes strongly negative for high debt situations in the group of the GIIPS economies. These results clearly reflect the difficult situation of these economies being hit hard by the debt crisis. This also becomes visible by the threshold value, which distinguishes the low debt regime from the high debt regime: it has risen to 96.5% of GDP for this group of countries. These results are in line with Legrenzi and Milas (2011) also finding high debt ratio thresholds for Italy and Greece of about 90%, but, their results refer to a historical time series and consider only single countries.

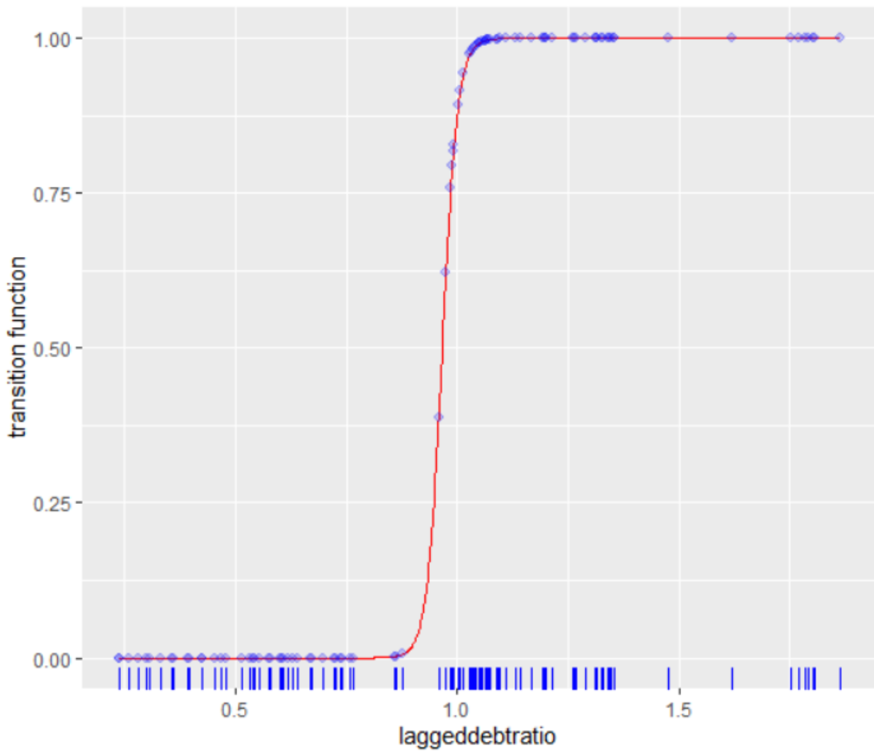
Figure 4 captures the transition function for the model estimated in table 8 using PSTR. It can be realised that most of the distribution of the transition variable are concentrated in the high debt regime with the threshold value close to 100%.

Table 8: PSTR estimation results for sub-sample - GIIPS group of countries

Variables	Coefficient	Standard error
Regime I - Low debt ratio		
Laggeddebt ratio	-0.0466***	0.0099
GVAR	-0.0032***	0.000
YVAR	0.0001**	0.000
Inflation	0.0403***	0.0148
Net export	0.00008***	0.000
Regime II - High debt ratio		
Laggeddebt ratio	0.1266***	0.0039
GVAR	-0.0007***	0.0002
YVAR	0.0001***	0.000
Inflation	-0.133***	0.0134
Net export	0.0003**	0.000
Threshold value(c)	0.965	0.009
Number of observations	100	

Estimation of $y_{it} = \mu_i + \lambda_t + \beta_0 x_{it} + \beta_1 x_{it} g(q_{it}; \gamma, c) + u_{it}$ using PSTR. Where *** indicates statistical significance of at most 5% in the strict sense.

Figure 4: Transition function for GIIPS group of countries



5 Conclusion

This paper empirically studies for debt sustainability for euro area economies based on the fiscal response approach by applying PSTR. This method allows to detect the existence of a threshold in the behaviour of the reaction function and refrains from the country-wise perspective. It applies a regime-switching model to detect non-linearities in the data generating process, where data is segregated data-driven into different regimes endogenously via a logistics regression.

