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Autonomous vehicles policy and safety investment: an equilibrium analysis with endogenous demand

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Abstract

The safety concerns for autonomous vehicles (AV) is shown a to be roadblock to their adoption. This paper addresses these concerns by studying a unified, game-theoretic framework (leader-follower game) of mixed traffic in which AVs and human-driven vehicles (HV) coexist, with endogenous vehicle demand and different types of accidents emerging in mixed traffic as crucial building blocks. We study the interaction between three types of players: (i) a policymaker, who decides on the liability regime and the level of V2I connectivity infrastructure, (ii) an AV producer, who decides on the AV price and safety level, and (iii) consumers, who differ in their preference for each vehicle type and choose the one they like best. Using both analytical and numerical tools, we analyze how the two policy variables, liability and V2I connectivity, affect behavior on the demand and supply side of the vehicle market and, in turn, AV market penetration and overall road safety. We also characterize optimal policies, thereby taking into account the market participants' behavioral responses. Our findings provide guidance for a fast adoption of AVs and a smooth transition from existing traffic conditions to a mixed traffic environment, and assist in decision making for policymakers, legal agencies, traffic operation and transportation planning agencies, as well as car manufacturers.

Keywords: Mixed traffic, autonomous vehicles, endogenous AV demand, road safety, AV liability, AV market penetration

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1 Introduction

1.1 Motivation

In today's transportation sector, vehicles already show a remarkable degree of automation and even fully autonomous vehicles (AVs) are widely believed to become available in the not too distant future (see e.g. European Commission, 2018).[1] AVs are perceived as potentially being much safer than conventional, human-driven vehicles (HVs) in the long run, in addition to further benefits such as improved traffic flows, better time use en route, and greater mobility of the elderly (see e.g. Fagnant and Kockelman, 2015).

The emergence of AVs gives rise to *mixed traffic*, the coexistence of AVs and HVs on the streets, potentially lasting for decades. This paper addresses several key properties of an automobile sector characterized by mixed traffic: First, mixed traffic gives rise to different types of accidents between AVs and HVs, some caused by the human drivers of HVs, others caused by the autonomous systems of AVs. For the latter, a currently topical question is how to apportion the damage between the AV producer, the AV owner/passenger and the victim(s) by resorting to (product) liability (see e.g Geistfeld, 2017; Shavell, 2020; Di et al., 2020). Moreover, the liability regime will also affect the incentives of AV producers to invest in the safety of their vehicles in the first place (see e.g. Dawid and Muehlheusser, 2022). Together, these two channels render AV liability a crucial task for policymakers.[2]

Second, a further key property of mixed traffic is that consumers have a choice between HVs and AVs. In particular, how quickly AVs will penetrate the market and become ubiquitous on the streets will not only depend on technological feasibility, but also on how much consumers like them. In this respect, a large body of empirical (survey) evidence documents that consumers differ vastly regarding their attitudes towards AV and their willingness to adopt them, and crucial determinants in this respect are liability, vehicle safety and price, and personal attributes (e.g. Kyriakidis et al., 2015; Shabanpour et al., 2018; Cunningham et al., 2019). This suggests that AV demand and market penetration and, consequently, overall road safety will crucially depend on these factors, and how they are addressed by manufacturers and policymakers.

Third, a higher AV market penetration can also be expected to foster road safety trough further channels. For example, AVs can better "communicate" with one another than with HVs thereby reducing the accident risk between AVs (*connectivity*). As individual consumers will tend to not fully take into account such positive spillover effects in their vehicle choice, this suggests a role for public investments in vehicle-to-infrastructure (V2I) connectivity infrastructure. Such investments enhance the attractivity of AVs for consumers, and thus will increase market demand. This in turn makes it more attractive for the AV producer to increase market supply.

In light of these inter-dependencies, it is important to gain a better understanding of how regulatory policies such as the liability regime and the availability of V2I infrastructure affect behavior on the demand and supply side of the vehicle market and, in turn, the mixed traffic structure and overall road safety. In this paper, we study a unified game-theoretic framework of mixed traffic that allows us to take all of these building blocks into account, thereby providing a set of novel results.

[1] Tesla already offers a "Full Self Driving" package since several years; however, drivers must always be ready to immediately take over control. According to the classification system of the Society of Automotive Engineers (SAE), this corresponds to autonomy level 2 (out of 5), see SAE International (2021). In 2021 Mercedes introduced its "Drive Pilot" system, where the human driver is not obliged to monitor the driving at all times, but must only be ready to take over after being prompted by the system (level 3). The system is currently approved for motorways and with a speed of up to 60 km/h. In June 2022, the *United Nations Economic Commission for Europe* (UNCE) has extended the maximum speed to 130 km/h (effective as of 2023) for vehicles which satisfy the respective requirements, see `https://unece.org/sustainable-development/press/un-regulation-extends-automated-driving-130-kmh-certain-conditions`.

[2] A further factor might be the imposition of a minimum safety standard (set by the policymaker) which AVs must satisfy in order to be allowed to be launched on the market (see e.g. Dawid and Muehlheusser, 2022).

1.2 Related work

Our paper contributes to various strands of literature related to AVs that have either studied these building blocks in isolation, or have focused on other aspects of AV behavior in mixed traffic.

First, an extensive empirical literature studying attitudes towards AVs has documented that people differ strongly with respect to their willingness to adopt AVs (see e.g. Schoettle and Sivak, 2014; Kyriakidis et al., 2015; Shabanpour et al., 2018; Cunningham et al., 2019; Wu et al., 2020). Many consumers are concerned about AV safety, vehicle price, liability, and data security, while the perceived benefits from AVs include a higher fuel efficiency, or a more productive use of time during travel. All in all, people have quite diverse perceptions on these factors (see e.g. the surveys by Haboucha et al., 2017; Gkartzonikas and Gkritza, 2019; Jing et al., 2020) that, importantly, are also affected by the choices made by AV producers (e.g. vehicle price and safety) and policymakers (e.g. liability). Dawid and Muehlheusser (2022) are the first to explicitly model the demand side, using a standard framework in industrial economics due to Hotelling (1929) and Salop (1979) where consumers have different preferences regarding horizontally differentiated products (AVs and HVs). As a result, the market shares of AVs and HVs arise endogenously from the players' choices in the game. Their (dynamic) framework focuses on the impact of product liability on the incentives to invest in AV safety as well as the timing of AV market introduction and AV market penetration over time. Feess and Muehlheusser (2022) study a game-theoretic model where the choice between AVs and HVs depends on behavior of AVs in situations of moral dilemma (swerving in avoidable accidents), but they do not consider a full-fledged market setting.

Second, with respect to the literature on product liability, apart from compensating victims for their harm suffered, one crucial question is whether the threat of liability increases firms' incentives to improve product safety. McGuire (1988) provides supportive (survey) evidence in this respect.[3] Polinsky and Shavell (2010) stress that firms have a high incentive to invest in product safety even in the absence of product liability; otherwise, the higher liability costs borne by consumers reduces their willingness to pay for the product, inducing a downward shift of demand. In the context of AVs, legal scholars have since long argued that the emergence of AVs raises important questions regarding liability (see e.g. Geistfeld, 2017; Smith, 2017; Wagner, 2018; Gless et al., 2016).

In recent years the impact of AV liability has also been studied in formal (game-theoretic) models, focusing on the comparison of the two core liability regimes in tort law, strict liability and fault-based liability, and variants thereof. For example, Shavell (2020) considers a case of full AV market penetration (i.e. no mixed traffic) and proposes a liability rule that holds the AV owner strictly liable for all accidents involving the AV, but the damage payments are made to the state, rather than to parties harmed in the accidents. The underlying rationale for this "double liability" rule is the possibility of aligning privately and socially optimal behavior with respect to both (driver) precaution and activity levels. Guerra et al. (2022) study the role of manufacturer's residual liability in this respect. Schweizer (2023) generalizes the analysis of Shavell (2020) and Guerra et al. (2022), thereby stressing the potential benefits of AVs in making vehicle behavior observable ex post in court. In settings of mixed traffic, Chatterjee and Davis (2013) and Chatterjee (2016) analyze how varying the loss share with contributory or comparative negligence would distort human's interaction with AVs. Friedman and Talley (2019) employ a multilateral precaution framework to explore how tort law should adapt to the emergence of AVs in mixed traffic. The potentially optimal legal rules include no fault, strict liability, and a family of negligence-based rules. Di et al. (2020) further study how

[3]Rather than improving the safety of existing products, the literature has also analyzed (both theoretically and empirically) how product liability affects firms' incentives to develop new, and potentially safer, products (see e.g. McGuire, 1988; Viscusi and Moore, 1993; Galasso and Luo, 2017, 2022; Schwartzstein and Shleifer, 2013).

AV manufacturers could strategically select AVs' safety level using a hierarchical game-theoretical model. In all of these models, AV demand is either not considered nor exogenously given. By contrast, in their framework with endogenous demand, Dawid and Muehlheusser (2022) compare strict and fault-based liability with respect to firms' incentives to invest in product safety and AV market penetration over time, thereby also capturing the channel emphasized by Polinsky and Shavell (2010). De Chiara et al. (2021) consider different liability rules in a static framework in which consumers choose between HVs and AVs, thereby not facing any liability risk when choosing the latter.

A third strand of literature studies the design of autonomous driving strategies in mixed traffic. In particular, using game-theoretic models to design algorithmic decision-making processes for AVs has gained increasing traction in various car encounters, namely, driving (Yoo and Langari, 2012; Huang et al., 2019, 2020a,b, 2021), merging (Yoo and Langari, 2013), lane-changing (Yu et al., 2018; Zhang et al., 2019), and unprotected left-turning behavior (Rahmati and Talebpour, 2017), with the game models categorized as either a two-person non-zero-sum non-cooperative game under (in)complete information (Talebpour et al., 2015), a Stackelberg game (Yoo and Langari, 2012, 2013; Yu et al., 2018; Zhang et al., 2019), or a dynamic mean field game (Huang et al., 2019, 2020a,b, 2021). A detailed survey of mixed traffic modeling using game theory and artificial intelligence methods is provided by Di and Shi (2021). These studies, however, primarily focus on improving traffic efficiency, and they abstract from the possibility of accidents and from economic considerations.

1.3 Framework and results

Against this background, this paper is the first to analyze the interaction among three crucial players in a mixed traffic setting – policymakers, vehicle manufacturers, and consumers – employing a unified, game-theoretic approach.

As for the vehicle demand side, we follow Dawid and Muehlheusser (2022) where consumers can choose between HVs and AVs, and each consumer's preferred vehicle depends on idiosyncratic preferences, price, safety, and potential liability costs in case of accidents. There are four different accident types, AV-AV, AV-HV, HV-AV, and HV-HV (e.g., AV-HV refers to an accident between an AV and an HV that is caused by the AV), and consumers' vehicle choice affects the mixed traffic composition and hence the prevalence of each accident type. As for the supply side, we consider a monopolistic AV producer who decides on the AV's price and safety (a higher safety level is costly, but makes accidents involving AVs less likely), both of which affect consumers' demand for AVs. The HV is provided by a competitive fringe of producers which we do not explicitly model. Finally, we consider a policymaker who decides on (i) the stringency of (product) liability for the AV producer for accidents caused by AVs, and (ii) how much to invest to improve V2I connectivity that reduces the likelihood of accidents in AV-AV interactions.

Consumers and the AV producer aim at maximizing their utility and profit, respectively, while the policymaker aims at minimizing the sum of the social costs from accidents and the costs of providing V2I connectivity infrastructure. From a methodological point of view, we employ a game-theoretic approach by considering a leader-follower game in which the policymaker moves first, followed by the AV producer and the consumers. We use backward induction to determine the subgame perfect equilibrium of the game, thereby applying both analytical and numerical tools.[4]

The equilibrium analysis reveals how the AV market penetration, the mixed traffic structure, and overall road safety depends on the choices of all players, as well as on the model parameters.

[4]The concept of subgame perfection is a standard tool in the analysis of dynamic games with complete information, see e.g. Fudenberg and Tirole (1991).

The remainder of the paper is organized as follows. Section 2 presents the game-theoretic model that accounts for the policymaker, the AV producer, and consumers. With respect to equilibrium behavior at different stages of decision-making, Sections 3 and 4 contain our findings based on analytical and numerical analysis, respectively. Section 5 concludes and discusses potential future extensions. All proofs are in the Appendix.

2 The Model

2.1 General setup

We consider a setup with four types of agents: a policymaker, a producer of AVs, producers of HVs, and consumers. HVs are produced by a representative (or competitive) producer and sold at price $p_H \geq 0$, which we take as exogenously given. AVs are produced by a monopolistic firm, the *AV producer*, and sold at price $p_A \geq 0$, which is set by the AV producer. Apart from the price, the AV producer also decides on the level of AV safety, $x \geq 0$, which determines the frequency of accidents. There is a unit-mass of consumers which differ with respect to their preference between the HV and the AV. Each consumer purchases one vehicle, and the choice between the AV and the HV depends, apart from preferences, on the price as well as on the expected liability costs arising from accidents. The quantity of AVs on the street is denoted by Q, and therefore the quantity of HVs is given by $1 - Q$. The policymaker aims at minimizing the sum of the costs generated by accidents and by infrastructure investments. She has two instruments at her disposal: First, the allocation of liability between the AV producer and consumers, where we denote the share of accidental damage covered by the producer by $\beta \in [0, 1]$. Second, the level of connectivity infrastructure, denoted by $c \geq 0$, which fosters connectivity between AVs (*V2I connectivity*) and allows them to communicate with each other while en route (see e.g. USDOT, 2019).

2.2 Vehicular encounters in mixed traffic

Before delving into each agent's decision making, we first model accident rates in various vehicular encounters in mixed traffic. Both types of cars can cause accidents, each leading to a damage $D > 0$. Subsequently, we introduce how to formulate accident rates for AV-HV, AV-AV, HV-AV, and HV-HV scenarios.

- **AV-HV and AV-AV accidents**: We denote by $k(x) > 0$ the probability that an AV causes an accident when meeting an HV, which depends on the AV safety level (x), where $k'(x) < 0$ and $k''(x) > 0$. This leads to an expected damage of $k(x)D$ from AV-HV accidents. The probability that an AV causes an accident when meeting another AV is $k(x) - h(c)$, where $h'(c) > 0, h''(c) < 0$ and $k(x) > h(c)$ for all x and c. The function $h(c)$ captures the impact of the degree of (V2I) *connectivity* of AVs, making it less likely that an AV causes an accident when meeting an AV compared to an HV.
- **HV-AV accidents**: The probability that an HV causes an accident with an AV is $g(x)$, where $g'(x) < 0$ and $g''(x) > 0$. Intuitively, $g(x)$ depends on the safety level of the AV (x), because a safer AV can prevent some accident which might have been caused by the HV, e.g. due to inattention or careless behavior of the HV's driver.[5] Importantly, this will imply that any investment into AV safety also improves safety of the HV, the AV's rival product. This is a specific feature of AVs, and throughout we refer to it as the *rival externality*.

[5]Note that we do not explicitly model the care level of HV drivers. See the frameworks of Di, Chen and Talley (2020) and De Chiara et al. (2021), where this aspect is explicitly considered.

- **HV-HV accidents**: The probability that an HV causes an accident with another HV is $\overline{g}$, which is independent of the level of AV safety, since there are no AVs involved in these types of accidents.

Throughout we make the following assumption on the two accident functions $k(x)$ and $g(x)$:

Assumption 1. *(i)* $\overline{g} \leq g(0) < k(0)$, *(ii)* $\lim_{x\to\infty} \frac{k(x)}{g(x)} < 1$, *(iii)* $\lim_{x\to\infty} \frac{k(x)}{\overline{g}} < 1$, *(iv)* $|g'(0)| < |k'(0)|$, *(v)* $\frac{k'(x)}{g'(x)}$ *is strictly decreasing in* x *and* $\lim_{x\to\infty} \frac{k'(x)}{g'(x)} = 0$.

Figure 1: AV and HV safety depending on safety investment x.

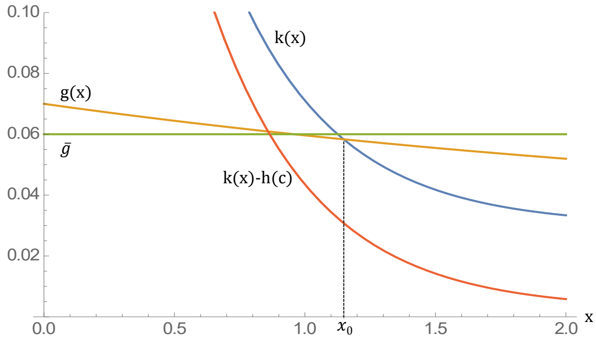


Figure 1 depicts the different accident probabilities, with the qualitative characteristics based on Assumption 1. Intuitively, for small AV safety the probability is higher for the AV to cause the accident than an HV in interactions between AVs and HVs (part (i)). Moreover, for large values of x the accident rate of an AV is smaller than that of an HV regardless of whether it interacts with an AV or an HV (parts (ii) and (iii)). Part (iv) formalizes that as long as the safety level of the AV is low a marginal increase in x more strongly reduces the accident rate of the AV itself than that of an HV. Finally, part (v) captures that for sufficiently large x, AVs are already so far advanced that further increasing x hardly reduces the probability that AVs cause accidents, but rather improves their ability to deal with errors of human drivers.

Below, we present the agents' decisions at the stage in which they are taken (see Figure 2 for an illustration). Since the focus of our paper is on the interplay between the AV producer producing the AV, the consumers, and the policymaker, we treat the (representative) HV producer as a passive party and take the HV price p_H as exogenously given.

2.3 Agents and their decisions

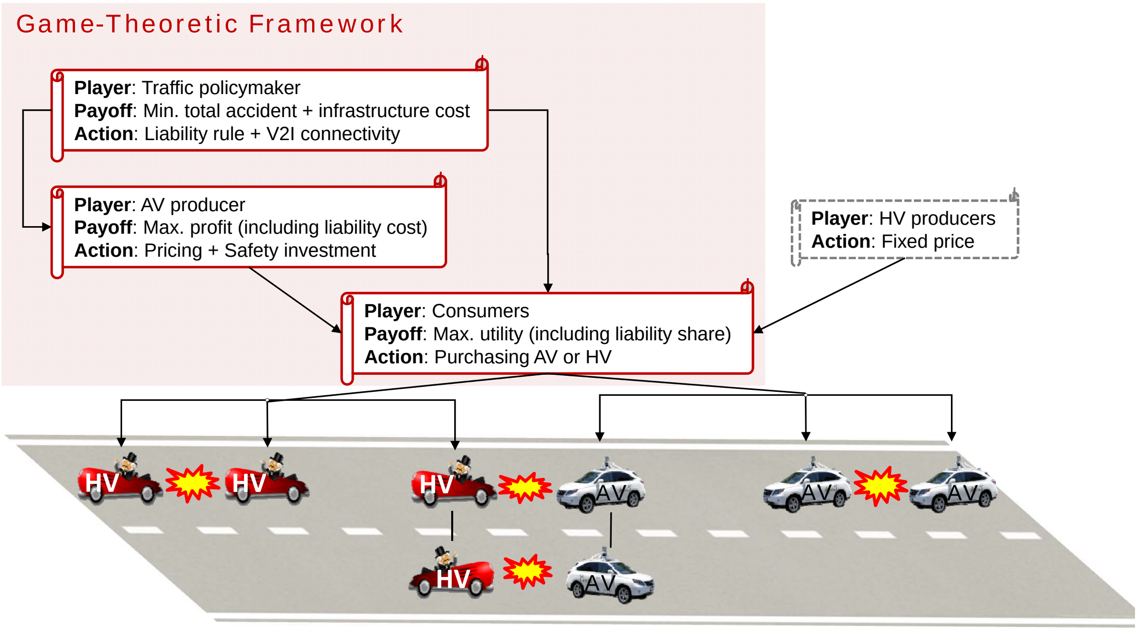
2.3.1 The policymaker

The policymaker has two policy variables at her disposal, the liability regime and the amount of infrastructure investment into V2I connectivity.

Liability rule A liability rule determines how the legal system allocates the damage from accidents between the parties involved. Thereby, we assume that the owner of an HV is responsible for the entire damage D caused by her vehicle. This is meant to reflect that, in the current situation with almost only HVs on the street, product liability plays only a minor role for apportioning the damage resulting from accidents.[6]

[6]While car manufacturers do face a (product) liability risk when a car model exhibits systematic technical defects, the vast majority of accidents are the result of erroneous driver behavior (see e.g. the 2008 *National Motor Vehicle Crash Causation Survey*). Hence, the damage from an accident are usually apportioned between the driver/owner of the vehicle causing the accident (and potentially their insurance company) and the parties harmed.

Figure 2: Illustration of the model structure



By contrast, for accidents caused by AVs, producers are expected to face substantially higher liability costs than they currently do with conventional cars. We follow Dawid and Muehlheusser (2022) and assume that a share β of the damage D is borne by the AV producer (*product liability*), where $\beta \in [0,1]$, while the AV owner/passenger is responsible for the remaining amount $(1-\beta)D$. In light of the current legal discussion summarized in Section 1, the design of liability regimes for AVs is an important policy variable.

Thereby, one has to keep in mind that consumers often do not fully internalize their liability share, for example, due to insurance policies with deductibles, or under-insurance (or even no insurance at all) in combination with wealth constraints. Also in line with Dawid and Muehlheusser (2022), AV owners actually only cover an amount of $\gamma((1-\beta)D) < (1-\beta)D$ with $\gamma' > 0, \gamma'' \leq 0, \gamma(0) = 0, \gamma'(0) \leq 1$.[7] Intuitively, whereas consumers strongly (or even fully) internalize small damages, the degree of internalization decreases as the damage becomes larger. Similarly, the owner of an HV covers an amount $\gamma(D) < D$.

V2I connectivity investment We assume that V2I connectivity reduces the probability of AV-AV accidents compared to AV-HV accidents. The cost of providing a level of AV connectivity c is giving by $\zeta(c)$, that is increasing and convex. For simplicity, throughout we consider a quadratic specification $\zeta(c) = \zeta_0 \cdot c^2$, with $\zeta_0 > 0$.