Introducing PSTR to the analysis of the fiscal response function is new and overcomes the pooling problem where one coefficient fits all and where the data is separated country-wise. PSTR allows to distinguish debt regimes based on the data generating process across all states of the debt ratios, independent of the country. The advantage is obvious: the amount of regimes is determined by the data, the coefficients are estimated for each regime separately (here for high and low debt) and the response coefficients lead to a better fit than in regular panel regression models. In particular, they are not linked to countries but determined by the prevailing debt situations. This implies that in one country different responses are feasible according to the respective debt situations, i.e. the response turns out to be different in low debt periods compared to periods characterised by high public debt.

The application of PSTR to euro area economies for the years 2000 to 2019 reveals that there are two regimes, a low debt regime and a high debt regime. The relationship between the primary balance ratio and public debt is not homogeneous across the distribution of the debt ratios. A positive reaction coefficient indicating sustainability has been found for the high debt regime while it is insignificant for the low debt regime. The threshold value that separates the low from the high debt regime is about 60 %, in accordance with the requirements of the Maastricht Treaty. Several robustness tests support these findings. Interestingly, the situation for a sub-sample of the highly indebted GIIPS economies, that suffered heavily during the debt crisis, is different: the low debt regime is characterised by a statistically significant negative reaction coefficient, indicating an unsustainable debt policy, while the estimated response for the high debt regime is statistically significant and positive suggesting a sustainability of public debt.

The PSTR addresses the debt sustainability issues well. It does provide a tool to analyse the relationship between the primary balance ratio and debt to GDP. The method does not account for a dynamic relationship in the data, however, it is not a problem in this setting because the primary balance and debt do not constitute a dynamic relationship. So further research should try to overcome these limits if it is required for the particular study design.

The policy implications derived from our analysis suggest that governments should carefully assess their prevailing debt situations in order to conduct fiscal policies that guarantee sustainable levels of public debt. The results also indicate that the initial debt level matters for the fiscal response and influences the relationship, this can be seen particularly in the results of the GIIPS countries. We observe a relatively high debt ratio in the low debt regime (with threshold of about 100%). This implies that at a very high debt level, even larger primary balances must be generated in order to infer sustainability. This is rather difficult if policy makers do not have enough fiscal space to respond to the very high debt levels, in which case they would have to solicit for external support for instance from the IMF, World Bank etc.

References

Afonso, A. (2005) “Fiscal Sustainability: The Unpleasant European Case." *FinanzArchiv/Public Finance Analysis*, Vol. 61 (1): 19-44

Afonso, A. and J.T. Jalles (2019) “Fiscal Reaction Functions Across the World: In Quest of Statistical (In)significance.” *FinanzArchiv/Public Finance Analysis*, Vol. 75 (3): 207-228

AMECO (2021), “European Commission’s Directorate General for Economic and Financial Affairs macro-economic database”, Available at `https://ec.europa.eu/info/business-economy-euro/indicators-statistics/economic-databases/macro-economic-database-ameco/ameco-database_en`, [Accessed on 08.03.2021]

Banerjee, A., Marcellino, M., & Osbat, C. (2004). "Some cautions on the use of panel methods for integrated series of macro-economic data" *Econometrics Journal,* Vol. 7(2), pp. 322-34.

Barro, R. (1979), “On the Determination of Public Debt”, *Journal of Political Economy,* vol. 87, no. 5, pp. 940 - 971

Beqiraj, E., Fedeli, S. and Forte, F. (2018), "Public debt sustainability: An empirical study on OECD countries", *Journal of Macroeconomics*, vol. 58: 238-248

Bohn, H. (1995) “The sustainability of budget deficits in a stochastic economy.” *Journal of Money, Credit and Banking*, Vol. 27 (1): 257-271.

Bohn, H. (1998) “The behavior of U.S. public debt and deficits.” *Quarterly Journal of Economics*, Vol. 113: (3) 949-963.

Checherita-Westphal, C., Zdarek, V. (2017). "Fiscal reaction function and fiscal fatigue: evidence for the euro area" . *ECB Working Paper* (No. 2036).

Fournier, J.-M., Fall, F. (2015) "Limits to government debt sustainability". OECD, OECD Economics Department Working Papers, No. 1229, July 2015.

Ghosh, A.R., J.I. Kim, E.G. Mendoza, J.D. Ostry and M.S. Qureshi (2013) “Fiscal fatigue, fiscal space and debt sustainability in advanced economies.” *The Economic Journal*, Vol. 123 (566): F4-F30.