The objective of the policymaker is to minimize the sum of infrastructure and accident costs. Thereby, the policymaker takes into account the effect these decisions will have on the behavior of the AV producer and consumers. In particular, with Q AVs and $1-Q$ HVs on the street, the expected total number of

[7]For example, while insurance is mandatory in most U.S. states, uninsured driving is an empirically relevant phenomenon (see e.g. a recent study of the Insurance Research Institute, `https://www.insurance-research.org/sites/default/files/downloads/UMNR1005.pdf`). Moreover, for insurance contracts with a deductible, the marginal liability effect is equal to one for damages below the amount of the deductible, and zero above it. Finally, a plaintiff might even be *judgement-proof* when the damages owed exceed the amount covered by the insurance policy (plus eventual own funds) (see e.g., Gilles, 2006). In all of these cases (or combinations thereof), the actual liability costs is lower than the damage caused in the course of the accident. All we need for our analysis is that a consumer's expected liability cost increases under-proportionally in the damage amount. As will become clear below, when consumers face no restrictions with respect to their ability to make liability payments (i.e. when $\gamma((1-\beta)D) = (1-\beta)D$), then under linear demand, any shift of liability between the AV producer and the consumers is offset one-to-one by a respective price change.

accidents, denoted by $A(x)$, is given by

$$A(x, Q; \beta, c) = Q \cdot ((k(x) - h(c))Q + k(x)(1 - Q)) + (1 - Q) \cdot (g(x)Q + \bar{g}(1 - Q)) \,. \qquad (1)$$

Each term of (1) captures the expected number of accidents for each of the four interaction types (i.e. AV-HV, AV-AV, HV-AV, and HV-HV). The objective of the policymaker is to minimize the sum of accident and infrastructure costs, and is hence given by

$$\Psi(x, Q; \beta, c) = A(x, Q; \beta, c) \cdot D + \zeta(c). \qquad (2)$$

2.3.2 The AV producer

The AV producer chooses the AV price, p_A, and safety level, x, in order to maximize expected profit. The associated cost is $\xi(x) = \xi_0 \cdot x^2$, with $\xi_0 \geq 0$. For simplicity we set the (marginal) AV production costs to zero. Denoting by $Q^D(p_A, x; \beta, c)$ the AV demand for a given price p_A, the profit function of the AV producer is given by

$$\Pi(p_A, x; \beta, c) = Q^D(\cdot) \cdot \left[p_A - \beta D \cdot \left(Q^D(\cdot) \cdot (k(x) - h(c)) + (1 - Q^D(\cdot)) \cdot k(x)\right)\right] - \xi(x). \qquad (3)$$

The second term in the square bracket captures the AV producer's expected liability cost, taking into account the different probabilities for accidents caused by AVs when interacting with another AV or with an HV.

2.3.3 The consumers

One key contribution of our paper is to explicitly incorporate consumers' (utility-maximizing) choice between the different types of vehicles. In doing so, we consider a setting of *horizontal product differentiation*, i.e. consumers differ in their personal taste with respect to the ideal properties of a vehicle, expressed by their "bliss point". We follow a standard approach in industrial organization due to Salop (1979), in which a unit mass of consumer is uniformly distributed on a circle with circumference 1 with respect to their bliss points.[8] Without loss of generality, the HV and the AV are located at distance one-half on the top and bottom position of the circle, respectively (see Figure 3).[9] Each consumer has the same gross valuation $v > 0$ for each vehicle type.[10] The optimal purchasing decision will therefore depend on the (individual) relative attractiveness of each type of vehicle, which depends on vehicle prices, the expected costs from accidents, and the *preference* costs, i.e. the reduction in a consumer's utility when the vehicle characteristics do not match her bliss point. Formally, for a consumer with bliss point y, we denote by $d_A(y)$ and $d_H(y)$ the "distance" along the Salop circle between the bliss point and the product position of the AV and HV, respectively. The reduction in utility is proportional to this distance with a sensitivity parameter $t > 0$. The smaller (larger) this distance, the higher (lower) is ceteris paribus the consumer's willingness to purchase the respective vehicle type.

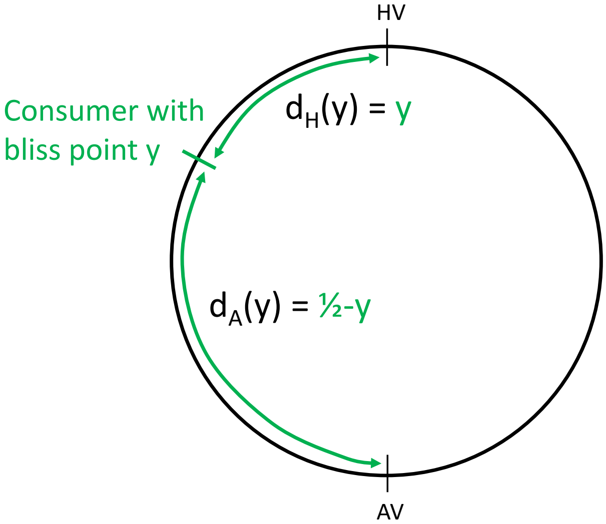
Denoting by $u_A(y, Q)$ and $u_H(y, Q)$ the expected utility of a consumer with bliss point y when pur-

[8]The concept of horizontal product differentiation goes back at least to Hotelling (1929)'s seminal model with competing ice-vendors on a beach, the famous "Hotelling line". The "Salop circle" is a well-established variant of the Hotelling line which, for our purpose, is slightly more convenient from an analytical point of view. Both models are canonical textbook material in industrial organization, see e.g. Tirole (1988).

[9]That is, we do not model here the location decision of the innovator in the product space.

[10]As is standard in the literature, assuming v to be sufficiently large ensures that each consumer purchases one of the two vehicles types, so that the total vehicle demand is always equal to one.

Figure 3: Consumer choice in the horizontally differentiated market for AVs and HVs



chasing the AV and the HV, respectively, we have

$$\begin{aligned} u_A(y,Q) &= v - td_A(y) - p_A - [Q \cdot (k(x) - h(c))\gamma((1-\beta)D) + (1-Q) \cdot k(x)\gamma((1-\beta)D)] \\ u_H(y,Q) &= v - td_H(y) - p_H - [Q \cdot g(x)\gamma(D) + (1-Q)\overline{g}\gamma(D)]. \end{aligned} \tag{4}$$

Note that, through the expected liability costs, individual utility depends on the overall number of AVs (Q) and HVs ($1-Q$) on the street.

Each consumer chooses the product which gives her the higher utility. Denoting by $P_A(y)$ the probability that a consumer with bliss point y optimally chooses an AV, we have[11]

$$P_A(y,Q,p_A,x) = \begin{cases} 1 & u_A(y,Q) > u_H(y,Q), \\ 1/2 & u_A(y,Q) = u_H(y,Q), \\ 0 & u_A(y,Q) < u_H(y,Q). \end{cases} \tag{5}$$

Finally, the AV market demand for a given AV price and safety level is then obtained by aggregating the probability of purchasing an AV over all consumers. That is $Q^D(p_A, x; \beta, c)$ solves

$$\int_y P_A(y,Q,p_A,x)dy = Q \tag{6}$$

with respect to Q.

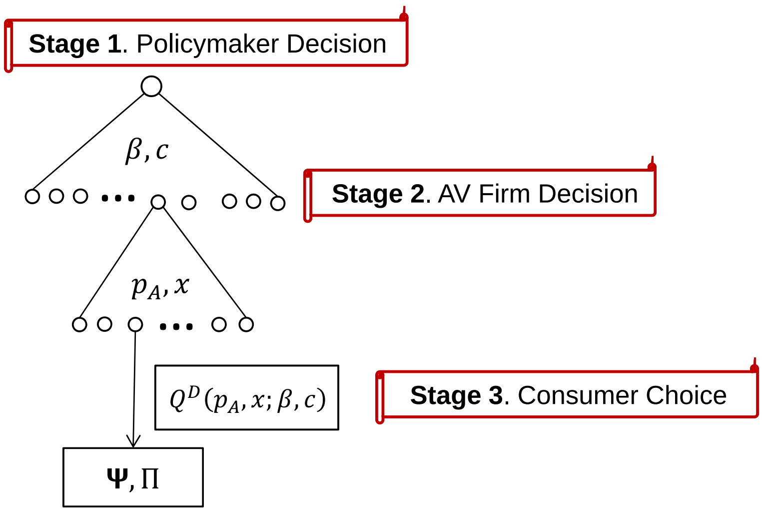
Unlike standard models of horizontal product differentiation, in our setup already the determination of AV demand constitutes a fixed point problem. Again this is due to the fact that, through the expected liability costs, each consumer's utility from each vehicle type depends on the total number of AVs and HVs on the street. Since we assume that the AV producer can deliver an arbitrary quantity of AVs at the posted price p_A the actual quantity sold always coincides with the demand Q^D.

[11] If consumers would observe their utility from purchasing an AV respectively HV only with some noise, this would (under certain assumptions on the distribution of the noise) give rise to a standard logit model, where the probability of purchasing an AV is given by $P_A(y,Q,p_A,x) = e^{\lambda u_A(y)}/(e^{\lambda u_A(y)} + e^{\lambda u_H(y)})$ for some intesity of choice parameter λ. Our model can therefore also be seen as the limit of such a logit model for $\lambda \to \infty$.

2.4 Model summary

In the above analysis, we capture that the policymaker takes long term decisions, which influence the AV producer's choice of AV price and safety level. Consumers then make their purchasing decisions taking into account the environment set by the policymaker as well as the properties of the AV chosen by the AV producer. Formally, we consider three stages. At stage 1, the policymaker decides on the liability regime (β) and on the investment in V2I connectivity (c). At stage 2, the AV producer decides on the AV price (p_A) and safety level (x). At stage 3, each consumer chooses her preferred type of vehicle. Formally, the three stage interaction can be described as an extensive form game with complete information, that is illustrated by Figure 4. Among the four types of agents in our model only two, the policymaker and the AV producer, are strategic players. The interaction has the form of a leader-follower game with the policymaker as the leader and the AV producer as the follower. The decision of a single consumer has no measurable impact on the objective functions of all other agents, and hence consumers are not strategic players. Since we take the price of the HV as given, also HV producers are no players in the game. Figure 4 illustrates the extensive-form game tree for these 3 stages.

Figure 4: Extensive-form game tree



2.5 Equilibrium

In our analysis, we characterize the subgame perfect equilibrium of the game, thereby following the principle of backward induction (see e.g., Fudenberg and Tirole, 1991). Following this approach, we determine the AV producer's optimal level of AV safety investment and AV price for given policy variables β and c. Furthermore, in setting these variables, the policymaker takes into account how they affect the subsequent optimal behavior of the AV producer.

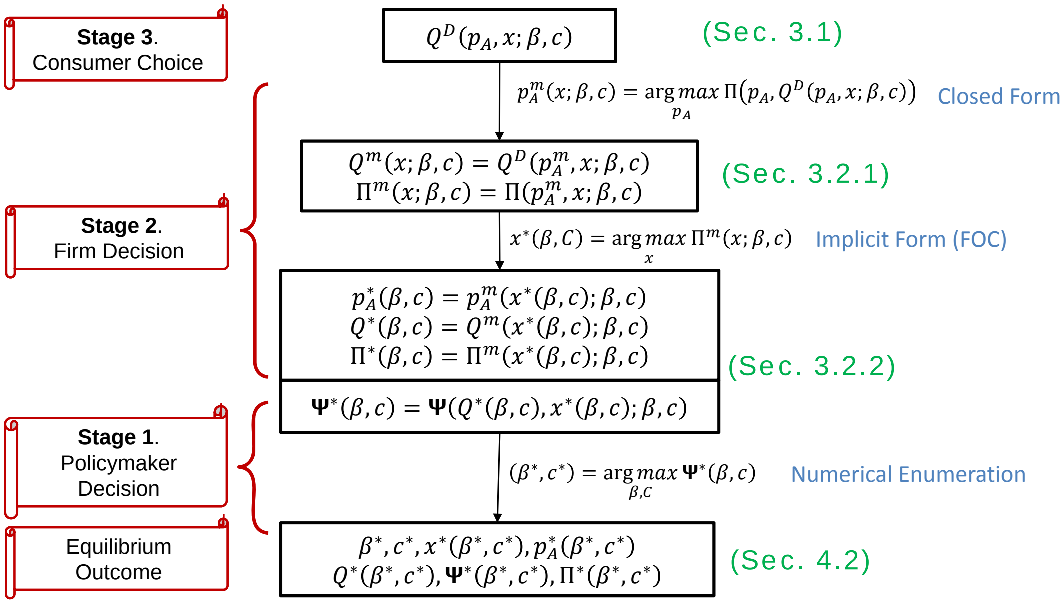
At equilibrium,

1. no consumer can improve her utility by switching the car purchase choice (for given values of β, c, p_A, x);

2. the AV producer cannot improve its net profit by changing the investment in AV safety and the price of the AV (for given values of β, c);

3. the policymaker cannot further reduce the total accident and infrastructure cost by switching the liability regime and the level of connectivity investment.

Figure 5: Equilibrium analysis using backward induction



In Figure 5 we outline the different steps in our equilibrium analysis. First, we determine the AV quantity $Q^D(p_A, x; \beta, c)$ sold (stage 3). Using this, we formulate the AV producer's problem and determine the optimal price function $p_A^m(x; \beta, c)$ and the optimal level of investment $x^*(\beta, c)$ (stage 2). Finally, using a numerical approach, we calculate the values β^* and c^* minimizing the policymaker objective function (stage 1).[12] This then gives rise to the equilibrium outcomes in terms of actions and payoffs of the players.

Due to the sequential structure of the game, equilibrium existence can be established by showing that the maximization problems at stages 2 and 3 have a (finite) solution. For stage 2, this follows immediately from the continuity of the profit function $\Pi^m(\cdot)$ and the compactness of the relevant range of values of p^A and x.[13] As for stage 1, the set $[0, 1] \times [0, \bar{c}]$ of relevant values of (β, c) is compact such that the Weyerstrass extreme value theorem implies the existence of an optimal solution for each (compact) segment of this set. Although, in general $\Psi(\cdot)$ is not necessarily continuous with respect to (β, c) (which is due to potential jumps of $x^*(\beta, c)$), abstracting from pathological cases where $x^*(\beta, c)$ jumps infinitely often on $[0, 1] \times [0, \bar{c}]$, the existence of a maximizer of $\Psi(\cdot)$ is ensured.[14]

We split the equilibrium analysis into two parts. First, in Section 3 we derive analytical results characterizing the optimal consumer choice and the resulting demand function (stage 3) as well as the optimal behavior of the AV producer (stage 2). In Section 4 we then employ numerical methods to analyze the sensitivity of optimal behavior of the AV producer with respect to the two policy variables β and c, and to determine the policymaker's optimal choice (stage 1).

[12] Before determining β^* and c^*, in Section 4.1, we also use numerical methods to perform a sensitivity analysis of the AV producer's optimal AV safety investment with respect to β and c.

[13] Since the AV producer's revenue is bounded from above and the cost function $\xi(x)$ is quadratic, there is an upper bound $\bar{x}$ such that $\Pi^m < 0$ for all $x > \bar{x}$. Similarly, there is an upper bound $\bar{c}$ such that the optimal value of c is always below $\bar{c}$.

[14] Ruling out infinitely many jumps of $x^*(\beta, c)$, the set $[0, 1] \times [0, \bar{c}]$ can be partitioned into finitely many subsets, where the function $\Psi(\cdot)$ is continuous in each subset and therefore has a finite maximizer. Comparing the maximal values of $\Psi(\cdot)$ across these subsets then yields the global maximizer.

3 Analytical findings

3.1 The demand for AVs

In a first step, we determine the demand for AVs for given vehicle prices (p_A and p_H), a given AV safety level (x) and a given liability regime (β) and level of connectivity c. To ease notation, throughout this subsection we drop the arguments β and c in all functions. Taking into account (5) and (6) we obtain that the quantity $Q^D(p_A, x)$ has to satisfy

$$\int_y \mathbf{1}_{u_A(y,Q^D(.))\geq u_H(y,Q^D(.))} dy = Q^D(.). \tag{7}$$

It follows from (4) that if a consumer with bliss point y prefers the AV, this is also the case for all consumers with bliss points y' satisfying $d_A(y') < d_A(y)$. Hence, consumers choosing AVs are located symmetrically around the location of the AV producer (see Figure 3). If the quantity $Q^D(.)$ is in the interior of its range $[0,1]$, the indifference condition $u_A(\tilde{y}, Q^D(.)) = u_H(\tilde{y}, Q^D(.))$ has to hold for the consumer whose bliss point $\tilde{y}$ has the property $d_A(\tilde{y}) = \frac{Q^D(.)}{2}$. Inserting $d_A(\tilde{y}) = \frac{Q^D}{2}$ and $d_H(\tilde{y}) = \frac{1-Q^D}{2}$ into the utility function and solving $u_A(\tilde{y}, Q^D) = u_H(\tilde{y}, Q^D)$ for Q^D then yields the function $\tilde{Q}^D(p_A, x)$. This denotes the total AV demand in all cases where it is in the interior of $[0,1]$:

$$\tilde{Q}^D(p_A, x) = \frac{1}{t - (r_1(x) - r_2(x))} \cdot \left(\frac{t}{2} + r_2(x) + p_H - p_A\right), \tag{8}$$

which is linear in the AV price p_A and where $r_1(x) = g(x)\gamma(D) - (k(x) - h)\gamma((1-\beta)D)$ and $r_2(x) = \bar{g}\gamma(D) - k(x)\gamma((1-\beta)D)$. Intuitively, $r_1(x)$ captures the change in the incentive to buy an AV if there is one additional AV on the street: in this case, when purchasing an HV, the consumer's expected costs from accidents increases by $g(x)\gamma(D)$. When purchasing an AV instead, the expected accident costs increases by $(k(x) - h)\gamma((1-\beta)D)$. Similarly, $r_2(x)$ captures the change in the incentive to buy an AV if there is one additional HV on the street: in this case, when purchasing an HV, the consumer's expected costs from accidents increases by $\bar{g}\gamma(D)$. When purchasing an AV instead, the expected accident costs increases by $k(x)\gamma((1-\beta)D)$. Hence, for both $r_1(x)$ and $r_2(x)$, when the difference between the two respective types of accident costs is positive (negative), the incentive to buy an AV increases (decreases). Taken together, the difference $r_1(x) - r_2(x)$ in (8) captures the externality induced by one consumer, i say, who switches from the HV to the AV, on all other consumers. In particular, the expected liability costs of an HV owner changes by $(g(x) - \bar{g})\gamma(D)$. If x is sufficiently small then this expression is positive and hence there is a negative AV consumer externality for HV users. Moreover, the switch of consumer i leads to a reduction of the expected liability cost for AV owners of $h\gamma((1-\beta)D)$. This effect is due to V2I connectivity between AVs and the resulting fewer accidents in AV-AV compared to AV-HV interactions. Taking into account that AVs demand cannot exceed the market size, normalized to one, and has to be non-negative, we obtain the demand function $Q^D(p_A, x) = \max[0, \min[1, \tilde{Q}^D(p_A, x)]]$.

Given that our focus is on mixed traffic, the following assumption ensures that in equilibrium HVs are not driven out of the market:

Assumption 2.

(i) $k(0)\gamma(D) > \frac{t}{2} + p_H + \bar{g}\gamma(D)$,

(ii) $\max[g(0)D, p_H + hD] < \frac{t}{2}$.

Part (i) ensures that if there is no investment in AV safety ($x = 0$), even the consumer with the strongest preference for the AV prefers the HV regardless of the AV price. Hence, AV demand is zero in such a situation. Part (ii) is a technical condition that guarantees that the two vehicle types are sufficiently differentiated in the eyes of the consumers such that regardless of AV safety, some consumers always prefer the HV (see proof of Proposition 1).[15]

3.2 The optimal behavior of the AV producer

3.2.1 AV pricing and resulting AV quantity

Given optimal consumer choice, we now determine the profit maximizing AV price p_A for the AV producer for a given AV safety level x. Taking into account that x is fixed, the objective of the firm is to maximize (3) with respect to p_A. This leads to the following result:

Proposition 1. *Assume that x is sufficiently large such that it is profitable for the firm to sell the AV on the market. Then the optimal AV price and the resulting AV quantity are*

$$p_A^m(x) = \frac{1}{2(z_1(x) - \beta h D)} \left[z_1(x) z_2(x) + \beta D(z_1(x)k(x) - 2z_2(x)h) \right], \tag{9}$$

$$Q^m(x) = \frac{z_2 - \beta k(x) D}{2(z_1(x) - \beta h D)} < 1, \tag{10}$$

where $z_1(x) = t - (r_1(x) - r_2(x))$ and $z_2(x) = \frac{t}{2} + r_2(x) + p_H$. This yields a maximized profit of

$$\Pi^m(x) = \tilde{\Pi}(p_A^m(x), x) = \frac{1}{2} Q^m(x)^2 (z_1(x) - \beta h D) - \xi(x). \tag{11}$$

Using this Proposition we can also infer that under our assumptions the AV producer needs some strictly positive level of AV safety for (profitably) selling the AV on the market. More precisely we have the following Corollary.

Corollary 1. *There exists a safety stock $x^l > 0$ such that the monopoly quantity $Q^m(x)$ is strictly positive if and only if $x > x^l$.*

Together, Proposition 1 and Corollary 1 imply that for all levels of AV safety investments $x > x^l$ there is mixed traffic in equilibrium, i.e. both vehicle types have a strictly positive market share.

In a next step, we investigate in more detail how the optimal AV quantity varies in the AV safety level, x. Taking the derivative of $Q^m(x)$ yields

$$\begin{aligned}\frac{\partial Q^m(x)}{\partial x} &= \frac{[-k'(x)(\gamma((1-\beta)D) + \beta D)(z_1(x) - \beta h D) + g'(x)\gamma(D)(z_2(x) - \beta D k(x))]}{2(z_1(x) - \beta h D)^2} \\ &= \frac{1}{2(z_1(x) - \beta h D)} \left[-k'(x)\gamma((1-\beta)D) - k'(x)\beta D + g'(x)\gamma(D) 2 Q^m(x) \right], \end{aligned} \tag{12}$$

where in the second line we use (10).