Gonzalez, A. T. Teräsvirta, Van Dijk, D., Yang, Y. (2017). “Panel smooth transition regression models." *CREATES Research Paper 2017-36*, pp. 1-47.

Granger, C.W.J. (2008) “Non-linear models: Where do we go next - Time varying parameter models?” *Studies in Nonlinear Dynamics & Econometrics,* Vol. 12, Issue 3, Article 1: 1-9.

Greiner, A. and B. Fincke (2015) *Public Debt, Sustainability and Economic Growth. Theory and Empirics.* Springer Verlag, Cham.

Hamilton, J.D. (2018). “ Why you should never use the Hodrick-Prescott filter" *Review of Economics and Statistics,* Vol. 100(5), pp. 831-843.

Legrenzi, G. and C. Milas (2011). "Debt Sustainability and Financial Crises: Evidence from the GIIPS". CESifo Working Paper Series No. 3594, Available at SSRN: https://ssrn.com/abstract=1938926 or http://dx.doi.org/10.2139/ssrn.1938926

Owusu B., B. Bökemeier and Greiner, A. (2021), "Assessing Non-Linearities and And Heterogeneity in Debt Sustainability Analysis: A Panel Spline Approach," *Bielefeld Working Papers in Economics and Management*, No. 10-2021, https://doi.org/10.4119/unibi/2954860

Teräsvirta, T. (1994), “Specification, estimation, and evaluation of smooth transition autoregressive models," *Journal of the American Statistical Association,* 89 (425), 208 - 218.

Teräsvirta, T. (1998), “Modelling economic relationships with smooth transition regressions," *in Handbook of applied economic statistics, ed. by A. Ullah, and D. E. A. Giles,* pp. 507 - 552. New York: Marcel Dekker.

Zeileis, A., Kleiber, C., Krämer, W., and Hornik, K (2003). "Testing and dating structural dates in practice" *Computational Statistics and Data Analysis*, Vol. 44, pp. 109 -123.

Appendix

Table 9: Misspecification test (No remaining heterogeneity)

	LM_{χ}		LM_F	
m	test	p-value	test	p-value
$m = 1$	33.59	0.000	3.163	0.000

Results of LM sequence of homogeneity test based on chi-square and F distribution. The hypothesis is based on the following; $H_0^* : \beta_3^* = \beta_2^* = \beta_1^* = 0$, $H_{03}^* := 0$,$H_{02}^* : \beta_2^* = 0| = \beta_3^* = 0$ and finally $H_{01}^* : \beta_1^* = 0| = \beta_3^* = \beta_2^* = 0$