Since the denominator is positive (see the proof of Corollary 1), the sign of $\frac{\partial Q^m(x)}{\partial x}$ coincides with that of the square bracket in (12). Intuitively, an increase in x affects the optimal AV quantity $Q^m(x)$ through three channels, indicated by the three terms in that square bracket: First, it reduces the expected liability

[15] Part (i) also implies that the denominator of (8) is positive. This follows, since $g(x)\gamma(D) < t/2$ and $h\gamma((1-\beta)D) < \lim_{x\to\infty} k(x)(1-\beta)D < g(0)D < t/2$.

cost for AV drivers in accidents caused by the AV.[16] Second, it reduces the AV price since also the expected liability costs of the AV producer decreases. Third, it makes the HV safer and hence more attractive for consumers, thereby generating a *rival externality*, the size of which depends on $Q^m(x)$.

Whereas the rival externality makes HVs more attractive and therefore has a negative effect on AV demand, the other two effects have a positive effect on AV demand (recall that $k'(x) < 0$ and $g'(x) < 0$). Taking into account that the rival externality becomes more pronounced as $Q^m(x)$ increases, for $Q^m(x)$ close to zero, the negative rival externality is essentially non-existent, so that $Q^m(x)$ is increasing in x in this range. However, as $Q^m(x)$ increases, the rival externality becomes relatively more important. The following proposition shows that under mild conditions, the rival externality dominates for sufficiently large x, so that $Q^m(x)$ decreases with x for x sufficiently large.

Proposition 2. *The optimal AV quantity $Q^m(x)$ increases with x for sufficiently small $x > x^l$. Furthermore, there exists an $\tilde{x} > x^l$ such that $Q^m(x)$ decreases with x if and only if $x \geq \tilde{x}$.*

From an economic point of view, the key insight from the proposition is that a costly investment into AV safety potentially has a detrimental effect on AV demand. For this reason, one might think that it will never be optimal for the AV producer to choose a level of x, which leads to that segment of AV demand. However as we show next, this is not necessarily the case.

3.2.2 AV safety investment

The AV producer maximizes its profit $\Pi(x)$ with respect to x, anticipating how x will affect optimal AV pricing and demand $Q(x)$ as determined above. The maximization problem of the AV producer is hence

$$\max_x \Pi(x) = \frac{1}{2} Q^m(x)^2 (z_1(x) - \beta D h) - \xi(x). \tag{13}$$

Lemma 1. *For sufficiently small ξ_0, the AV producer's maximization problem* (13) *has an interior solution, x^*, that satisfies the condition*

$$Q^m(x^*) Q^m(x^*)' \cdot (z_1(x^*) - \beta D h) + \tfrac{1}{2} Q^m(x^*)^2 z_1(x^*)' - 2\xi_0 x^* = 0. \tag{14}$$

To gain an intuition for the lemma, consider the first order condition (14). The first term represents the marginal revenue generated by the change in quantity induced by a marginal increase of x via $Q^m(x)'$. The second term is the direct effect of an increase of the safety level on the AV producer's profit for a fixed quantity. The third term is the marginal cost of investment. The second term becomes more important relative to the first one the larger $Q^m(x^*)$ is. The third term increases with x^*.

Recall from Proposition 2 above that AV demand increases (decreases) with the AV safety level x for small (large) x. As shown next, for sufficiently small investment costs, the AV producer optimally chooses a large value of x, thereby indeed locating in the decreasing segment of AV demand:

Proposition 3. *For any given values of β and c, the optimal investment level x^* decreases with ξ_0. Furthermore, there exists a threshold $\bar{\xi}$ such that for all $\xi_0 < \bar{\xi}$, the optimal investment level x^* satisfies $Q^m(x^*)' < 0$ for all values of $\beta \in [0, 1]$ and $c \geq 0$.*

Intuitively, if the investment cost parameter ξ_0 is small, it is optimal for the firm to choose a high investment level x^* and, correspondingly the AV quantity is high.[17] Hence the interplay of the second and

[16] While an AV is less likely to cause an accident with another AV than with an HV (where the difference in the accident probabilities is just h), the marginal change of accident probability with respect to x is $k'(x)$ in both cases.

[17] In spite of the result of Proposition 2 that $Q^m(x)' < 0$ for large x, the AV quantity still stays at a high level.

the third term in the first order condition are crucial for determining the optimal value x^*. In particular, this implies that for sufficiently small ξ_0, it is optimal for the AV producer to invest a lot, thereby pushing the AV quantity into a region where it is already decreasing in x, i.e. more investment into AV safety increases demand for the HV, the AV's rival product (formally, x^* is above the threshold $\tilde{x}$ defined in Proposition 2). Doing so is nevertheless optimal for the AV producer since the resulting large AV quantity generates a strong incentive to reduce the expected liability costs, even if this comes along with a dampening effect on AV demand.

As the optimal investment level x^* of the AV producer cannot be given in closed form and due to the complexity of the underlying first order condition, the further steps in the equilibrium analysis require a numerical approach.

4 Numerical experiments

In this section, we further analyze the developed modeling framework using numerical experiments, accompanied by various sensitivity analyses. Algorithm 1 contains the pseudo-code summarizing our approach to calculate the equilibrium values in these experiments.

Algorithm 1 Algorithm for equilibrium solution

```
Input: A finite set of grid points (β^i, c^i), i = 1, ··· , I.
First-order condition for max_x Π^m(x; β, c): FOC(x; β, c)
Set Ψ_min = 10^6;
for i ← 1 to I do
    Determine numerically X = {x : FOC(x; β^i, c^i) = 0};
    x_temp(β^i, c^i) = x ∈ X, s.t.  Π^m(x; β^i, c^i) ≥ Π^m(y; β^i, c^i), ∀y ∈ X;
    if Ψ(Q^*(x_temp(β^i, c^i)), x_temp(β^i, c^i); β^i, c^i)) < Ψ_min then
        Ψ_min = Ψ(Q^m(x_temp(β^i, c^i)), x_temp(β^i, c^i); β^i, c^i);
        β^* = β^i, c^* = c^i;
        x^* = x_temp(β^i, c^i);
    end if
end for
Output: β^*, c^*, x^*
```

Throughout the numerical analysis we use a parameter setting that satisfies the conditions stated in Assumptions 1 and 2.[18] Moreover, it also has the property that the AV is always safer than the HV under the resulting optimal AV safety investment, i.e. $k(x^*(\beta, c)) < g(x^*(\beta, c))$. Finally, we consider the marginal cost of AV safety investment ξ_0 as a key parameter in addition to the two policy variables β and c.

4.1 Impact of policy variables on AV safety investment, market penetration and road safety

We next analyze how the optimal AV safety investment, the AV quantity and the resulting number of accidents vary with the two policy variables, i.e. the liability regime (β) and the level of communication infrastructure (c). While we have suppressed this dependence in the previous analysis for notational convenience, from now on we take it explicitly into account and write $x^* = x^*(\beta, c)$, $Q^m(x^*(\beta, c)) = Q^*(\beta, c)$, and $A^*(\beta, c) = A(x^*(\beta, c), Q^*(\beta, c); \beta, c)$.

[18] In particular, we set $g(x) = \underline{g} + g_0 e^{-\mu x}$, $k(x) = \underline{k} + k_0 e^{-\nu x}$ and $\gamma(x) = \frac{\tau x}{\tau + x}$ where $k_0 > g_0$ and $\nu > \mu$. Moreover, $\underline{k} = 0.03$, $k_0 = 0.5$, $\nu = 2.5$, $\underline{g} = 0.03$, $g_0 = 0.05$, $\mu = 0.3$, $D = 12$, $t = 2$, $p_H = 0$, $v = 5$, $\bar{g} = 0.06$, $\tau = 100$, and $\zeta_0 = 53$.

1. Impact of the liability regime Consider first the impact of the liability regime (β), which is illustrated in Figure 6 for the case of low (left column) and high (right column) marginal costs of AV safety investment (ξ_0), respectively. An increase in β has two effects: First, the *safety effect* induces the AV producer to invest more in AV safety (see first row of Figure 6). A similar effect has been identified in the literature on product liability (e.g. McGuire, 1988; Viscusi and Moore, 1993; Galasso and Luo, 2022). Intuitively, the main effect is that a higher β increases the liability costs per unit of AV sold, which in turn leads to a higher investment incentive. Second, the *quantity effect* leads to a lower AV quantity as β increases (see second row of Figure 6). Intuitively, a higher β increases the expected liability costs of the AV producer per unit of AV, which leads to a higher price. At the same time, since consumers face now less liability, this increases their willingness to pay for the AV and hence AV demand. However, due to limited liability, captured by $\gamma(\cdot)$, this effect is smaller than the price effect, so that the AV quantity decreases in β. Note that the cost parameter ξ_0 does not affect the overall shape of the safety and the quantity effect, but their relative importance. In particular, when ξ_0 is low, the AV producer's optimal investment x^* is large (see top left panel in Figure 6). In this case, since the accident probability $k(x)$ decreases in a convex way, the marginal effect of a further increase of x on AV safety (induced by a higher β) is hence relatively small compared to the quantity effect. By contrast, when ξ_0 is high, the optimal x^* is small (see top right panel in Figure 6), and hence the marginal effect of an increase on AV safety is relatively large. As a consequence, whether an increase in β reduces or increases the expected number of accidents crucially depends on ξ_0 (see the two bottom panels of Figure 6). This also has implications for the policymaker's optimal choice of β which are discussed in detail below.

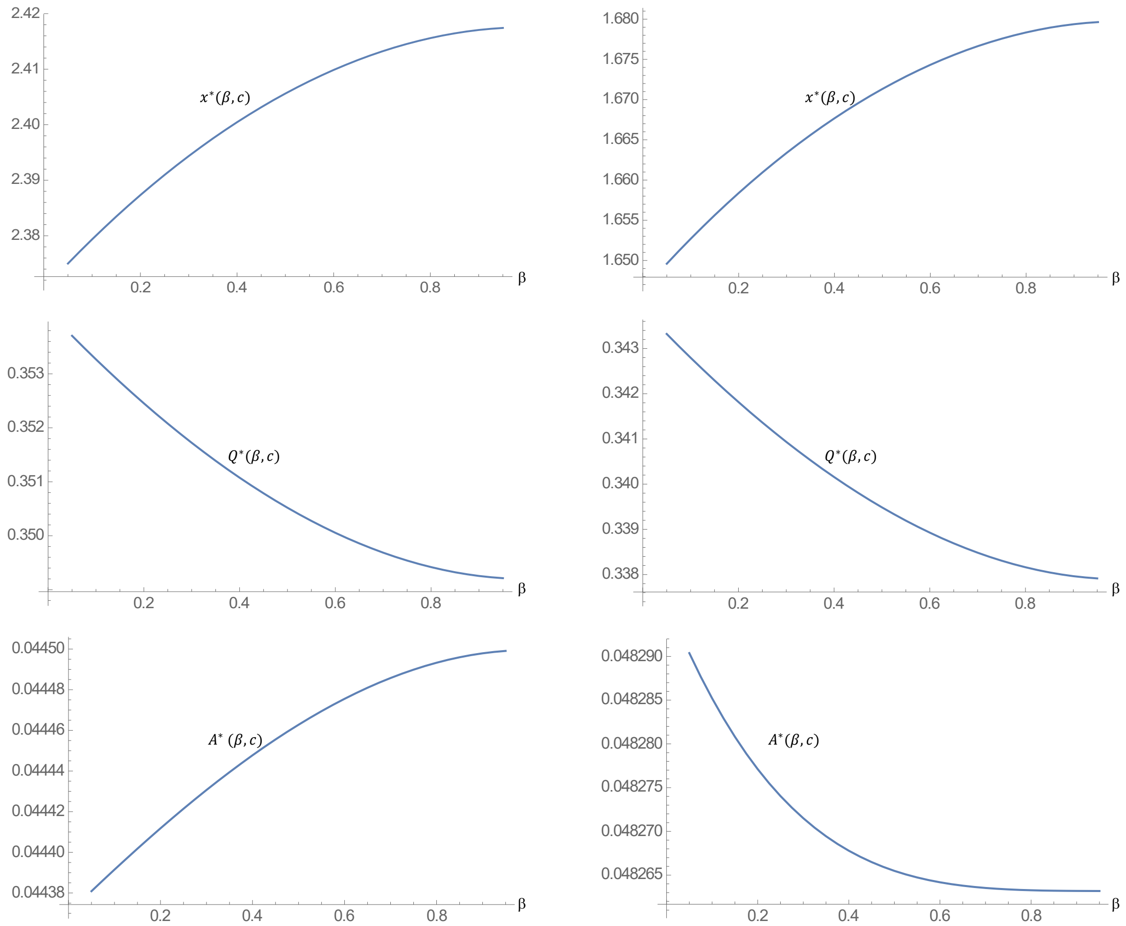
2. Impact of AV connectivity Consider next the effect of an increase in the level of AV connectivity (C) on the optimal behavior of the AV producer. This is illustrated in Figure 7, again separately for the case with low (left column) and high (left column) marginal costs of improving AV safety, ξ_0. Intuitively, an increase in c lowers the AV producer's expected liability costs per unit of AV sold, which in turn provides an incentive to increase the AV quantity. Whether this involves a higher or lower investment in AV safety depends on the marginal costs of doing so (see Proposition 3 above): if ξ_0 is sufficiently low, then $x^*(\beta, c)$ is large and lies in a region where AV demand is decreasing in x. Hence, an increase in Q is accompanied by a reduction of x (see upper left panel in Figure 7). By contrast, for ξ_0 sufficiently large, $x^*(\beta, c)$ is small and lies in a region where AV demand is increasing in x. In this case, increasing Q requires an increase in x (see upper right panel in Figure 7). Note also that in both cases, the total number of accidents $A^*(\beta, c)$ is negatively related to the AV quantity (see bottom row of Figure 7).

4.2 The optimal AV policy

In the final step of the backwards procedure of analysis, we consider the policymaker's optimal choice of the liability regime (β) and the level of AV connectivity (c), thereby taking into account the subsequent optimal behavior of consumers and the AV producer as characterized in the previous analysis. Recall from (2) above that the policymaker chooses β and c to minimize the total accident and connectivity infrastructure costs, denoted by $\Psi^*(\beta, c) = \Psi^*(x^*(\beta, c), Q^*(\beta, c); \beta, c)$.

To develop an intuition, Figure 8 illustrates the effects of β and c on $\Psi^*(\beta, c)$ separately, again for low and high marginal cost of AV safety investment (ξ_0), respectively. With respect to the optimal liability regime, whether the policymaker's objective increases or decreases with β depends on ξ_0, and hence on the relative importance of the safety and the quantity effect as described above. If ξ_0 is small, the AV producer optimally chooses a high level of AV safety. In this case, the marginal effect of a further increase of x induced by β on the accident probability $k(x)$ is relatively small (see Figure 1), and the negative effect of

Figure 6: Effect of product liability (β) on optimal AV investment $x^*(\beta, c)$ (first row), AV quantity $Q^*(\beta, c)$ (second row), and expected number of accidents $A^*(\beta, c)$, (third row), for low (left column) and high (right column) marginal costs of safety investment (ξ_0)

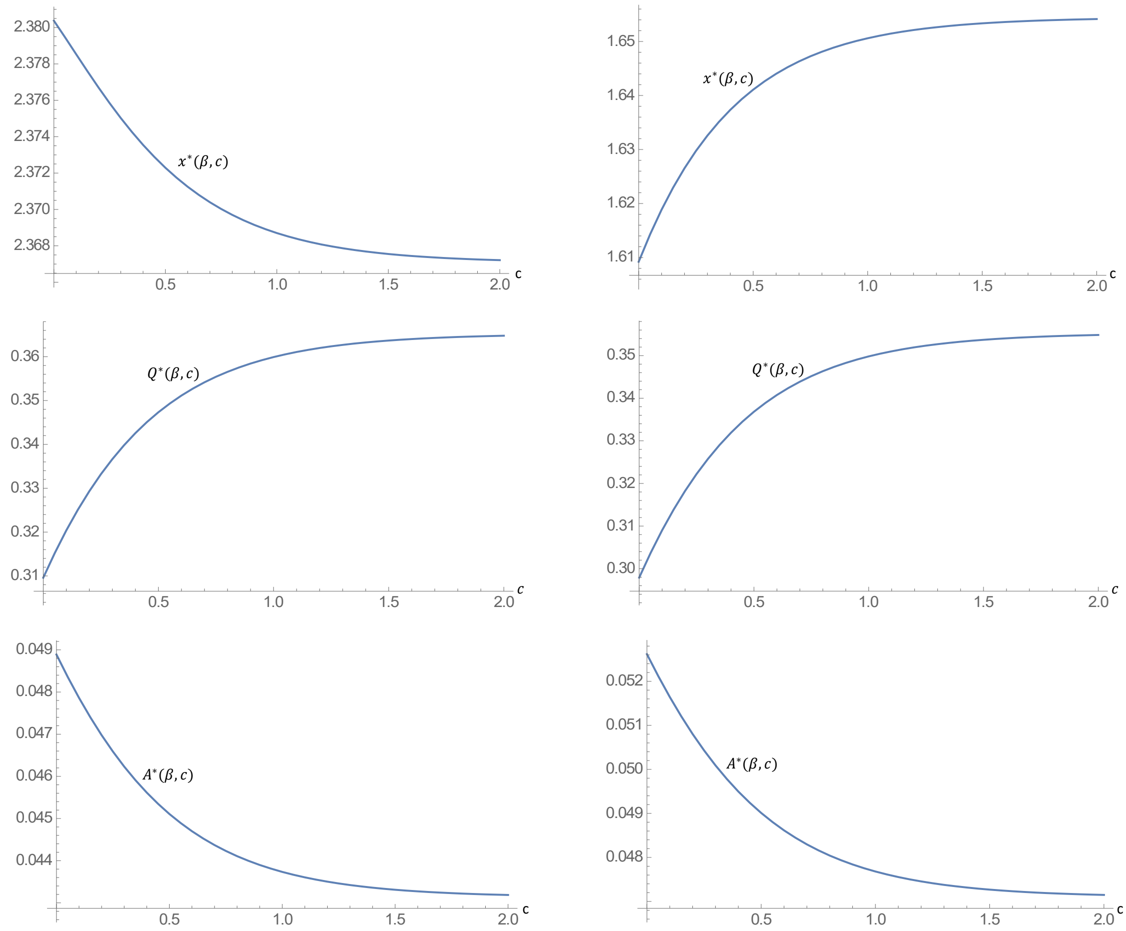


Note: The basic parameter setting applies (see Footnote 18). The low and high value for ξ_0 is 0.001 and 0.02, respectively. In addition, we set $c = 0.7$.

β on the AV quantity dominates the positive effect on AV safety investment. As a result, the total number of accidents $A^*(\beta, c)$ increases with β and so does the policymaker's cost $\Psi^*(\beta, c)$ (see upper left panel of Figure 8). This leads to $\beta^* = 0$, i.e. the AV producer should not be subject to product liability. By contrast, when ξ_0 is large, then $x^*(\beta, c)$ is low, and increasing it has a large effect in lowering $k(x)$. In this case, the safety effect dominates the quantity effect. As a result, both the total number of accidents ($A^*(\beta, c)$) and policymaker's cost $\Psi^*(\beta, c)$ decrease with β (see upper right panel of Figure 8), leading to $\beta^* = 1$, i.e. full liability for the AV producer. Figure 9 (in red) shows the optimal liability regime as a function not only for two values of ξ_0, but for a whole interval. As can be seen there also exists an intermediate range of ξ_0 where neither the safety nor the quantity effect dominates, and where the safety effect becomes relatively more important as ξ_0 increases. In this range, β^* is interior and increases with ξ_0. Overall, the analysis indicates that the optimal liability policy strongly depends on the marginal costs of improving AV safety.

Consider next the optimal investment in AV connectivity infrastructure c^*. First, as long c is not too high, there exists a negative relationship between $\Psi^*(\beta, c)$ (see Figure 8, bottom row) and the AV quantity $Q^*(\beta, c)$ (see Figure 7, second row). That is, an increase in c leads to both a higher AV quantity $Q^*(\beta, c)$ and a lower cost $\Psi^*(\beta, c)$ for the policymaker. Intuitively, an increase in c directly reduces the likelihood of accidents caused by each AV, $k(x) - h(c)$, so that the overall benefit from a higher c scales with the

Figure 7: Effect of V2I connectivity (c) on optimal AV safety investment $x^*(\beta, c)$ (first row), AV quantity $Q^*(\beta, c)$ (second row), and expected number of accidents $A^*(\beta, c)$, (third row), for low (left column) and high (right column) marginal costs of safety investment (ξ_0)



Note: The basic parameter setting applies (see Footnote 18). The low and high value for ξ_0 is 0.001 and 0.02, respectively. In addition, we set $\beta = 0$.

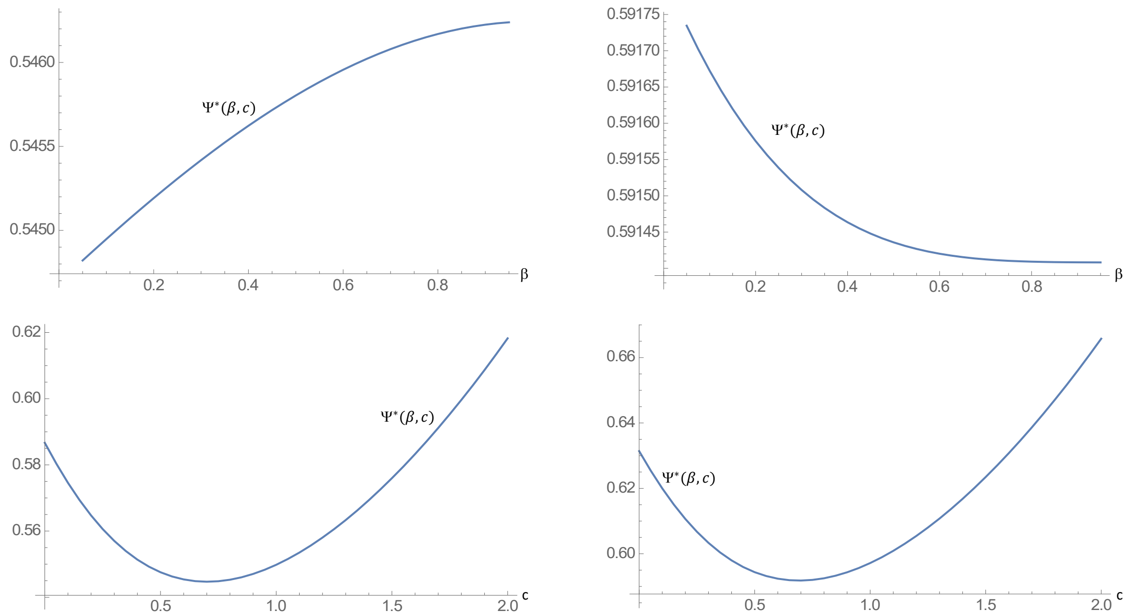
AV quantity $Q^*(\beta, c)$. As a result, the expected accident costs, and hence $\Psi^*(\beta, c)$, decrease.[19] Moreover, a higher c decreases the marginal costs per unit of AV (through lower expected liability payments), which also leads to a higher AV quantity. The policymaker weighs these benefits of higher AV connectivity against the (increasing and convex) costs, so that $\Psi^*(\beta, c)$ eventually increases for for c sufficiently large.