Figure 5: Residual diagnostics

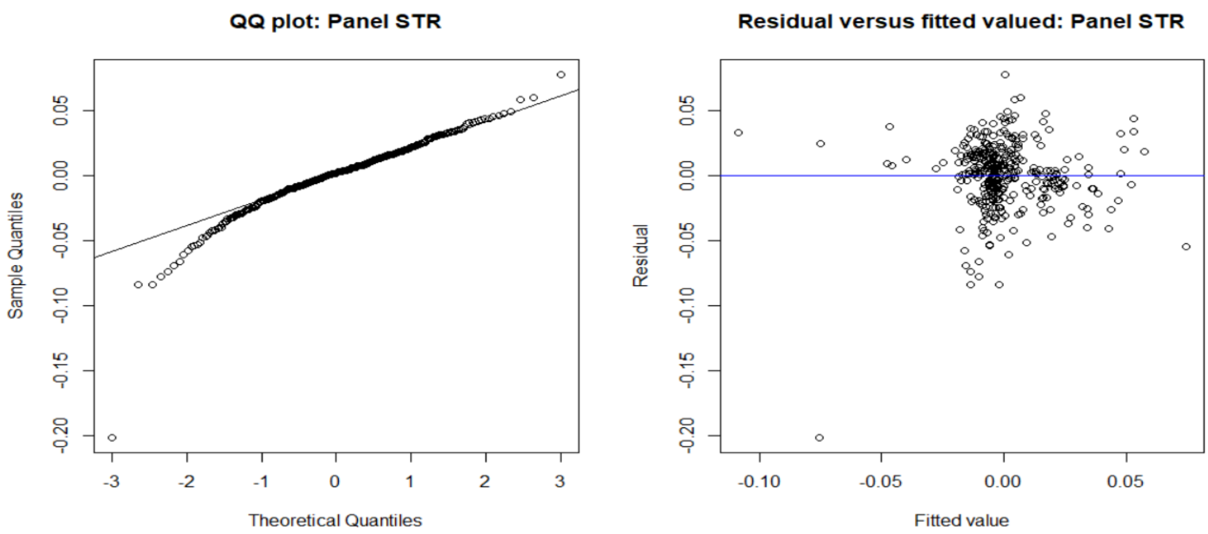


Figure 6: Country-specific residual plot

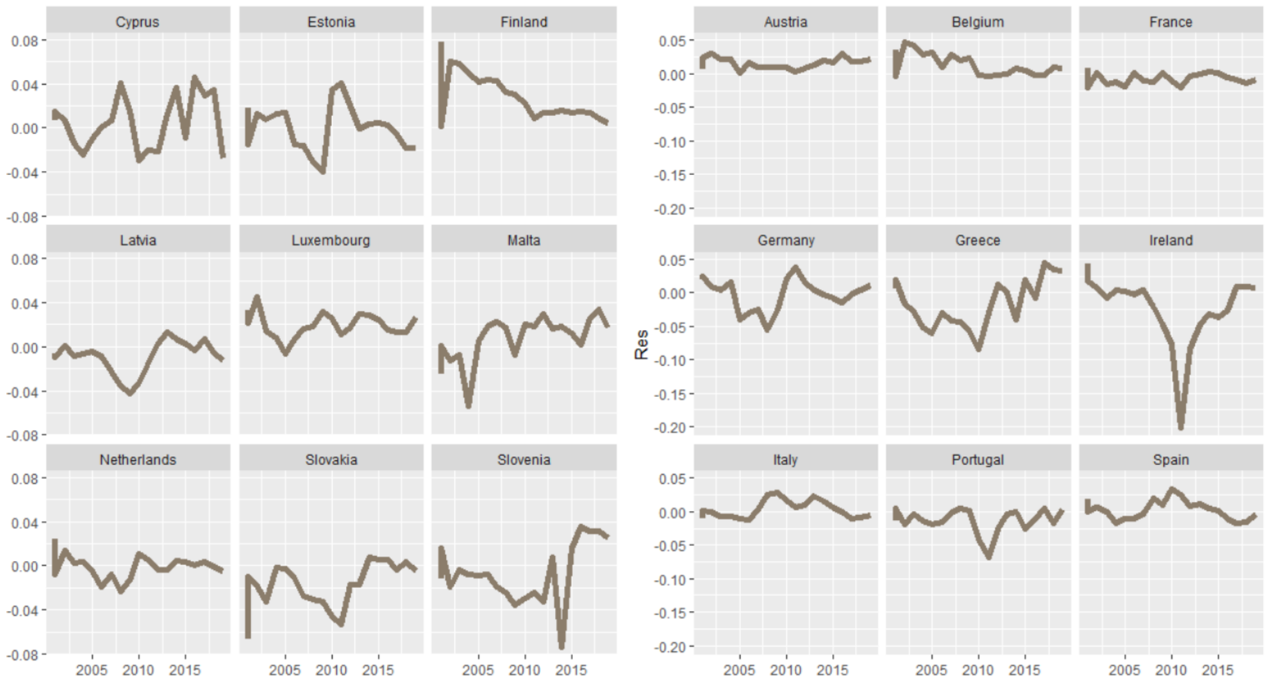


Figure 7: Actual and fitted primary balance

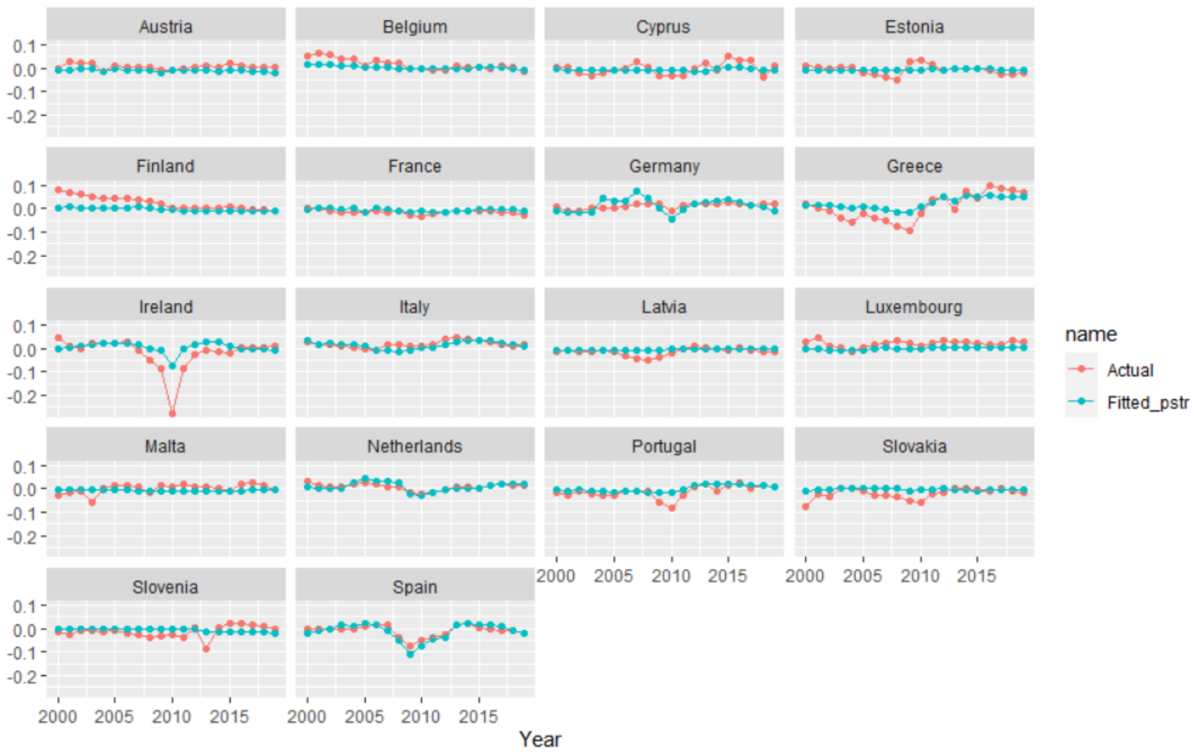


Table 10: Sensitivity Analysis

Variables	Coefficient	Std error		Coefficient	Std error
		Model A			
Regime I			**Regime II**		
Laggeddebt ratio	0.0112	0.0095		0.0171***	0.001
Number of observations	360			360	

Variables	Coefficient	Std error		Coefficient	Std error
		Model B			
Regime I			**Regime II**		
Laggeddebt ratio	0.0124	0.2391		0.06975***	0.0212
Net export	0.0003	0.0011		0.00017	0.00037
Inflation	-0.0019	0.0082		-0.0696***	0.0383
Number of observations	360			360	

Variables	Coefficient	Std error		Coefficient	Std error
		Model C			
Regime I			**Regime II**		
Laggeddebt ratio	0.0119	0.1461		0.0687***	0.0153
Net export	0.0003	0.0007		0.00015	0.00022
Inflation	-0.0017	0.02054		-0.0683***	0.0210
YVAR	0.00001	0.0001		-0.0001***	0.000
Number of observations	360			360	

Variables	Coefficient	Std error		Coefficient	Std error
		Model D			
Regime I			**Regime II**		
Laggeddebt ratio	0.0119	0.0230		0.0179***	0.0051
Net export	0.0005***	0.0003		-0.00003	0.00006
YVAR	0.0003	0.0002		-0.0002***	0.0001
GVAR	-0.0032***	0.0006		-0.0009	0.0006
Number of observations	360			360	

Estimation of $y_{it} = \mu_i + \lambda_t + \beta_0 x_{it} + \beta_1 x_{it} g(q_{it}; \gamma, c) + u_{it}$ using PSTR. Where *** indicates statistical significance of at most 5% in the strict sense.

Table 11: Structural break dates based on test by Zeileis et al (2003) - Debt to GDP ratio

Countries	Break date
Austria	2008
Belgium	2009
Bulgaria	2011
Cyprus	2011
Estonia	2008
Finland	2011
France	2008
Germany	2008
Greece	2009
Ireland	2008
Italy	2011
Latvia	2008
Luxembourg	2007
Malta	2014
Netherlands	2008
Portugal	2009
Slovakia	2011
Slovenia	2011
Spain	2011