How the optimal connectivity c^* policy depends on the marginal costs of AV safety investment (ξ_0) is illustrated in Figure 9 (in blue). As long as the optimal liability policy β^* is constant in ξ_0 (which is the case when ξ_0 is either low or high), an increase in ξ_0 has two main effects on the policy maker's optimal choice of level of infrastructure. First, since an increase in c reduces the accident probability for each interaction between two AVs on the street, the incentive to increase c is positively related to AV quantity.[20] Second, since it is in the interest of the policy maker that the AV quantity is high, her incentive to invest in connectivity is positively related to $\partial Q^*(\beta, c)/\partial c$. For small ξ_0 the reduction in the optimal safety investment induced by an increase in ξ_0, leads to an increase in $Q^*(\beta, c)$ (see Proposition 3) and also in $\partial Q^*(\beta, c)/\partial c$. Hence, optimal connectivity investment c^* increases with ξ_0. As ξ_0 increases further the effect on $Q^*(\beta, c)$ becomes negative, but initially the indirect effect through $\partial Q^*(\beta, c)/\partial c$ dominates such

[19]Recall that in our numerical analysis, under the optimal safety investment of the AV producer, the AV is safer than the HV, i.e. $k(x^*(\beta, c)) < g(x^*(\beta, c))$.

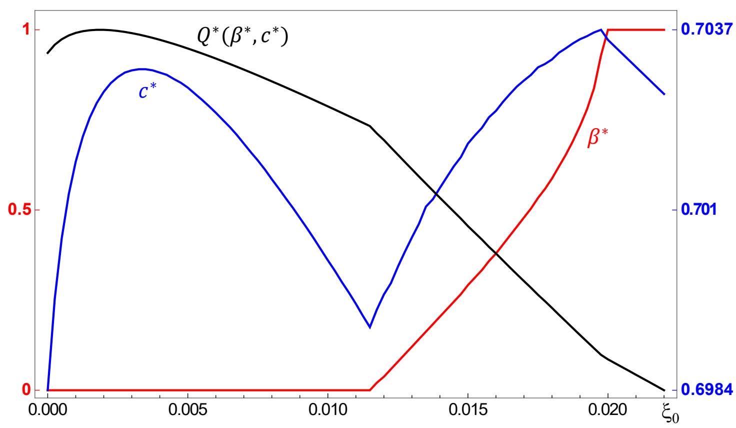
[20]As can be seen from taking the derivative of (2) with respect to c, this effect scales with Q^2.

Figure 8: Effect of product liability (β, top panels) and V2I connectivity (c, bottom panels) on policymaker's objective function $\Psi^*(\beta, c)$ for low (left column) and high (right column) marginal costs of safety investment (ξ_0).



Note: The basic parameter setting applies (see Footnote 18). The low and high value for ξ_0 is 0.001 and 0.02, respectively. In addition, we set $c = 0.7$ (top row) and $\beta = 0$ (bottom row).

Figure 9: Effect of costs of safety investment on firm liability (β^*, red), V2I connectivity (c^*, blue) and the resulting AV quantity (Q^*, black).

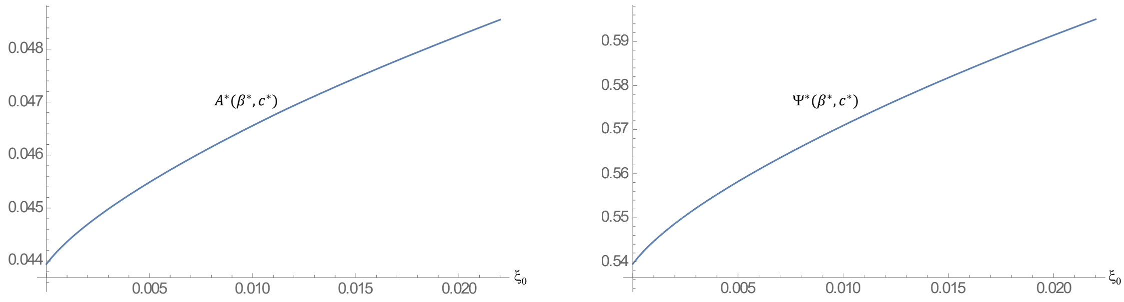


that c^* still increases with ξ_0. For even larger ξ_0 the direct quantity effect dominates such that c^* decreases with ξ_0. In the region where β^* increases with ξ_0, the optimal value c^* goes up. The intuition is that a higher c stimulates AV quantity in order to compensate the negative effect on AV quantity induced by the increase of β (see Figure 6). Intuitively, a higher β fosters connectivity investments by increasing $\partial Q^*(\beta, c)/\partial c$.

Again, our analysis shows that the optimal AV policy crucially depends on the cost of AV safety investments. In addition, the findings highlight the interplay between the optimal liability connectivity policies, which cannot be observed when looking at these policies in isolation.

Finally, Figure 10 illustrates behavior and payoffs along the equilibrium path as a function of ξ_0. We observe that, taking into account the optimal reactions of the AV producer and the policymaker in response

Figure 10: Effect of marginal costs of AV safety investment (ξ_0) on equilibrium outcomes: accidents ($A^*(\beta^*, c^*)$, left) and total costs ($\Psi^*(\beta^*, c^*)$, right).



to an increase in the marginal cost of AV safety investment, a positive relationship between ξ_0 and both the total number of accidents and total costs arises. This is driven by a combination of the decrease of the AV producer's safety investment and the reduction in AV quantity, as discussed above.

5 Conclusion

This paper studies a unified, game-theoretic framework (leader-follower game) of mixed traffic, thereby explicitly taking into account the fact that consumers have a choice between AVs and HVs, and the different types of accidents emerging in mixed traffic. We focus on the interaction between three crucial types of players: (i) a policymaker, who decides on the liability regime and the level of V2I connectivity, (ii) an AV producer, who decides on the AV price and safety level, and (iii) consumers, who differ in their preferences for each vehicle type.

Our analysis identifies two novel types of spillover effects: (i) An individual consumer's expected liability cost when purchasing an AV, say, depends on the total number of AVs on the street (through the different types of accidents that may occur), which in turn results from all consumers' purchasing decisions. Therefore, each consumer's vehicle choice creates a spillover effect on all other consumers, and the determination of AV demand constitutes a fixed point problem. (ii) A higher level of AV safety might actually *reduce* the demand for AVs. Intuitively, a safer AV renders not only the AV more attractive, but also the HV, as HV-AV accidents become less likely. In this case, the AV producer's (costly) investment into AV safety creates a positive spillover on its competitors (the HV producers) by making their product more appealing to consumers (*rival externality*).

Furthermore, we show that the AV has a positive market share only if its safety level is above a minimum level. Moreover, when the marginal cost of AV safety investment is sufficiently small, the AV producer's optimal investment level is so large that this has a negative marginal effect on AV demand. In this case, despite the rival externality, the AV producer's benefit (the reduction of liability cost) outweighs the loss due to lower AV demand.

From a policy perspective, we study how the equilibrium behavior of consumers and the AV producer is affected by the two policy variables, and we highlight the crucial role of the marginal cost of AV safety investment. Our first main result in this respect is that more stringent AV (product) liability induces the AV producer to invest more in AV safety (*safety effect*), but also leads to a lower AV market penetration (*quantity effect*). The relative importance of these two effects, and whether the social harm from accidents increases or decreases as liability becomes more stringent, depends on the marginal costs of AV safety investment. Second, an increase in V2I connectivity makes AVs more attractive for consumers and reduces the expected costs of the AV producer. In equilibrium, this leads to a higher AV market penetration. Whether this increase is accompanied with higher or lower of AV safety investment depends, again, on the marginal costs of AV safety investment.

Taking these effects of the two choice variables into account, the policymaker optimally chooses a liability share for the AV producer that, starting from zero, weakly increases with the marginal cost of AV safety investment, and even full liability for the AV producer can be optimal when this cost is sufficiently large. The optimal investment in V2I connectivity is positively related with the AV market penetration, in the parameter range of marginal cost of AV safety investment where the optimal liability rule is constant. By contrast, in the (intermediate) range where the liability share of the AV producer increases, the optimal connectivity investment increases although AV market penetration decreases. Hence, from the perspective of the policymaker there is a complementarity between these two policy variables.

A further policy implication emerging from our analysis is that policymakers should carefully consider the incentives of AV producers to invest in AV safety. If these incentives are high (e.g. because the marginal investment cost is low), then the policymaker should shift the burden of liability to consumers rather than AV producers. Also, public investment into V2I connectivity need not be at very high levels, because AVs are safe enough already to limit the number of accidents. If, however, AV producers' (marginal) cost of investment are high, and hence their incentive to invest in AV safety is low, the policymakers should hold AV producers liable to a larger degree. Moreover, due to the complementarity between the policy instruments, this policy should be complemented by a large investment into V2I connectivity.

All in all, our findings provide a number of novel insights that are relevant for a fast adoption of AVs and a smooth transition from existing traffic situation to a mixed traffic environment. They provide guidance for decision making for policymakers, legal agencies, traffic operation and transportation planning agencies, as well as car manufacturers.

In future work it would be interesting to extend the model in several directions. First, V2I connectivity that can improve traffic safety for both AVs and HVs could be considered. Second, one could augment our framework to include exclusive AV lanes, which reduce interactions between AVs and HVs, as a further potentially important instrument for improving road safety in mixed traffic scenarios. Third, with respect to the supply side one could further explore the role of (imperfect) competition between different AV manufacturers. Finally, another interesting extension is to study a dynamic model of AV market penetration, where all agents make decisions as time progresses. This could provide further insights into how policymakers should regulate the market and decide on infrastructure investment, and how AV manufacturers should invest in AV safety, as the AV market evolves.

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Appendix

A Proofs

Proof of Proposition 1:

Using the definition of $z_1(x)$ and $z_2(x)$ as well as (8) and taking into account that x is fixed, the optimization problem of the firm can be rewritten as

$$\max_{p_A} \left[\left(\frac{z_2(x)}{z_1(x)} - \frac{1}{z_1(x)} p_A \right) \left(p_A - \beta D k(x) + \beta h D \left(\frac{z_2(x)}{z_1(x)} - \frac{1}{z_1(x)} p_A \right) \right) \right]$$

From the first order condition we obtain after collecting terms and multiplication with $z_1(x)^2$, which is strictly positive due to Assumption 2(ii), we obtain

$$-p_A 2(z_1(x) - \beta h D) + \beta D(z_1(x)k(x) - z_2(x)h) + z_2(x)(z_1(x) - \beta h D) = 0$$

Solving for p_A yields expression (9). Inserting $p_A^m(x)$ into (8) yields

$$\begin{aligned} Q^m(x) &= \frac{z_2(x)}{z_1(x)} - \frac{1}{z_1(x)} \frac{z_1(x)z_2(x) + \beta D(z_1(x)k(x) - 2z_2(x)h)}{2(z_1(x) - \beta h D)} \\ &= \frac{1}{2z_1(x)(z_1(x) - \beta h D)} \left[2z_2(x)(z_1(x) - \beta h D) - (z_1(x)z_2(x) + \beta D(z_1(x)k(x) - 2z_2(x)h)) \right] \\ &= \frac{z_1(x)(z_2(x) - \beta k(x) D)}{2z_1(x)(z_1(x) - \beta h D)} \\ &= \frac{z_2(x) - \beta k(x) D}{2(z_1(x) - \beta h D)}. \end{aligned}$$

In order to show that optimal price and quantity are indeed determined by the first order condition, we still have to verify that $Q^m(x) < 1$. This inequality is equivalent to

$$\begin{aligned} & 2(z_1(x) - \beta h D) > z_2(x) - \beta D k(x) \\ \Leftrightarrow \quad & 0 < \frac{3t}{2} + \bar{g}\gamma(D) - 2g(x)\gamma(D) + (k(x) - 2h)(\gamma((1-\beta)D) + \beta D) - p_H \\ \Leftrightarrow \quad & 0 < \underbrace{t - 2g(x)\gamma(D)}_{>0} + \underbrace{\frac{t}{2} - p_H - h(\gamma((1-\beta)D) + \beta D)}_{>0} + \bar{g}\gamma(D) \\ & \quad + (k(x) - h)(\gamma((1-\beta)D) + \beta D), \end{aligned}$$

which holds, since all terms in the sum are positive. For the first two terms this is due to Assumption 2(ii).

The expression for (11) follows directly form inserting $p_A^m(x) = z_2(x) - z_1(x)Q^m(x)$ into $\tilde{\Pi}$ and simplifying terms. ■

Proof of Corollary 1:

Assumption 2(ii) guarantees that $z_1(x) - \beta h D = t - g(x)\gamma(D) - h\gamma((1-\beta)D) - \beta h D + \bar{g}\gamma(D) > 0$. This follows from $g(x)\gamma(D) < \frac{t}{2}$, as shown above, in combination with $h\gamma((1-\beta)D) + \beta h D < hD < g(0)D < \frac{t}{2}$.

Taking into account that $\gamma((1-\beta)D) + \beta D > \gamma(D)$, Assumption 2(i) implies directly that for $x = 0$, the numerator in (10) is negative and therefore it is optimal for the firm not to sell any AVs. Furthermore, the numerator in (10) is strictly increasing in x. Since $\lim_{x\to\infty} k(x) < g(0)$ (see Assumption 1(iii)), it follows from Assumption 2(ii) that the numerator is positive for sufficiently large x. Together, these observations yield the claim of the corollary. ■

Proof of Proposition 2:

Expression (12) can be rewritten as

$$\frac{dQ^m(x)}{dx} = \frac{-g'(x)(\gamma((1-\beta)D)+\beta D)}{2(z_1(x)-\beta DC)}\left[\frac{k'(x)}{g'(x)} - \frac{2Q^m(x)\gamma(D)}{\gamma((1-\beta)D)+\beta D}\right].$$

Taking into account that $z_1(x) - \beta DC > 0$ and $g'(x) < 0$, this implies that $Q^m(x)$ is increasing in x if and only if

$$\frac{k'(x)}{g'(x)} > \frac{2Q^m(x)\gamma(D)}{\gamma((1-\beta)D)+\beta D}. \tag{15}$$

We define $w(x) = \frac{2Q^m(x)\gamma(D)}{\gamma((1-\beta)D)+\beta D}$ and hence for any x with $Q^m(x) \in (0,1)$ we have $(Q^m)'(x) < 0$ if and only if $\frac{k'(x)}{g'(x)} < w(x)$. It follows from Assumption 1 that $\frac{k'(0)}{g'(0)} > 1$ and due to $Q^m(0) = 0$ we have $lim_{x\to 0} w(x) = 0$. Hence $\frac{k'(x)}{g'(x)} > w(x)$ for sufficiently small x (i.e. $Q^m(x)$ is increasing in x). Furthermore, according to Assumption 1, $\frac{k'(x)}{g'(x)}$ strictly decreases with x, whereas $w(x)$ strictly increases with x for any x with $\frac{k'(x)}{g'(x)} > w(x)$. The last observation follows since $w(x)$ increases for increasing $Q^m(x)$ and $(Q^m)'(x) > 0$ for all values of x with $\frac{k'(x)}{g'(x)} > w(x)$.

We now show that there has to exist a value $\tilde{x}$ with $\frac{k'(\tilde{x})}{g'(\tilde{x})} = w(\tilde{x})$ and, as a next step, we then show that $(Q^m)'(x) < 0$ for almost all $x > \tilde{x}$.

Assume that no value $\tilde{x}$ with $\frac{k'(\tilde{x})}{g'(\tilde{x})} = w(\tilde{x})$ exists. Then we must have $\frac{k'(x)}{g'(x)} > w(x)$ and therefore $w'(x) > 0$ for all $x \geq 0$ with $Q^m(x) > 0$. From Corollary 1 it follows that $Q^m(x) > 0$ for all $x > x^l$ and therefore there exists some $\epsilon > 0$ with $h(x) > \epsilon$ for all $x > x^l$. Under our assumption $\frac{k'(x)}{g'(x)} > w(x)$ for all $x > 0$ this implies $\frac{k'(x)}{g'(x)} > \epsilon$ for all $x < 0$. This contradicts Assumption 1, which requires that $\lim_{x\to\infty} \frac{k'(x)}{g'(x)} = 0$. It follows that there must exist a value $\tilde{x} > x^l$ with $\frac{k'(\tilde{x})}{g'(\tilde{x})} = w(\tilde{x})$.

As a third step we show that $w(x)$ can never cross $\frac{k'(x)}{g'(x)}$ from above at any value $x > \tilde{x}$. At $\tilde{x}$ we have $(Q^m)'(x) = 0$, which implies that $w'(x) = 0$. Taking into account that $\frac{k'(x)}{g'(x)}$ is a strictly decreasing function of x this implies that $\frac{d}{dx}\left(\frac{k'(\tilde{x})}{g'(\tilde{x})} - w(\tilde{x})\right) < 0$ and therefore $\frac{k'(x)}{g'(x)} < w(x)$ for $x \in [\tilde{x}, \tilde{\tilde{x}}]$, where $\tilde{\tilde{x}}$ is either the smallest intersection point between $\frac{k'(x)}{g'(x)}$ and $w(x)$ above $\tilde{x}$, or, if no such second intersection point exists, $\tilde{\tilde{x}} = \infty$. If a finite point $\tilde{\tilde{x}} > \tilde{x}$ with $\frac{k'(\tilde{\tilde{x}})}{g'(\tilde{\tilde{x}})} = w(\tilde{\tilde{x}})$ exists, then the same arguments as applied to $\tilde{x}$ show that $\frac{k'(x)}{g'(x)} < w(x)$ holds also for all x between $\tilde{\tilde{x}}$ and the next intersection point. Overall, this shows that $\frac{k'(x)}{g'(x)} < w(x)$ for almost all $x \geq \tilde{x}$. Hence $(Q^m)'(x) < 0$ for almost all $x \geq \tilde{x}$ and therefore $Q(x)$ is a (weakly) decreasing function of x for $x \geq \tilde{x}$. ■

Proof of Lemma 1:

It follows from Corollary 1 that $Q^m(x) > 0$ if and only if $x > x^l > 0$. Hence $\Pi(x) \leq 0$ for all $x \leq x^l$. This directly implies that $x^* > x^l$ has to hold. For $x > x^l$ the expression $\Pi(x)$ is continuous and continuously differentiable in x. Furthermore, as shown in Corollary 1 $z_1(x) - \beta Dh > 0$ for all $x > x_l$. Therefore, $\frac{1}{2}Q^m(x)^2(z_1(x) - \beta Dh) > 0$ for all $x > x_l$. This implies that for sufficiently small values of $\xi_0 > 0$, there exist values of $x > x_l$ such that $\Pi(x) > 0$. Consider any such value of ξ_0, then

$$\Pi'(x) = Q^m(x)Q^m(x)' \cdot (z_1(x) - \beta Dh) + \frac{1}{2}Q^m(x)^2 z_1(x)' - 2\xi_0 x.$$

Since $Q^m(x)' < 0$ for x is sufficiently large (see Proposition (2)), the first and the third term in this sum are strictly negative, where the third term goes to $-\infty$ for $x \to \infty$. The second term is positive, but it follows from Assumption 1 that $\lim_{x\to\infty} g'(x) = 0$. Hence $\Pi'(x) < 0$ for sufficiently large x, which implies that the value x^* maximizing $\Pi(x)$ is in the interior of the interval (x^l, ∞). Taking into account that $\Pi(x)$ is continuously differentiable on this entire interval, it follows that the optimal value of x has to satisfy the first order condition (14). ■

Proof of Proposition 3:

Using the first order condition (14) we obtain by implicit differentiation with respect to ξ_0 that

$$\frac{\partial x^*}{\partial \xi_0} = -\frac{-2x^*}{\Pi''(x^*)}.$$

Since x^* is a (local) maximum of $\Pi(x)$, we must have $\Pi''(x^*) < 0$ and therefore $\frac{\partial x^*}{\partial \xi_0} < 0$.

To show the second claim of the proposition we prove that $Q^m(x^*)' < 0$ for $\xi_0 = 0$ for any value of β and c. By continuity this property also holds for positive values of ξ_0 close to 0. For ξ_0 sufficiently small we have $x^* > x^l$ and therefore $Q^m(x^*) > 0$. Taking this into account and setting $\xi_0 = 0$ we obtain from the first order condition (14) that x^* has to satisfy

$$Q^m(x^*)' \cdot (z_1(x^*) - \beta Dh) + \frac{1}{2}Q^m(x^*) z_1(x^*)' = 0.$$

Hence,

$$Q^m(x^*)' = -\frac{Q^m(x^*) z_1(x^*)'}{2(z_1(x^*) - \beta Dh)}.$$

Since $z_1(x^*) - \beta Dh > 0$ (see proof of Corollary 1) and $z_1'(x^*) = -g'(x^*)\gamma(D) > 0$ we directly obtain that $Q^m(x^*)' < 0$.

It follows that for any value of $\beta \in [0, 1]$ and $h(c), c \geq 0$ there exists a threshold $\bar{\xi}_{\beta,h(c)} > 0$ with the property that $Q^m(x^*)' < 0$ for all $\xi_0 < \bar{\xi}_{\beta,h(c)}$. Since we have assumed that $k(x) > h(c)\ \forall x, c$, we have $h(c) \leq \bar{h} = \lim_{x\to\infty} k(x)$. Due to the compactness of $[0, 1] \times [0, \bar{h}]$, there exists a value $\bar{\xi} = \min[\bar{\xi}_{\beta,h} | (\beta, h) \in [0, 1] \times [0, \bar{h}]] > 0$ and $Q^m(x^*)' < 0$ for all $\beta \in [0, 1]$ and $c \geq 0$. ■